COMPRESSIVE SENSING OF ECG SIGNALS BASED ON MIXED PSEUDONORM OF THE FIRST- AND SECOND-ORDER DIFFERENCES

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ABSTRACT

An improved algorithm for the reconstruction of electrocardiogram signals in compressive sensing is proposed. The algorithm is based on the minimization of a mixed pseudonorm of first- and second-order differences of the signal. Locations of QRS segments are estimated using a technique based on signal derivatives and the Hilbert transform, and they are used to implement the mixed pseudonorm. Simulation results demonstrate that the proposed algorithm offers approximately 23.5%, 11.4%, 4.4%, and 2.1% improvement in signal-to-noise ratio for a compression ratio of 90%, 80%, 70%, and 60%, respectively, relative to several competitive state-of-the-art algorithms.

Index Terms— Compressive sensing, signal reconstruction, electrocardiogram, QRS, mixed lp pseudonorm

1. INTRODUCTION

Compressive sensing (CS) is a signal processing technique that can be used to represent a sparse signal in terms of a small number of measurements [1, 2, 3]. CS involves signal reconstruction, which can be carried out by using an algorithm that can be based on the ℓ_1-minimization [4], ℓ_p-minimization [5, 6, 7], ℓ_0-minimization [8, 9], greedy approximation [10, 11], iterative shrinkage [12, 13], or Bayesian approximation [14]. These algorithms would be suitable for the reconstruction of electrocardiogram (ECG) signals, if a basis which can be used for the sparse representation of ECG signals is available. Unfortunately, such basis is not known.

By using block structure to promote temporal correlation, the block-sparse Bayesian learning bound-optimization (BSBL-BO) algorithm [15] yields ECG signals with significantly improved quality relative to several algorithms discussed above [16]. The ℓ_p^d-regularized least-squares (ℓ_p^d-RLS) [17] and the ℓ_p^{2d}-regularized least-squares (ℓ_p^{2d}-RLS) [18] algorithms also promote temporal correlation, but they do so by encouraging sparsity on the first- and second-order differences, respectively, of the signal. Consequently, they offer improved signal-to-noise ratio (SNR) and reduced computational effort relative to the BSBL-BO algorithm.

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In this paper, an improved algorithm, called as the $\ell_p^{(1,2)d}$-regularized least-squares ($\ell_p^{(1,2)d}$-RLS) algorithm, for the reconstruction of ECG signals is proposed. The ℓ_p^d and ℓ_p^{2d} pseudonorms used in the ℓ_p^d-RLS and ℓ_p^{2d}-RLS algorithms, respectively, are combined to formulate a new regularization function called as the $\ell_p^{(1,2)d}$ pseudonorm. By minimizing the $\ell_p^{(1,2)d}$-pseudonorm regularized squared-error, the $\ell_p^{(1,2)d}$-RLS algorithm promotes sparsity on the second-order difference of QRS segments and on the first-order difference of the rest of the segments of an ECG signal. Locations of QRS segments used in the $\ell_p^{(1,2)d}$ pseudonorm can be estimated by using a technique based on signal derivatives and the Hilbert transform. The $\ell_p^{(1,2)d}$-RLS algorithm offers improved SNR relative to the ℓ_p^d-RLS, ℓ_p^{2d}-RLS, and BSBL-BO algorithms. The improvement in SNR is attained at the cost of increased computational effort relative to the ℓ_p^d-RLS and ℓ_p^{2d}-RLS algorithms. Proposed algorithm would facilitate the development of power efficient and cheap sensing devices designed based on sparse binary measurement matrices. In resource-constrained personal and home-care telemonitoring systems, such sensing devices would be more efficient than the devices designed by implementing conventional discrete-wavelet transform based embedded ECG compression algorithms [19].

2. BACKGROUND AND PREVIOUS WORK

In CS, a signal vector $\boldsymbol{x}$ of length N and its measurement vector $\boldsymbol{y}$ of length M are interrelated as

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{w}$$

where $\boldsymbol{\Phi}$ is a measurement matrix of size $M \times N$ typically with $M \ll N$ and $\boldsymbol{w}$ is a noise vector of length M.

A sparse signal $\boldsymbol{x}$ can be recovered from its measurement $\boldsymbol{y}$ by solving the ℓ_1-regularized least-squares (ℓ_1-RLS) prob-

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$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \tfrac{1}{2}\,||\boldsymbol{\Phi x}-\boldsymbol{y}||_2^2+\lambda\sum_{n=1}^{N}|x_n| \tag{1}$$

where λ is a regularization parameter, if a condition which requires signal $\boldsymbol{x}$ to be sufficiently sparse is satisfied [1, 2, 3].

The ℓ_p-regularized least-squares algorithms, which solve the optimization problem

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \tfrac{1}{2}\,||\boldsymbol{\Phi x}-\boldsymbol{y}||_2^2+\lambda\sum_{n=1}^{N}|x_n|^p \ , \tag{2}$$

have been shown to offer improved signal reconstruction performance relative to an ℓ_1-RLS algorithm [5, 6, 7].

For the reconstruction of ECG signals, the ℓ_p^d-RLS and ℓ_p^{2d}-RLS algorithms which are based on solving the optimization problem

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \tfrac{1}{2}\,||\boldsymbol{\Phi x}-\boldsymbol{y}||_2^2+\lambda f_r(\boldsymbol{x}) \tag{3}$$

with the regularization function

$$f_r(\boldsymbol{x})=\sum_{n=1}^{N-1}\left[(x_n-x_{n+1})^2+\epsilon^2\right]^{p/2} \tag{4a}$$

and

$$\begin{aligned} f_r(\boldsymbol{x}) \;=\; & \left[(x_1-x_2)^2+\epsilon^2\right]^{p/2}+\\ & \sum_{n=2}^{N-1}\left[(x_{n-1}-2x_n+x_{n+1})^2+\epsilon^2\right]^{p/2}+\\ & \left[(x_{N-1}-x_N)^2+\epsilon^2\right]^{p/2}, \end{aligned} \tag{4b}$$

respectively, for a small value of ϵ and $0<p\leq 1$, have been found to yield improved SNR and reduced computational effort relative to the BSBL-BO algorithm [17, 18].

3. $\ell_p^{(1,2)d}$-RLS ALGORITHM

3.1. Problem formulation and $\ell_p^{(1,2)d}$-RLS algorithm

Consider a discrete time ECG signal $\boldsymbol{x}$ of length N which has total G QRS segments. Let $n_1, n_2, \ldots, n_G$ be the indices of the peaks of QRS segments in $\boldsymbol{x}$, and let the indices of the gth QRS segment span from $n_{gl}=n_g-w_d$ to $n_{gr}=n_g+w_d$, where w_d is a positive integer.

We consider the ℓ_q^{2d}-pseudonorm of the gth QRS segment given by

$$f_g^{2d}=\mu\sum_{n=n_{gl}+1}^{n_{gr}-1}\left[(x_{n-1}-2x_n+x_{n+1})^2+\upsilon^2\right]^{q/2} \tag{5a}$$

for $g=1,2,\ldots,G$, and the ℓ_p^d-pseudonorm of the ECG segment between the gth and $g+1$th QRS segments given by

$$f_{g,g+1}^d=\lambda\sum_{n=n_{gr}}^{n_{(g+1)l}-1}\left[(x_n-x_{n+1})^2+\epsilon^2\right]^{p/2} \tag{5b}$$

where υ and ϵ are small positive smoothing parameters, μ and λ are positive regularization parameters, $0<q\leq 1$, $0<p\leq 1$, and

$$n_{gl}=\begin{cases}1 & \text{for } g=1 \text{ and } n_g-w_d<1\\ n_g-w_d & \text{otherwise}\end{cases}$$

$$n_{gr}=\begin{cases}N & \text{for } g=G \text{ and } n_g+w_d>N\\ n_g+w_d & \text{otherwise}\end{cases}.$$

Using (5), we formulate a new regularization function $f_{(1,2)d}(\boldsymbol{x})$ given by

$$f_{(1,2)d}(\boldsymbol{x})=b_0 f_0^d+\sum_{g=1}^{G}\left(f_g^{2d}+b_g f_{g,g+1}^d\right) \tag{6}$$

where

$$b_0=\begin{cases}0 & \text{if } n_{1l}\leq 1\\ 1 & \text{otherwise}\end{cases},$$

$$b_g=\begin{cases}0 & \text{if } g=G \text{ and } n_{gr}\geq N\\ 1 & \text{otherwise}\end{cases},$$

and

$$f_0^d=\lambda\sum_{n=1}^{n_{1l}-1}\left[(x_n-x_{n+1})^2+\epsilon^2\right]^{p/2} \tag{7a}$$

$$f_{G,G+1}^d=\lambda\sum_{n=n_{Gr}}^{N-1}\left[(x_n-x_{n+1})^2+\epsilon^2\right]^{p/2} \tag{7b}$$

We propose to recover signal $\boldsymbol{x}$ by solving the problem

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad f(\boldsymbol{x})=\tfrac{1}{2}\,||\boldsymbol{\Phi x}-\boldsymbol{y}||_2^2+f_{(1,2)d}(\boldsymbol{x}) \tag{8}$$

where $f_{(1,2)d}(\boldsymbol{x})$ is the regularization function given in (6). The regularization parameter λ appearing in the similar problems in (1), (2), and (3) has been incorporated as parameters μ and λ in the function $f_{(1,2)d}(\boldsymbol{x})$, see (5) and (7).

In [18], the ℓ_p^{2d}-RLS algorithm was found to yield better QRS segments than the ℓ_p^d-RLS algorithm in [17]. It can also be shown that the segments other than the QRS segments are more accurate in an ECG signal recovered by using the ℓ_p^d-RLS algorithm. This difference in performance is caused by the fact that the ℓ_p^{2d}-RLS and ℓ_p^d-RLS algorithms are based on the minimization of different pseudonorms, namely, the ℓ_p^{2d} and ℓ_p^d pseudonorms, respectively. Since the function $f_{(1,2)d}(\boldsymbol{x})$ in (8) is a weighted summation of ℓ_p^{2d} pseudonorms of the QRS segments and ℓ_p^d pseudonorms of the other segments of signal $\boldsymbol{x}$, an improved ECG recovery can be expected by solving the problem in (8).

From the above discussion and the discussion in [17] and [18], it follows that that the objective function $f(\boldsymbol{x})$ in (8) is non-differentiable for $p\leq 1$ and/or $q\leq 1$. Optimization of such function can be facilitated by using a sequential strategy in conjunction with parameter continuation [6, 7, 8, 17, 18, 20]. To solve the problem in (8), we use a similar sequential strategy described as follows. First, select sufficiently large initial values ϵ_1, υ_1, λ_1, μ_1 and sufficiently small target values ϵ_T, υ_T, λ_T, μ_T of parameters ϵ, υ, λ, μ, and generate total $T-2$ values of these parameters in between the initial and target values using

$$(*)_t = (*)_1 \exp\left(-\alpha(t-1)\right) \quad \text{for} \quad t = 2, 3, \ldots, T-1 \quad (9)$$

with $\alpha = \log((*)_1/(*)_T)/(T-1)$, where $(*)$ is equal to either ϵ, υ, λ, or μ. For example, to evaluate ϵ_t replace $(*)$ in (9) by ϵ. Next, solve the problem in (8) sequentially for the parameters equal to $\{\epsilon_1, \upsilon_1, \lambda_1, \mu_1\}$, $\{\epsilon_2, \upsilon_2, \lambda_2, \mu_2\}, \ldots$, $\{\epsilon_{T-1}, \upsilon_{T-1}, \lambda_{T-1}, \mu_{T-1}\}$, and $\{\epsilon_T, \upsilon_T, \lambda_T, \mu_T\}$ using the basic conjugate-gradient (BCG) technique described in [21].

The $\ell_p^{(1,2)d}$-RLS algorithm based on the technique discussed above can be constructed as summarized in Table 1. Parameters E_t, L_b, and r are a sufficiently small error threshold, a positive integer, and a positive real number, respectively. Vector $\boldsymbol{d}_k$ and α_k in Step 3.iv.d are conjugate direction and step size, respectively, evaluated in Step 3.iv.a.

Table 1. $\ell_p^{(1,2)d}$-RLS Algorithm

Step 1: Input T, p, ϵ_1, ϵ_T, υ_1, υ_T, λ_1, λ_T, μ_1, μ_T,$\boldsymbol{\Phi}$, $\boldsymbol{y}$, E_t, L_b, r, and set $\boldsymbol{x}_s = \boldsymbol{0}$.
Step 2: Compute ϵ_t, υ_t, λ_t, μ_t for $t = 2, \ldots, T-1$ using (9)
Step 3: Repeat the following for $t = 1, \ldots, T$
i) Set $\epsilon = \epsilon_t$, $\upsilon = \upsilon_t$, $\lambda = \lambda_t$, $\mu = \mu_t$.
ii) Set $k = 0$, $\boldsymbol{x}_0 = \boldsymbol{x}_s$, $E_r = E_t + 1$.
iii) Compute $L = L_b + \text{round}(t/r)$.
iv) Repeat the following while $E_r > E_t$,
a) Compute $\boldsymbol{x}_{k+1}$ using BCG technique [21].
b) Set $k = k+1$.
c) Exit loop if $k > L$.
d) Compute $E_r = \|\|\alpha_k \boldsymbol{d}_k\|\|_2$.
v) Set $\boldsymbol{x}_s = \boldsymbol{x}_k$.
Step 4: Output $\boldsymbol{x}^* = \boldsymbol{x}_s$ and stop.

3.2. Estimation of QRS indices

The BCG technique used in the sequential optimization procedure described in Sec. 3.1 involves evaluation of the gradient and Hessian of function $f_{(1,2)d}(\boldsymbol{x})$ in (6). For an evaluation of the gradient and Hessian, an information about the indices of $\boldsymbol{x}$ where QRS segments are located, called as the QRS indices, is necessary. Below, we present a technique to estimate QRS indices from the measurements $\boldsymbol{y}$.

Given the measurements $\boldsymbol{y}$, a rough estimate of signal $\boldsymbol{x}$, say, $\boldsymbol{x}^*$, can be readily obtained by running the ℓ_p^{2d}-RLS algorithm [18] with a small number iterations T. The obtained vector $\boldsymbol{x}^*$ can be expected to have peaks with significant amplitude value at the QRS indices. A popular technique for the detection of QRS segments is based on the signal derivatives where a linear combination of the first- and second-order differences of the signal can be used directly to estimate QRS indices [22]. Since $\boldsymbol{x}^*$ is erroneous, we smooth its first-order difference $\boldsymbol{x}^{*d}$ and second-order difference $\boldsymbol{x}^{*2d}$, given by

$$\boldsymbol{x}^{*d} = \left[x_1^{*d}\ x_2^{*d}\ \cdots\ x_{N-1}^{*d}\right]^T \quad (10a)$$

$$\boldsymbol{x}^{*2d} = \left[x_1^{*2d}\ x_2^{*2d}\ \cdots\ x_{N-2}^{*2d}\right]^T \quad (10b)$$

where $x_i^{*d} = x_i^* - x_{i+1}^*$ and $x_i^{*2d} = x_i^* - 2x_{i+1}^* + x_{i+2}^*$, before evaluating their linear combination. Also, we determine envelope of the linear combination using Hilbert transform and use the envelope for estimating QRS indices. The detail procedure can be summarized as an algorithm as shown in Table 2.

Table 2. An Algorithm to Find QRS Indices

Step 1: Run ℓ_p^{2d}-RLS algorithm in [18] with $\boldsymbol{y}$ as an input and with a small number of iterations T. Denote the resulting solution as $\boldsymbol{x}^*$.
Step 2: Compute $\boldsymbol{x}^{*d}$ and $\boldsymbol{x}^{*2d}$ using (10) and perform temporal averaging to smooth these vectors.
Step 3: Compute $\hat{\boldsymbol{x}}^* = \boldsymbol{x}^{*d} + \boldsymbol{x}^{*2d}$, evaluate Hilbert transform of $\hat{\boldsymbol{x}}^*$, and denote the magnitude of resulting vector as $\boldsymbol{x}^{*E_{nv}}$.
Step 4: Find set of indices of vector $\boldsymbol{x}^{*E_{nv}}$ for which its value is greater than a small threshold τ.
Step 5: Arrange each set of consecutive indices obtained from Step 4 into a subset. Remove the subsets with less than d indices.
Step 6: For each subset, determine mean index by computing mean of its indices and rounding it to the nearest integer.
Step 7: Output the mean indices obtained from Step 6 as the QRS indices and stop.

4. SIMULATION RESULTS

An ECG signal $\boldsymbol{x}$ of length $N = 720$ was constructed by selecting total N consecutive samples from a random location of an ECG signal from *MIT-BIH arrhythmia* database from Physionet [23] and arranging them as the consecutive components of $\boldsymbol{x}$. Total sixteen values of the number of measurements were chosen as $M = round(t \times N)$ where $t = 0.1, 0.12, 0.14, \ldots, 0.4$. A sparse measurement matrix $\boldsymbol{\Phi}$ of size $M \times N$ was constructed by first constructing a matrix with all zero components and setting randomly chosen $round(0.06 \times N)$ components in each column to unity. Such matrix can have energy-efficient implementation for CS [16] [19]. Matlab implementation of the BSBL-BO algorithm was downloaded from [24]. The ℓ_p^d-RLS [17], ℓ_p^{2d}-RLS [18], and BSBL-BO [16] algorithms have been found to be more effective for the reconstruction of ECG signals relative to several state-of-the-art algorithms including basis pursuit (BP) [4], smoothed ℓ_0 (SL0) [8], compressive sampling matching pursuit (CoSaMP) [25], and block orthogonal matching pursuit (BOMP) [26] algorithms.

The proposed $\ell_p^{(1,2)d}$-RLS algorithm was run with $T = 30$, $p = 1$, $q = 0.1$, $\epsilon_1 = 1$, $\epsilon_T = 10^{-3}$, $\upsilon_1 = 1$, $\upsilon_T = 10^{-3}$, $\lambda_1 = 1$, $\lambda_T = 10^{-3}$, $\mu_1 = 0.1$, $\mu_T = 10^{-7}$, $E_t = 10^{-25}$, $L_b = 15$, $r = 4$ and the algorithm in Table 2 was run with $T = 10$ and $p = 0.1$ in Step 1, $\tau = 0.009$ in Step 4, and $d = 5$ in Step 5. The ℓ_p^d-RLS algorithm was run with $p = 1$, $\epsilon_1 = 1$, $\epsilon_T = 10^{-3}$, $\lambda_1 = 1$, $\lambda_T = 10^{-3}$, $T = 30$, $E_t = 10^{-25}$, $L_b = 15$, $r = 4$ and the ℓ_p^{2d}-RLS algorithm was run with $p = 0.5$, $\epsilon_1 = 1$, $\epsilon_T = 10^{-3}$, $\lambda_1 = 1$, $\lambda_T = 10^{-3}$, $T = 30$, $E_t = 10^{-25}$, $L_b = 15$, $r = 4$. SNR was measured as $20\log_{10}(||\boldsymbol{x}||_2/||\boldsymbol{x} - \hat{\boldsymbol{x}}||_2)$, where $\hat{\boldsymbol{x}}$ is a vector representing reconstructed signal. Compression ratio (CR) for a given value of M was determined as $(N - M) \times 100/N$.

Average SNR for the four algorithms over the reconstruction of 1000 different ECG signals is shown in Fig. 1. As can be seen, the $\ell_p^{(1,2)d}$-RLS algorithm offers approximately 23.51%, 11.36%, 4.38%, and 2.08% improvement in SNR for $CR = 90\%$, 80%, 70%, and 60%, respectively, relative to the other algorithms.

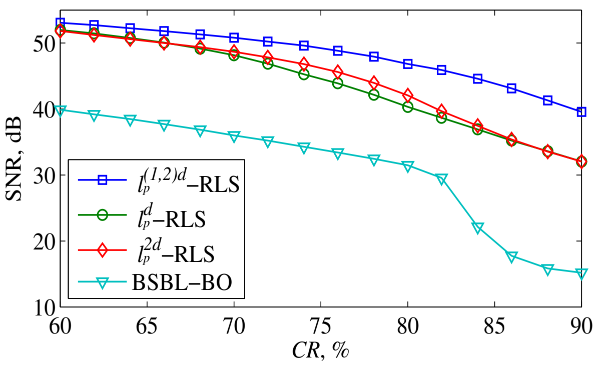

Fig. 1. Average SNR for $\ell_p^{(1,2)d}$-RLS, ℓ_p^d-RLS, ℓ_p^{2d}-RLS, and BSBL-BO algorithms.

Computational effort was measured in terms of actual CPU time required by the algorithms in a laptop with Intel Core i7 2.2 GHz CPU, 8 GB RAM, Windows 8, and MATLAB ver. 7.11.0.584. Average CPU time for the four algorithms over the reconstructions of 1000 different ECG signals is shown in Fig. 2. As can be seen, the $\ell_p^{(1,2)d}$-RLS algorithm requires 37% to 39% more CPU time relative to the ℓ_p^{2d}-RLS algorithm, but it continues to be 25% (for $CR = 90\%$) to 50% (for $CR = 60\%$) more efficient than the BSBL-BO algorithm. The increase in the computational effort relative to the ℓ_p^d-RLS and ℓ_p^{2d}-RLS algorithms is primarily caused by the effort required to estimate the QRS indices using the algorithm in Table 2.

A typical original signal and signals reconstructed using the four algorithms with $M = 130$ are shown in Fig. 3. As can be seen, the signal reconstructed using the $\ell_p^{(1,2)d}$-RLS algorithm, $x_{(1,2)d}(n)$, is much better than that recovered using the other algorithms, $x_d(n)$, $x_{2d}(n)$, $x_{\text{BSBL-BO}}(n)$.

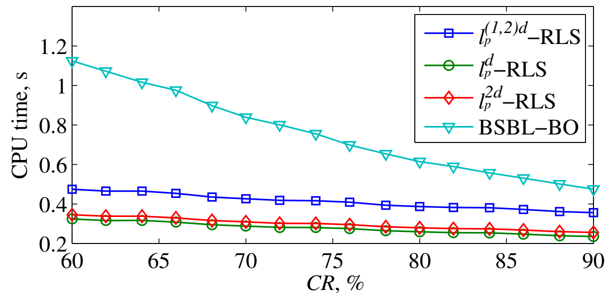

Fig. 2. Average CPU time for $\ell_p^{(1,2)d}$-RLS, ℓ_p^d-RLS, ℓ_p^{2d}-RLS, and BSBL-BO algorithms.

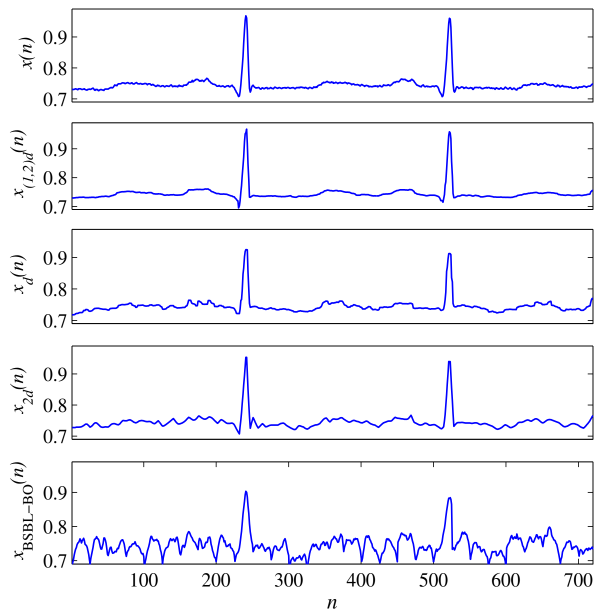

Fig. 3. Original signal, $x(n)$, and signals reconstructed using using $\ell_p^{(1,2)d}$-RLS algorithm, $x_{(1,2)d}(n)$; using ℓ_p^d-RLS algorithm, $x_d(n)$; using ℓ_p^{2d}-RLS algorithm, $x_{2d}(n)$; and using BSBL-BO algorithm, $x_{\text{BSBL-BO}}(n)$.

5. CONCLUSIONS

An improved algorithm, namely, the $\ell_p^{(1,2)d}$-RLS algorithm, for compressive sensing of ECG signals has been proposed. It is based on the minimization of $\ell_p^{(1,2)d}$ pseudonorm which promotes sparsity on the second-order difference of the QRS segments and on the first-order difference of the rest of the segments of ECG signal. QRS indices are estimated by using a technique based on signal differences and Hilbert transform, and they are used to implement the $\ell_p^{(1,2)d}$ pseudonorm. As demonstrated using simulation results, the $\ell_p^{(1,2)d}$-RLS algorithm yields significantly improved SNR for CS with high compression ratio relative the the state-of-the-art ℓ_p^d-RLS, ℓ_p^{2d}-RLS, and BSBL-BO algorithms.

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