A WEIGHTED ℓ_1 MINIMIZATION ALGORITHM FOR COMPRESSED SENSING ECG

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ABSTRACT

Compressive sensing has recently been applied to electrocardiogram (ECG) acquisition and reconstruction with the aim of lowering energy consumption and sampling rates in wireless body area networks for ambulatory ECG monitoring. However, most current methods only adopt a sparse prior on the ECG wavelet representation. In this paper, we propose to further exploit the wavelet representation structure by incorporating two properties in the formulation of the optimization problem: the exponentially decaying magnitude of the detail coefficients across scales and the accumulation of signal energy in the approximation subband. We derive a weighted ℓ_1 minimization algorithm, based on a maximum a posteriori (MAP) approach, that leads to a significant reduction in the number of measurements and superior reconstruction performance compared to current CS-based methods with application to wireless ECG systems.

Index Terms— Compressed sensing, electrocardiogram, wavelet transform, wireless body area networks (WBAN).

1. INTRODUCTION

Due to recent technological advances in the area of wireless body area networks, long-term and ubiquitous real-time ECG monitoring are becoming increasingly popular. However, such systems face a large number of constraints, such as limited memory, energy, computation and communication capabilities. The WBAN energy consumption can be divided into three main processes: sensing, wireless communication and data processing. The cost to wirelessly transmit data is greater than for any other function [1], which suggests that some data reduction strategy at the sensor node should be employed. With this aim, Mamaghanian et al. [2] recently proposed compressed sensing (CS) to lower energy consumption and complexity in WBAN-enabled ECG monitors. Their results show that CS outperforms state-of-the-art wavelet transform-based ECG compression methods in terms of energy efficiency.

Compressed sensing is a groundbreaking paradigm that enables the reconstruction of sparse or compressible signals from a small number of linear projections [3, 4]. Even though the application of CS in WBAN-enabled ECG monitors is still at its infancy, it has already led to important results. For example, Chen *et al.* [5] demonstrated the ability of CS to continually and blindly compress ECG signals at compression factors of 10X, without the need for any general purpose memory or processing at the sensor node. Dixon *et al.* [6] studied several design considerations for CS-based ECG telemonitoring via a WBAN, including the encoder architecture and the design of the measurement matrix. Their results show high compression ratios using a 1-bit Bernoulli measurement matrix.

Our previous contribution to the area includes CS-based algorithms for ECG compression with a focus on algorithms enabling joint reconstruction of ECG cycles by exploiting correlation between adjacent heartbeats [7, 8, 9]. In addition, we also proposed a CS-based method to reconstruct ECG signals in the presence of electromyographic (EMG) noise using symmetric α -stable distributions to model the EMG interference [10].

A latent problem when trying to reconstruct ECG signals using CS-based methods is the inability to accurately recover the low-magnitude coefficients of the wavelet representation [11]. To alleviate this problem, we propose to incorporate prior information about the magnitude decay of the wavelet coefficients across subbands in the reconstruction algorithm. More precisely, we derive a weighted ℓ_1 minimization algorithm with a weighting scheme based on the standard deviation of the wavelet coefficients at different scales. In addition, the weighting scheme also takes into consideration the fact that the approximation subband coefficients accumulate most of the signal energy. Experimental results on ECG records from the MIT-BIH Arrhythmia database validate the superior performance of the proposed algorithm in terms of reconstruction quality and number of measurements.

2. BACKGROUND AND MOTIVATION

2.1. Compressed Sensing Review

Let $x \in \mathbb{R}^N$ be a signal that is either K-sparse or compressible in some orthogonal basis Ψ . Thus, the signal x can be well approximated by a linear combination of a small set of vectors from Ψ , *i.e.* $x \approx \sum_{i=1}^{K} s_i \psi_i$, where $K \ll N$. Let Φ be an $M \times N$ sensing matrix, M < N. Compressed sensing [3, 4] deals with the recovery of x from undersampled linear measurements of the form $y = \Phi x = \Phi \Psi s$. If we define $\Theta = \Phi \Psi$, then the measurement vector becomes $y = \Theta s$. Compressed sensing addresses the signal x can be recovered from $M = \mathcal{O}(K\log(N/K))$ measurements if the matrix Θ satisfies the restricted isometry property (RIP) [4]. In practical scenarios with noise, the signal s can be recovered from y by solving the convex optimization problem

$$\min_{s} \frac{1}{2} \|y - \Theta s\|_{2}^{2} + \lambda \|s\|_{1}, \tag{1}$$

with λ a parameter that controls the trade-off between sparsity and reconstruction fidelity. Problem (1), known as basis pursuit denoising (BPDN) [12], can be viewed as a maximum a posteriori (MAP) estimate for *s* under the assumption that each component of *s* is drawn i.i.d. from a Laplace prior [13].

2.2. Wavelet representation

Consider a signal x of length N. Given a scaling function φ and a wavelet function ψ , the wavelet representation of x can be expressed in terms of shifted versions of φ and shifted and dilated versions of ψ

$$x = \sum_{i=0}^{N_1 - 1} a_{1,i} \varphi_{1,i} + \sum_{j=1}^{L} \sum_{i=0}^{N_j - 1} d_{j,i} \psi_{j,i}, \qquad (2)$$

where j denotes the scale of analysis and L indicates the finest scale. $N_j = N/2^{L-j+1}$ corresponds to the number of coefficients at scale $j \in \{1, \ldots, L\}$ and i represents the position, $0 \le i \le N_j - 1$. The wavelet transform consists of the scaling coefficients $a_{1,i}$ and wavelet coefficients $d_{j,i}$. Using the previous notation, we write $x = \Psi s$, where $s = [a_{1,1} \ldots a_{1,N_1-1}d_{1,0} \ldots d_{1,N_1-1} \ldots d_{L,0} \ldots d_{L,N_L-1}]^T$ is the vector of scaling and wavelet coefficients and Ψ is the orthogonal matrix containing the wavelet and scaling functions as columns. The vector s can be decomposed into L+1 subvectors. The first subvector corresponds to the scaling coefficients and is denoted as a_1 . The next L subvectors are denoted by d_j , $j = 1, \ldots, L$, and the jth subvector contains all of the wavelet coefficients for scale j. Thus, s can also be written as $s = [a_1d_1d_2 \ldots d_L]^T$.

2.3. Motivation

The optimization problem in (1) only considers the sparsity of the ECG wavelet representation, and therefore, it does not exploit all of the rich structure present in the ECG wavelet representation. Recent efforts on CS show that the incorporation of prior knowledge into standard sparse recovery algorithms can boost their performance [14, 15]. Along these lines, we aim to find properties of the ECG wavelet representation that can be incorporated into compressive sensing-based algorithms to improve the reconstruction and reduce the number of necessary measurements.



Fig. 1. Reconstruction of ECG signals using BPDN (a) Original signal. (b) Wavelet representation. (c) Relative error signal.

An experiment is performed to evaluate the recovery of ECG signals when only the sparsity property is exploited in the reconstruction. The selected signal, denoted as x and illustrated in Fig. 1(a), corresponds to a sequence of the record 117 from the MIT Arrythmia database, formed by N = 2048samples. Figure 1(b) refers to its corresponding wavelet representation, using Daubechies-4 and a decomposition level L = 5. The dotted lines indicate the separation between consecutive wavelet subbands. We note that the approximation subband accumulates most of the signal energy. The sparsity level is selected as the number of coefficients that accumulates 99.99% of the signal energy, which corresponds to K = 210 for the selected sequence. The sequence is sensed with a random CS matrix satisfying the RIP, and with a number of measurements M = 630. The reconstruction is performed with the traditional BPDN algorithm presented in (1) and the evaluation performance, in terms of the relative reconstruction error, is illustrated in Fig. 1(c). The relative reconstruction error is defined as $(x - \hat{x})/x$, where \hat{x} is the recovered signal. From Fig. 1(c), we note that the error in the reconstruction of the detail subbands d_4 and d_5 has the highest magnitude, which is detrimental for medical diagnosis.

The unsatisfactory performance of BPDM in reconstructing ECG signals motivates the development of new CS-based algorithms for ECG reconstruction. This paper aims at advancing CS ECG by exploiting two properties of the ECG wavelet representation. More precisely, we exploit the fact that the approximation subband coefficients accumulate most of the signal energy and that the magnitude of the detail wavelet coefficients decreases across scales.

3. METHODS

3.1. Random sampling

A sliding window of size N is used to sample the ECG signal. Let x denote the ECG segment captured by the window. The information we gather about x can be described by $y = \Phi x$, where Φ is a $M \times N$ matrix, or equivalently as $y = \Phi \Psi s = \Theta s$, where Ψ is the orthogonal wavelet basis and s is the wavelet representation. In order to recover the best K-term approximation of the original signal, the matrix Φ needs to satisfy the RIP. It is known that sub-Gaussian matrices satisfy this condition with overwhelming probability [4]. Here we assume that the entries of the matrix Φ are independently sampled from a symmetric Bernoulli distribution (P($\Phi_{i,j} = \pm 1/\sqrt{M} = 1/2$)) to facilitate an efficient hardware implementation. The use of Bernoulli matrices, as compared to other sub-Gaussian matrices, results in simpler circuit complexity, data storage, and computation requirements [5].

3.2. Reconstruction

This section aims to reconstruct the vector s from the measurements $y = \Theta s + r$, where r is the noise unavoidably corrupting the data. A maximum a posteriori (MAP) approach is proposed for the reconstruction of s. To favor a sparse estimate, a Laplacian distribution with standard deviation σ_i is adopted for each entry s_i of s; that is

$$p(s_i) = \frac{1}{\sqrt{2}\sigma_i} \exp\left(-\frac{\sqrt{2}\|s_i\|_1}{\sigma_i}\right).$$
 (3)

The noise r is modeled as independent and Gaussian with zero mean and variance equal to σ_r . To infer s from y, we maximize the conditional probability distribution $p(s|y, \Theta)$, which can be expressed by means of Bayes's rule as

$$p(s|y,\Theta) \propto p(y|\Theta,s)p(s).$$
 (4)

Because the noise is assumed to be Gaussian, the likelihood function is given by $p(y|\Theta, s) \propto \exp\left(-\|y - \Theta s\|_2^2/(2\sigma_r^2)\right)$. Therefore, maximizing the posterior distribution $p(s|y,\Theta)$ leads to

$$s_{\text{MAP}} = \operatorname*{arg\,max}_{s} p(s|y,\Theta)$$
 (5)

$$= \arg\max_{s} \left(\log p(y|\Theta, s) + \sum_{i} \log p(s_{i}) \right)$$
(6)

$$= \arg\min_{s} \left(\frac{\|y - \Theta_s\|_2^2}{2\sigma_r^2} + \sum_{i} \frac{\sqrt{2}\|s_i\|_1}{\sigma_i} \right).$$
(7)

The problem in (7) can also be expressed as

$$s_{\text{MAP}} = \arg\min_{s} \left(\frac{1}{2} \| y - \Theta s \|_{2}^{2} + \lambda \| W s \|_{1} \right), \quad (8)$$

where W is a diagonal matrix, whose *i*th diagonal element is of the form $1/\sigma_i$, and λ is a tuning parameter. This problem is equivalent to BPDN when all σ_i are equal.

The solution of problem (8) requires the standard deviations for each coefficient of the wavelet representation, which are unknown in our case. To cope with this issue, we propose to model the variance variation across scales with exponential decay functions. As presented in Section 2.3, the detail wavelet coefficients of ECG signals tend to decrease across scales, and this behavior can be enforced by modeling the variances so that they decay exponentially as the scale becomes finer. This idea corresponds to the model proposed by Romberg *et al.* [16]:

$$\sigma_j^2 = C 2^{-j\alpha} \quad j = 1, \dots, L,\tag{9}$$

where C and α are the model parameters and j is the scale of analysis. In this model, the σ_i , i = 1, ..., N values are made equal for all coefficients within a scale, and therefore σ_i^2 refers to the variance of the coefficients at scale j.

Define $W^* = \sqrt{C}W$ and $\lambda^* = \lambda/\sqrt{C}$. Then, problem (8) can be reformulated as the following ℓ_1 weighted minimization

$$s_{\text{MAP}} = \arg\min_{s} \left(\frac{1}{2} \| y - \Theta s \|_{2}^{2} + \lambda^{\star} \| W^{\star} s \|_{1} \right), \quad (10)$$

where λ^* is regarded as a tuning parameter. By using eq. (9) and the fact that each diagonal entry of W satisfies $W_{i,i} = 1/\sigma_i$, we infer that diagonal elements of W^* corresponding to scale j are of the form $2^{j\alpha/2}$.

As presented in Section 2.3, the approximation subband coefficients accumulate most of the signal energy and, therefore, should be included in the sparse representation of the ECG. To exploit this property, we employ a similar approach as that of Vaswani *et al.* [15] to reconstruct a sparse signal when part of the support is known *a priori*, which consists of finding the signal that satisfies the data fidelity constraint and is the sparsest outside of the known support. In our approach, this idea is implemented by setting to zero the diagonal entries of W^* corresponding to the approximation subband, *i.e.*. $W_{i,i}^* = 0$ for $i = 1, ..., N_1$.

As the diagonal entries of W^* only depend on the value of α , problem (10) can be solved after α is calculated. This leads us to propose a training stage to estimate the value of α . The first part predicts the standard deviations σ_j , $j = 1, \ldots, L$ using maximum likelihood estimation. Once the variances are estimated, simple linear regression can be employed to solve for α in the following equation, derived from (9),

$$\log_2 \sigma_j^2 = \log_2 C - j\alpha, \ j = 1, \dots, L.$$
 (11)



Fig. 2. Scatter plot of *j* versus $\log_2 \sigma_j^2$ and fitted regression line.

4. EXPERIMENTAL RESULTS

To validate the proposed method, the MIT Arrythmia database [17] is employed for both training and testing. Every file in the database consists of two lead recordings sampled at 360Hz with 11 bits per sample of resolution. The data set proposed by Lu *et al.* [18] is used in our experiments. It consists of records 100, 101, 102, 103, 107, 109, 111, 115, 117, 118 and 119, which encompasses a variety of signals with different rhythms, QRS complex morphologies and ectopic beats. We set the length of the sliding window to N = 2048, a commonly used segment length value for ECG processing. The orthogonal Daubechies-4 wavelets is set as the sparsifying transform and the decomposition level is set to L = 5.

The reconstruction SNR (R-SNR) is used as the performance measure for experiments,

$$\text{R-SNR} = 10\log_{10} \frac{\|x\|_2^2}{\|x - \hat{x}\|_2^2},$$
 (12)

where x and \hat{x} denote the N-dimensional original and reconstructed signals, respectively.

The first experiment aims at finding the parameter α in (11) through simple linear regression. The training data consists of the wavelet representation of 330 ECG sequences of length N = 2048 from the selected set of records; 30 sequences per record. Let c_j denote the vector formed by the concatenation of the wavelet subbands from the training data at scale j. The standard deviations σ_j , $j = 1, \ldots, L$, are first calculated using the maximum likelihood estimate of the standard deviation,

$$(\sigma_j)_{\mathrm{ML}} = \frac{\sqrt{2} \|c_j\|_1}{P_j} \ j = 1, \dots, L,$$
 (13)

where P_j refers to the dimension of c_j . Figure 2 shows the scatter plot with scale *j* displayed on the horizontal axis and $\log_2 \sigma_j^2$ on the vertical axis. The continuous line in Fig. 2, with slope $-\alpha = -1.62$, corresponds to the fitted regression line.



Fig. 3. Comparison of the proposed method with BPDN. Reconstruction SNR averaged over all the records of the selected data set for different number of measurements.

The second experiment is performed to compare the proposed method with the BPDN algorithm, which is the reconstruction algorithm selected by Mamaghanian *et al.* [2], Dixon *et al.* [6], and Chen *et al.* [5] for the recovery of ECG signals. As described in Section 3.1, the measurements are obtained with a Bernoulli matrix. The experiment is carried out and averaged over 10-min long single leads extracted from the selected set of records. The reconstruction SNR is used to evaluate the quality of the recovered signals as a function of the number of measurements M. The measurements are corrupted by additive white Gaussian noise with $\sigma_r = 0.05$. Chen *et al.* [12] proposed to set the tuning parameter to the value $\lambda = \sigma_r \sqrt{2\log(N)}$. The same criteria is adopted for the selection of the tuning parameters in our experiments.

As shown in Fig. 3, the proposed algorithm outperforms the traditional BPDN algorithm as it requires fewer measurements while achieving superior reconstruction quality. These results are expected as the proposed method exploits prior knowledge of the signal structure, unlike BPDN, which only leverages the sparsity of the signals.

5. CONCLUSION

This paper proposes an ECG signal reconstruction scheme based on a weighted ℓ_1 minimization method. The proposed weighting scheme allows the efficient use of information on two important properties of the ECG wavelet representation: energy concentration in the approximation subband and exponential magnitude decay of the detail coefficients across scales. The proposed algorithm was evaluated for the reconstruction of a set of eleven ECG records from the MIT-BIH Arrhythmia database encompassing a variety of signals with different rhythms, QRS complex morphologies and ectopic beats. Results show significant performance gains are attained over the traditional basis pursuit denoising algorithm.

6. REFERENCES

- H. Cao, V. Leung, C. Chow, and H. Chan, "Enabling technologies for wireless body area networks: A survey and outlook," *IEEE Communications Magazine*, vol. 47, no. 12, pp. 84 –93, Dec. 2009.
- [2] H. Mamaghanian, N. Khaled, D. Atienza, and P. Vandergheynst, "Compressed sensing for real-time energyefficient ECG compression on wireless body sensor nodes," *IEEE TBME*, vol. 58, no. 9, pp. 2456 –2466, Sept. 2011.
- [3] D. Donoho, "Compressed sensing," *ITIT*, vol. 52, no. 4, pp. 1289–1306, Sept. 2006.
- [4] E.J. Candès and M.B. Wakin, "An introduction to compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, pp. 21 – 30, Mar. 2008.
- [5] F. Chen, A.P. Chandrakasan, and V.M. Stojanovic, "Design and analysis of a hardware-efficient compressed sensing architecture for data compression in wireless sensors," *IEEE Journal of Solid-State Circuits*, vol. 47, no. 3, pp. 744–756, Mar. 2012.
- [6] A.M.R. Dixon, E.G. Allstot, D. Gangopadhyay, and D.J. Allstot, "Compressed sensing system considerations for ECG and EMG wireless biosensors," *IEEE TBCAS*, vol. 6, no. 2, pp. 156–166, April 2012.
- [7] L.F. Polania, R.E. Carrillo, M. Blanco-Velasco, and K.E. Barner, "Compressed sensing based method for ECG compression," in *ICASSP*, May 2011, pp. 761 – 764.
- [8] L.F. Polania, R.E. Carrillo, M. Blanco-Velasco, and K.E. Barner, "Compressive sensing exploiting waveletdomain dependencies for ECG compression," in *SPIE Defense, Security, and Sensing*, April 2012, pp. 83650E–83650E.
- [9] L.F. Polania, R.E. Carrillo, M. Blanco-Velasco, and K.E. Barner, "On exploiting interbeat correlation in compressive sensing-based ECG compression," in *SPIE Defense, Security, and Sensing*, Apr. 2012, pp. 83650D– 83650D.
- [10] L.F. Polania, R.E. Carrillo, M. Blanco-Velasco, and K.E. Barner, "Compressive sensing for ECG signals in the presence of electromyographic noise," in 38th Annual Northeast Bioengineering Conference, Mar. 2012, pp. 295–296.
- [11] Z. Zhang, T. Jung, S. Makeig, and B.D. Rao, "Compressed sensing for energy-efficient wireless telemonitoring of noninvasive fetal ECG via block sparse bayesian learning," *IEEE TBME*, vol. 60, no. 2, pp. 300– 309, Feb. 2013.

- [12] S.S. Chen, D.L. Donoho, and M.A. Saunders, "Atomic decomposition by basis pursuit," *SIAM Journal on Scientific Computing*, vol. 20, no. 1, pp. 33–61, Dec. 1998.
- [13] M.S. Lewicki and B.A. Olshausen, "Probabilistic framework for the adaptation and comparison of image codes," *J. Opt. Soc. Am. A*, vol. 16, no. 7, pp. 1587– 1601, Jul 1999.
- [14] R. E. Carrillo, L. F. Polania, and K. E. Barner, "Iterative algorithms for compressed sensing with partially known support," in *Proc.*, *IEEE ICASSP*, Dallas, TX, Mar. 2010, pp. 3654–3657.
- [15] N. Vaswani and Wei Lu, "Modified-CS: Modifying compressive sensing for problems with partially known support," *ITSP*, vol. 58, no. 9, pp. 4595–4607, 2010.
- [16] J.K. Romberg, Hyeokho Choi, and R.G. Baraniuk, "Bayesian tree-structured image modeling using wavelet-domain hidden Markov models," *ITIP*, vol. 10, no. 7, pp. 1056–1068, 2001.
- [17] A.L. Goldberger et al., "Physiobank, Physiotoolkit, and Physionet: Components of a new research resource for complex physiologic signals," *Circulation*, vol. 101, no. 23, pp. e215–e220, 2000.
- [18] Z. Lu, D. Y. Kim, and W. A. Pearlman, "Wavelet compression of ECG signals by the set partitioning in hierarchical trees algorithm," *IEEE T-BME*, vol. 47, no. 7, pp. 849 –856, July 2000.