ROBUST BEAMFORMING UNDER UNCERTAINTIES IN THE LOUDSPEAKERS DIRECTIVITY PATTERN

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ABSTRACT

In this paper we propose a robust beamforming technique which takes into account uncertainties and variations in the radiation pattern of the loudspeakers. The proposed technique is based on the solution of a robust least-square problem in which the propagation matrix is to some extent unknown. Both simulations and experimental results prove the validity of the proposed methodology in terms of directivity index and white noise gain.

Index Terms— Loudspeaker array, robust beamforming, robust least-squares, radiation pattern, regularization.

1. INTRODUCTION

This paper concerns the problem of synthesizing directive sound fields by means of beamforming through loudspeaker arrays. In the last decades, controllable directivity patterns were adopted for sound reinforcement applications [1]. Recently, thanks to the availability of fast beam tracing techniques [2, 3], controlled directivity patterns have been used for immersive sound reinforcement in reverberant environments [4] and for sound field rendering [5, 6].

Many beamforming design techniques have been proposed for the purpose of spatially selective sound capture and, thanks to the reciprocity between microphones and loudspeakers [7], these techniques can be readily applied to the sound playback scenario [8, 9]. In these techniques, each loudspeaker is fed with a filtered version of the wideband signal to be reproduced; the derivation of the filters is what characterizes a specific technique.

Broadband and frequency-invariant beamformers, i.e. beamformers with a beamwidth constant among frequencies, have been first introduced in [10, 11] based on the spatial Fourier transform of a continuous aperture. Analytic approaches have been presented for specific discrete array deployments, e.g. linear [12], cylindrical [13], spherical [14, 15]. To overcome the restrictions on transducer locations imposed by analytical approaches, in [16] Parra introduces a numerical method based on the solution of a least-squares problem, originally conceived for microphone arrays but easily applicable to loudspeaker arrays. This is a two-step procedure, which first solves numerically a least-squares problem that decouples the frequency-behavior from the spatial-behavior through a change of basis, and then solves a second least-squares problem for the design of the beamforming filter. It has been noted, however, in [17], that such technique suffers from relevant errors when the knowledge of the propagation matrix $\mathbf{G}$ is not exact. In particular, the matrix $\mathbf{G}$, which comes into the picture in both steps of the procedure, is influenced by uncertainties in the speaker positions and/ or in their radiance pattern.

Several approaches have been proposed to address the sensitivity problem. In [17] Mabande *et al.* present a method which incorporates a constraint for the White Noise Gain (WNG) into a least-squares beamformer design and still leads to a convex optimization problem that can be solved directly. An extension to time-domain design has been presented in [18]. In [19] Lai states, however, that it is difficult to select an appropriate level of WNG for any given set of errors in speaker positions and/ or directivity. In [20] Trucco *et al.* present a beamforming technique that is inherently robust against errors in the propagation matrix. In [21] Lai *et al.* adopt a Farrow structure for the beamforming and they incorporate the probability density functions for the microphone error into the design formulation. In [22], in the context of mitigating the effect of interferers, Rübsamen and Gershman extend the 1D covariance matrix fitting approach to multiple dimensions, and the steering vectors are modeled by means of uncertainty sets. In [23] Doclo and Moonen propose two design procedures aimed at increasing the robustness. The first embeds the probability density functions of the steering vectors, whereas the second optimizes the worst-case performance through the minimax criterion. In [24] Levin *et al.* propose a technique that modifies the classical loading scheme incorporating a non-diagonal elements to attenuate the effect of sensor uncertainties. The method in [24], however, is data-dependent, i.e. the filters depend on the signal to be rendered, thus making its use for rendering purposes quite cumbersome.

In this paper we propose an extension of the data-independent beamformer in [16]. More specifically, in both problems of change of basis and beamforming, the uncertainty in the propagation matrix is modeled as in the Robust Least-Squares problem in [25]. A non-diagonal loading of the original least-squares solution is obtained as solution. Results show that the accuracy of the beamforming improves with respect to the Tikhonov regularization in terms of directivity index and white noise gain.

The rest of the manuscript is organized as follows: Section 2 states the problem and provides a short overview over the beamforming solution in [16]. Section 3 presents the proposed methodology. Section 4 shows some simulations and experimental results. Section 5 states the relation of the work in this paper with state-of-the-art techniques. Finally, section 6 draws some conclusive remarks.

2. BACKGROUND AND PROBLEM STATEMENT

Consider an array of N loudspeakers, placed at $\mathbf{x}_n = [x_n, y_n, z_n]^T$, $n = 1, \ldots, N$. Denote with $\mathbf{k}$ the wave vector pointing toward direction $\Omega = (\theta, \phi)$, where $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi)$ represent the elevation angle and the azimuth angle, respectively. The magnitude of the wave vector is $k = |\mathbf{k}| = \omega/c$, ω being the radial frequency and c being the speed of sound. Let us consider a grid of propagation directions denoted by Ω_q, $q = 1, \ldots, Q$. The array response on this

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grid is [26]

$$\mathbf{f}(k) = \mathbf{G}(k)\mathbf{h}(k), \tag{1}$$

where $[\mathbf{G}(k)]_{q,n} = g_n(k,\Omega_q)$ is the propagation function from the nth loudspeaker toward direction Ω_q, $\mathbf{h}(k)$ is a vector of N complex filter coefficients and $[\mathbf{f}(k)]_q = f(k,\Omega_q)$ is the array response in direction Ω_q. Notice that the propagation function contains the contributions related to the attenuation and to the directional loudspeaker behavior. It can therefore be factorized into [7]

$$g_n(k,\Omega_q) = g_{FF,n}(k,\Omega_q)\cdot d_n(k,\Omega_q), \tag{2}$$

where $g_{FF,n}(k,\Omega_q) = e^{-j(\mathbf{k}^T\mathbf{x}_n)}$ is the free-field propagation function and $d_n(k,\Omega_q)$ is the (complex) directivity of the loudspeaker.

In order to produce a frequency-invariant beamformer, in [16] Parra proposes a basis transformation, which leads from $\mathbf{G}(k)$ to a new basis $\widetilde{\mathbf{G}}$ through a transformation matrix $\mathbf{B}(k)$, which minimizes the approximation error in the least-squares sense

$$\underset{\mathbf{B}(k)}{\text{minimize}}\ \|\mathbf{G}(k)\mathbf{B}(k) - \widetilde{\mathbf{G}}\|_2^2. \tag{3}$$

In [16], the new basis $\widetilde{\mathbf{G}}$ is chosen to be frequency-invariant and easily steerable. For this reason, the new basis consists in the spherical harmonics and the approximation is truncated to order L and degree M. We can thus write the propagation matrix in direction Ω_q in the new basis as

$$[\widetilde{\mathbf{G}}]_q = \left[Y_0^0(\Omega_q),\ Y_1^{-1}(\Omega_q),\ \ldots,\ Y_L^M(\Omega_q)\right]. \tag{4}$$

In [16] design of the beamformer is accomplished in the transformed domain. In particular, the spatial filter in the transformed domain can be computed to approximate an objective array response $\mathbf{f}$ in the least-squares sense [26], i.e.

$$\underset{\widetilde{\mathbf{h}}(k)}{\text{minimize}}\ \|\mathbf{G}(k)\mathbf{B}(k)\widetilde{\mathbf{h}}(k) - \mathbf{f}\|_2^2 \tag{5}$$

and the filter coefficients in the original domain are obtained as [16]

$$\mathbf{h}(k) = \mathbf{B}(k)\widetilde{\mathbf{h}}(k). \tag{6}$$

The previous derivation of the spatial filters does not take into account uncertainty or variations in speaker locations and their directivity pattern [27]. In this manuscript we consider the geometrical uncertainty to be negligible with respect to the directivity pattern. We model the effect of the uncertainty on the pattern as a random variable $\delta(k,\Omega)$ of variance σ_D^2, which alters the nominal directivity pattern $\overline{d}(k,\Omega)$ usually specified by the loudspeaker manufacturer. We can thus write that

$$d_n(k,\Omega) = \overline{d}(k,\Omega) + \delta(k,\Omega), \tag{7}$$

where $\delta(k,\Omega) \sim \mathcal{N}(0,\sigma_D^2(k,\Omega))$. Notice that the same approach could also be used for the modeling of the uncertainty of the speaker locations. This aspect, however, goes beyond the scope of the paper. By substituting (7) into (2) we obtain an expression that relates the uncertainty on propagation function with the uncertainty on the directivity pattern

$$g_n(k,\Omega) = g_{FF,n}(k,\Omega)\overline{d}(k,\Omega) + g_{FF,n}(k,\Omega)\delta(k,\Omega), \tag{8}$$

or, in matrix notation,

$$\mathbf{G}(k) = \mathbf{G}_{FF}(k)\odot\overline{\mathbf{D}}(k) + \mathbf{G}_{FF}(k)\odot\mathbf{\Delta}(k) \tag{9}$$

where $[\mathbf{G}_{FF}(k)]_{q,n} = g_{FF,n}(k,\Omega_q)$, $[\overline{\mathbf{D}}(k)]_{q,n} = \overline{d}(k,\Omega_q)$, $[\mathbf{\Delta}(k)]_{q,n} = \delta(k,\Omega_q)$ and $\odot$ denotes Hadamard product. Finally, we can express $\mathbf{G}(k)$ as the random variable

$$\mathbf{G}(k) = \overline{\mathbf{G}}(k) + \mathbf{U}(k), \tag{10}$$

where $\overline{\mathbf{G}}(k) = \mathbf{G}_{FF}(k)\odot\overline{\mathbf{D}}(k)$ is the mean value of $\mathbf{G}(k)$ and $\mathbf{U}(k) = \mathbf{G}_{FF}(k)\odot\mathbf{\Delta}(k)$ describes its statistical variation.

In the next section we describe how a knowledge of the statistical model of $\mathbf{G}(k)$ can be exploited to derive a closed-form solution for the problem of designing a robust beamformer.

3. ROBUST LEAST-SQUARES BEAMFORMING

In [16] the propagation matrix appears in both steps of basis transformation and synthesis of the filter.

We first consider the basis transform problem in (3). We are interested in minimizing the mean case approximation error, i.e.

$$\underset{\mathbf{B}(k)}{\text{minimize}}\ \mathbb{E}[\|\mathbf{G}(k)\mathbf{B}(k) - \widetilde{\mathbf{G}}\|_2^2], \tag{11}$$

where $\mathbb{E}[\,\cdot\,]$ is the expectation operator. Recalling the factorization of the propagation matrix in (10) we can express the objective function as [25]

$$\begin{aligned}\mathbb{E}[\|\mathbf{G}\mathbf{B} - \widetilde{\mathbf{G}}\|_2^2] &= \mathbb{E}[(\overline{\mathbf{G}}\mathbf{B} - \widetilde{\mathbf{G}} + \mathbf{U}\mathbf{B})^T(\overline{\mathbf{G}}\mathbf{B} - \widetilde{\mathbf{G}} + \mathbf{U}\mathbf{B})]\\ &= (\overline{\mathbf{G}}\mathbf{B} - \widetilde{\mathbf{G}})^T(\overline{\mathbf{G}}\mathbf{B} - \widetilde{\mathbf{G}}) + \mathbb{E}[\mathbf{B}^T\mathbf{U}^T\mathbf{U}\mathbf{B}]\\ &= \|\overline{\mathbf{G}}\mathbf{B} - \widetilde{\mathbf{G}}\|_2^2 + \mathbf{B}^T\mathbf{P}\mathbf{B},\end{aligned}$$

where we have dropped the dependency on k for compactness of the notation, and $\mathbf{P} = \mathbb{E}[\mathbf{U}^T\mathbf{U}]$. Hence, the statistical robust least-squares problem can be written as a regularized least-squares problem

$$\underset{\mathbf{B}}{\text{minimize}}\ \|\overline{\mathbf{G}}\mathbf{B} - \widetilde{\mathbf{G}}\|_2^2 + \|\mathbf{P}^{1/2}\mathbf{B}\|_2^2, \tag{12}$$

whose solution is [25]

$$\mathbf{B} = (\overline{\mathbf{G}}^T\overline{\mathbf{G}} + \mathbf{P})^{-1}\overline{\mathbf{G}}^T\widetilde{\mathbf{G}}. \tag{13}$$

Let us now consider the problem in (5) for the computation of the beamforming filters. If we apply the same minimization criterion used in (11), we obtain

$$\underset{\widetilde{\mathbf{h}}}{\text{minimize}}\ \mathbb{E}[\|\mathbf{G}\mathbf{B}\widetilde{\mathbf{h}} - \mathbf{f}\|_2^2], \tag{14}$$

where the uncertainty lies in the product $\mathbf{GB}$. Recalling (9) and (10) we can write

$$\mathbf{G}\mathbf{B} = \overline{\mathbf{G}}\mathbf{B} + \mathbf{U}\mathbf{B} = \overline{\mathbf{G}\mathbf{B}} + \mathbf{\Upsilon}, \tag{15}$$

where $\overline{\mathbf{GB}} = \overline{\mathbf{G}}\mathbf{B}$ and $\mathbf{\Upsilon} = \mathbf{UB}$. Thus, the objective function in (14) can be written as

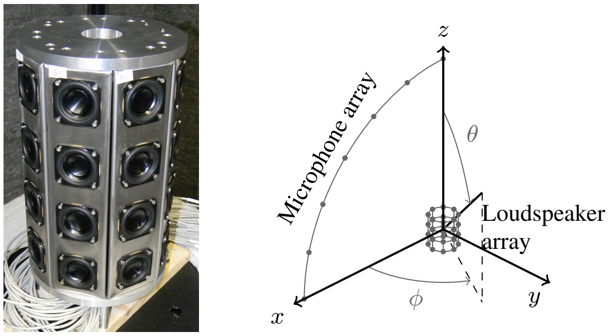
$$\begin{aligned}\mathbb{E}[\|\mathbf{G}\mathbf{B}\widetilde{\mathbf{h}} - \mathbf{f}\|_2^2] &= \mathbb{E}[(\overline{\mathbf{G}\mathbf{B}}\widetilde{\mathbf{h}} - \mathbf{f} + \mathbf{\Upsilon}\widetilde{\mathbf{h}})^T(\overline{\mathbf{G}\mathbf{B}}\widetilde{\mathbf{h}} - \mathbf{f} + \mathbf{\Upsilon}\widetilde{\mathbf{h}})]\\ &= (\overline{\mathbf{G}\mathbf{B}}\widetilde{\mathbf{h}} - \mathbf{f})^T(\overline{\mathbf{G}\mathbf{B}}\widetilde{\mathbf{h}} - \mathbf{f}) + \mathbb{E}[\widetilde{\mathbf{h}}^T\mathbf{\Upsilon}^T\mathbf{\Upsilon}\widetilde{\mathbf{h}}]\\ &= \|\overline{\mathbf{G}\mathbf{B}}\widetilde{\mathbf{h}} - \mathbf{f}\|_2^2 + \widetilde{\mathbf{h}}^T\mathbf{\Pi}\widetilde{\mathbf{h}},\end{aligned}$$

where

$$\mathbf{\Pi} = \mathbb{E}[\mathbf{\Upsilon}^T\mathbf{\Upsilon}] = \mathbf{B}^T\mathbf{P}\mathbf{B}. \tag{16}$$

Thus, the problem (14) can be reformulated as the statistical robust least-squares problem

$$\underset{\widetilde{\mathbf{h}}}{\text{minimize}}\ \|\overline{\mathbf{G}\mathbf{B}}\widetilde{\mathbf{h}} - \mathbf{f}\|_2^2 + \|\mathbf{\Pi}^{1/2}\widetilde{\mathbf{h}}\|_2^2, \tag{17}$$



(a) *Loudspeaker array.* (b) *Measurement setup.*

Fig. 1: Custom cylindrical loudspeaker array (Fig. 1a) and measurement setup (Fig. 1b).

whose solution is [25]

$$\widetilde{\mathbf{h}} = (\overline{\mathbf{GB}}^T\overline{\mathbf{GB}} + \mathbf{\Pi})^{-1}\overline{\mathbf{GB}}^T\mathbf{f}. \tag{18}$$

Finally, the filter coefficients $\mathbf{h}$ in the original domain are obtained by the change of basis in (6).

It is important to observe that both (13) and (18) correspond to a loading of the least-squares solution through matrices $\mathbf{P}$ and $\mathbf{\Pi}$, respectively. With respect to the Tikhonov regularization, however, the loading is not diagonal, as $\mathbf{P}$ and $\mathbf{\Pi}$ may have non-diagonal elements different from zero. A similar result was attained in [24] in the context of data-dependent beamformers. We also notice from (16) that $\mathbf{P}$ and $\mathbf{\Pi}$ are related through $\mathbf{B}$. Therefore, with respect to [16], the only additional knowledge required for a complete characterization of the beamforming filters is the matrix $\mathbf{P}$.

4. RESULTS

In this section we present simulations and experimental results to validate the beamforming design methodology presented in sec. 3.

4.1. Experimental setup

The measurements are conducted in a room with short reverberation time, being $T_{60} \approx 50$ ms in the frequency band 200 Hz ÷ 5 kHz. We adopt a custom cylindrical loudspeaker array, designed by B&C Speakers, mounting $N = 32$ 2-inch full-range drivers. The radius of the cylinder is 9.2 cm, while the distance between drivers along the z axis is 7.5 cm. This results in a maximum radiation mode $L = 3$, having Nyquist frequency $f_{\mathrm{Nyq},3} \approx 2.2$ kHz [28]. Fig. 1a shows the loudspeaker array.

The measurement setup, along with the reference frame, is shown in Fig. 1b. Notice that the origin of the reference frame coincides with the center of the loudspeaker array. In order to estimate the response of the array of speakers, $M = 7$ measurement microphones are deployed at angles $\Omega = (\theta_t, 0), t = 1, \dots, M$ and distance fixed and equal to 1.3 m, thus forming an arc of sensors on the xz plane. The loudspeaker array is mounted on a stepper turntable, which allows to rotate the array in the xy plane towards directions $\phi_p,\ p = 1, \dots, Q_\phi$. The microphones have been calibrated in such a way that their response to an omnidirectional source located in the center of the reference frame is equal in amplitude for all the $M = 7$ microphones. For each rotation angle of the turntable and for each microphone, the response of the array in the direction (θ_q, ϕ_p) is evaluated. As a consequence, the response is evaluated on a grid of $\Omega_q = (\phi_p, \theta_t)$ propagation directions, indexed by $q = Q_\phi(t-1) + p$, with resolution $180°/(Q_\theta - 1) = 15°$ for the elevation angle and $360°/Q_\phi = 5°$ for the azimuth angle.

In all the simulations and experiments shown in this section, the frequency-independent desired response of the loudspeaker array $\mathbf{f}(k)$ is a unit impulse in direction $\Omega = (90°, 0°)$, i.e.

$$\begin{cases} f(k,\Omega_q) = f(\Omega_q) = 1, & \text{if } q = Q_\phi\left(\frac{Q_\theta+1}{2} - 1\right) + 1, \\ f(k,\Omega_q) = f(\Omega_q) = 0, & \text{otherwise.} \end{cases}$$

We choose to truncate the spherical harmonics expansion to $L = 12$ in order to ensure a more numerically accurate basis transformation; we remark that this choice is not intended to increase the accuracy of the reproduced soundfield since radiation modes higher than $L = 3$ are not reproducible by our setup.

The time-domain filters $h_n(t), n = 1, \dots, N$ are derived from (6) through length-K Inverse Fourier Transform, being $K = 512$. The excitation signals are Golay complementary sequences $a(t)$ and $b(t)$ of length $L_G = 4096$ [29]. Let $r_a(t,\Omega_q) = a(t) * f(t,\Omega_q)$ and $r_b(t,\Omega_q) = b(t) * f(t,\Omega_q)$ be the array responses in direction Ω_q due to input $a(t)$ and $b(t)$, respectively. Through the definition of Golay complementary sequences [29], the array response is obtained by

$$f(t,\Omega_q) = (1/(2L_G))\left(a(t)r_a(t,\Omega_q) + b(t)r_b(t,\Omega_q)\right),$$

which is transformed through length-K Fourier transform to obtain the array response $f(k,\Omega_q)$ in the frequency domain. For each rotation p of the turntable the set $f(k,\Omega_q), q = Q_\phi(t-1) + p, t = 1, \dots, M$ of array responses is acquired, such that after a complete rotation of the turntable the array response is sampled on the whole northern hemisphere. Due to the symmetry of the measurement setup and of the loudspeaker array, the response on the southern hemisphere is obtained by symmetrizing the response in the northern hemisphere, i.e.

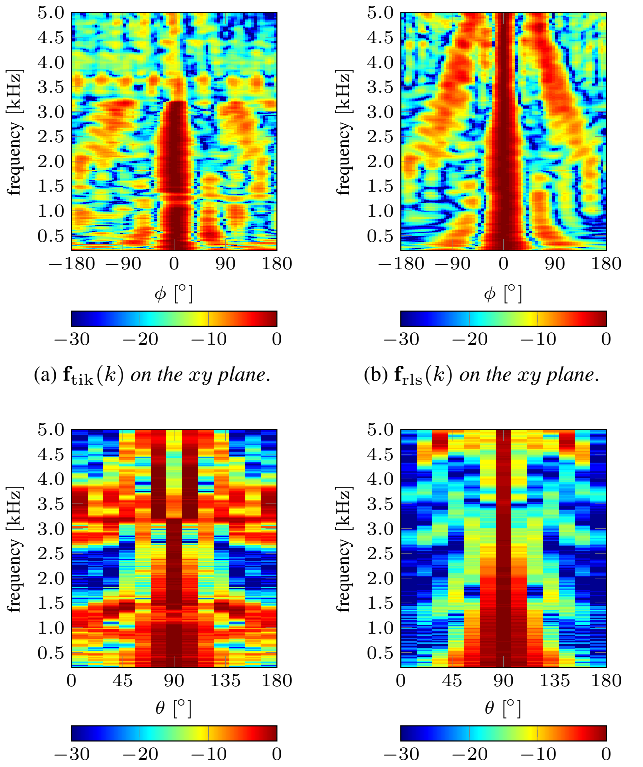
$$f(k,\Omega_q)\big|_{\substack{q=Q_\phi(t-1)+p,\\ t=M+1,\dots,Q_\theta}} = f(k,\Omega_q)\big|_{\substack{q=Q_\phi(t-1)+p,\\ t=M-1,\dots,1}}.$$

In the following paragraph, we compare the array response $\hat{\mathbf{f}}_{\mathrm{tik}}(k)$ produced by filters computed from (5) adopting Tikhonov regularization, as suggested in [16], with the array response $\hat{\mathbf{f}}_{\mathrm{rls}}(k)$ produced by filters computed from (18). The coefficient for the Tikhonov regularization has been set to -20 dB. For the proposed robust design, we set the variance of the loudspeaker directivity pattern to $\sigma_D^2(k,\Omega) = -20$ dB. The nominal directivity pattern $\overline{d}(k,\Omega)$, and the variance $\sigma_D^2(k,\Omega)$ used in both design methodologies, have been experimentally determined by measuring in a preliminary stage individual drivers mounted in the array structure.

4.2. Experimental results

Figs. 2a and 2b show the array responses on the xy plane, i.e. $\hat{f}_{\mathrm{tik}}(k,(90°,\phi_p))$ and $\hat{f}_{\mathrm{rls}}(k,(90°,\phi_p))$, $p = 1, \dots, Q_\phi$. Notice that with the proposed design methodology the beam is reasonably rendered in the whole frequency band of interest, despite the presence of aliasing starting from the Nyquist frequency, which for the considered loudspeaker configuration is $f_{\mathrm{nyq}} \approx 1.7$ kHz. On the other hand, with Tikhonov regularization, the beam is not rendered for frequencies above 3 kHz.

Figs. 2c and 2d show the array responses on the xz plane, i.e. $\hat{f}_{\mathrm{tik}}(k,(\theta_t,0°))$ and $\hat{f}_{\mathrm{rls}}(k,(\theta_t,0°))$, $t = 1, \dots, Q_\theta$. As already



(a) $\mathbf{f}_{\mathrm{tik}}(k)$ *on the* xy *plane.* (b) $\mathbf{f}_{\mathrm{rls}}(k)$ *on the* xy *plane.*

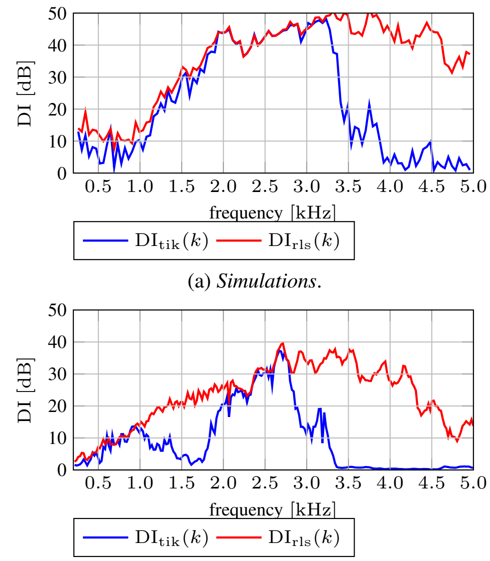
(c) $\mathbf{f}_{\mathrm{tik}}(k)$ *on the* xz *plane.* (d) $\mathbf{f}_{\mathrm{rls}}(k)$ *on the* xz *plane.*

Fig. 2: Measured array response on the xy plane (Figs. 2a and 2b) and on the xz plane (Figs. 2c and 2d). The amplitude response is normalized on a frequency basis and represented in a dB scale.

observed for the xy plane, also in this case $\hat{\mathbf{f}}_{\mathrm{rls}}(k)$ clearly exhibits a reasonably narrow beam, though widened at low frequencies due to the small number of loudspeakers available to control the sound beam along the direction of the elevation. On the other hand, $\hat{\mathbf{f}}_{\mathrm{tik}}(k)$ only matches the desired response in a small frequency range between 1.5 kHz and 3 kHz.

In order to quantitatively assess the performance of the proposed design methodology, Fig. 3 shows the directivity index $\mathrm{DI}(k)$ of the loudspeaker array as a function of frequency. $\mathrm{DI}(k)$ is defined as the ratio between the power radiated in the solid angle towards which the beam is steered and the average power radiated on the sphere, as in [26]. Fig. 3a shows the directivity index computed in a simulative setup that reproduces the real one, while Fig. 3b plots the directivity index for real data. Both results clearly show that the energy of $\hat{\mathbf{f}}_{\mathrm{tik}}(k)$ is similar to an omnidirectional pattern frequencies above 3.5 kHz, while a significant portion of the energy of $\hat{\mathbf{f}}_{\mathrm{rls}}(k)$ still remains concentrated in the desired region.

The robustness of the proposed design methodology is highlighted through the analysis of the White Noise Gain (WNG), defined as in [26] by $\mathrm{WNG}(k) = \left(\sum_{n=1}^{N} |h_n(k)|^2\right)^{-1}$. Fig. 4 compares the WNG resulting from the filters computed with Tikhonov regularization with the WNG resulting from filters computed from (18). Notice that the proposed methodology is able to guarantee an improved robustness in the whole frequency band of interest.



(a) *Simulations.*

(b) *Experiments.*

Fig. 3: Simulated (Fig. 3a) and measured (Fig. 3b) directivity, obtained with Tikhonov regularization ($\mathrm{DI}_{\mathrm{tik}}(k)$, blue curves) and with the proposed design methodology ($\mathrm{DI}_{\mathrm{rls}}(k)$, red curves).

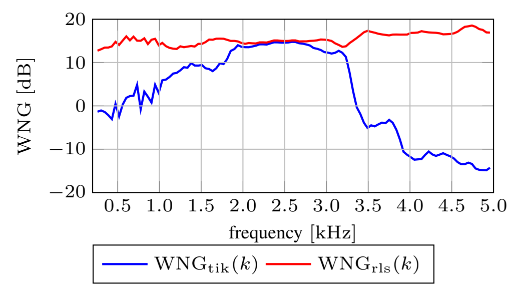


Fig. 4: White Noise Gain resulting from filters computed with Tikhonov regularization ($\mathrm{WNG}_{\mathrm{tik}}(k)$, blue curve) and from filters computed with the proposed methodology ($\mathrm{WNG}_{\mathrm{rls}}(k)$, red curve).

5. RELATION TO PRIOR WORK

The design methodology developed in this paper should be considered as an improved version of the beamforming algorithm originally presented in [16], where robustness against variations and errors of the speaker directivity is gained. Even if developed following a different strategy, the method in [24] attains similar conclusions for what concerns the loading of the least-squares solution. We remark, however, that the beamforming in [24] is data-dependent, whereas the technique presented in this paper is data-independent.

6. CONCLUSIONS

In this work we have presented a data-independent methodology for the design of beamfoming through loudspeakers under uncertainties of the directional behaviour of the acoustic drivers. Experimental results and simulations show that the novel technique is more robust than Tikhonov regularisation. Even if explicitly designed for loudspeaker arrays, the same framework can be adopted in the context of beamforming with microphone arrays.

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