ANALOG JOINT SOURCE CHANNEL CODING FOR GAUSSIAN BROADCAST CHANNELS

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ABSTRACT

We consider the problem of transmission of independent and correlated Gaussian sources over the two user Gaussian Broadcast Channel (GBC). We present a low complexity, low delay, analog joint source channel coding communication system based on extensions of existing Nested Quantization techniques. Simulations results show that the resulting performance is close to the optimal theoretical limits for several cases of interest.

***Index Terms*—** Analog Joint Source Channel Coding, Gaussian Broadcast Channels, Nested Quantization

1. INTRODUCTION

The use of digital communications systems based on the Shannon separation principle between source and channel coding [1] has led to ubiquitous communications in our society. In this framework, continuous signals are first acquired and source encoded. Then, capacity approaching channel codes are utilized. It is well known that this approach is optimal provided that there are no constraints in terms of complexity and delays. However, long block lengths are required, and these separated systems are not very robust to changes in the channel parameters.

Recently, systems based on analog joint source-channel coding have been discussed in the literature [2,3]. In this approach, the concatenation of the (vector) quantizer, source encoder and channel encoder characteristic of digital systems is substituted by an end-to-end analog encoder. This discrete-time, continuous-amplitude system directly processes the acquired samples using a non-linear transformation, whose output is transmitted directly through the channel after proper modulation. For the same performance, these schemes may present more robustness and require less encoding/decoding complexity than traditional digital systems.

The work proposed in this paper is based on analog joint source-channel coding and successive encoding/decoding techniques, in particular, on a scheme devised in [4] called Nested Quantization (NQ), which in [5] was adapted for analog joint source channel coding over the Gaussian Multiple Access Channel (GMAC). We extend this framework to the Gaussian Broadcast Channel by developing novel mappings that are appropriate in this environment.

The paper is organized as follows: Section 2 provides the problem statement and defines the optimal theoretical limits for the Gaussian Broadcast Channel. Section 3 explains NQ and introduces the proposed scheme for the GBC. Section 4 introduces transmitting correlated sources over the GBC and the proposed coding scheme in that case. Section 5 presents simulation results for the proposed scheme for several cases of interest and Section 6 concludes the paper.

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2. THE BROADCAST CHANNEL

We consider the two user Gaussian Broadcast Channel. In this channel, there is a common transmitter and two receivers experiencing independent zero-mean additive white Gaussian noise (n_1, n_2) with variances (N_1, N_2), respectively. Without loss of generality we assume that $N_1 \geq N_2$. The problem is to find an encoding function $g(.)$ that takes user 1 data, x_1, and user 2 data, x_2, to produce the channel output, y. Each receiver observes a corrupted version of y and each employs a decoding function f_i to obtain each user's data, x_i.

2.1. Optimal Distortion Region

First, we focus in the transmission of independent Gaussian sources to two users. We assume the sources to be zero mean with variance 1, i.e., $(x_1, x_2) \sim \mathcal{N}(\mathbf{0}, \mathbf{I_2})$. In that case, for a maximum transmission power P and under the Mean Squared Error distortion criterion, the minimum (optimal) distortion pair incurred by user 1 and 2 sources is given by [6]

$$D_1 = (1 + \frac{(1-\gamma)P}{\gamma P + N_1})^{-\kappa} \tag{1}$$

$$D_2 = (1 + \frac{\gamma P}{N_2})^{-\kappa}, \tag{2}$$

where $\gamma \in [0, 1]$. γ is a parameter that controls the rate allocated to each user, D_1, D_2 are the distortions experienced by sources 1 and 2, respectively, and κ is the number of source symbols sent to *each* user per signaling time (the information rate). In this paper we focus only in the case when the transmitter wishes to transmit one source symbol for each user using the channel once ($\kappa = 1$).

3. NESTED QUANTIZATION

The proposed scheme is based on Nested Quantization (NQ). NQ first appeared in [4] and was used for the Multiple Access Channel (MAC). A variant of NQ called *Scalar Quantizer Linear Coder* (SQLC) was used in [5], again for the MAC channel. The proposed scheme in this paper is based on SQLC.

In SQLC, the two users source symbols are encoded into one channel symbol. The first symbol, x_1, is passed through a uniform quantizer of step Δ to produce y_1. The second symbol, x_2, is scaled by α and clipped to force it to lie in the interval $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$ to produce y_2. The sum of $y_1 + y_2$ is sent through the channel after scaling it by a factor, β, that controls the power of the transmission system. That is

$$y_1 = \Delta \lceil \frac{x_1}{\Delta} \rfloor \quad (3)$$
$$y_2 = \mathcal{L}_{\pm\frac{\Delta}{2}}(\alpha x_2) \quad (4)$$
$$\eta = y_1 + y_2 \quad (5)$$
$$y = \beta\, \eta, \quad (6)$$

where $\lceil \cdot \rfloor$ rounds its argument to the nearest integer, α controls the spread of the second symbol and $\mathcal{L}_{\pm\frac{\Delta}{2}}[\cdot]$ forces its output to lie within the interval $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$. That is

$$\mathcal{L}_{\pm\frac{\Delta}{2}}(\lambda) = \begin{cases} \lambda & \text{if } -\frac{\Delta}{2} \leq \lambda \leq \frac{\Delta}{2} \\ \frac{\Delta}{2} & \text{if } \lambda > \frac{\Delta}{2} \\ -\frac{\Delta}{2} & \text{if } \lambda < -\frac{\Delta}{2} \end{cases} \quad (7)$$

Note that $\lceil \frac{x_1}{\Delta} \rfloor$ in (3) produces an *integer* (positive, negative or zero). That integer is then scaled by Δ to produce y_1.

The above system can be applied for the MAC because for fixed Δ and β, the x_1 data can be encoded separately from the x_2 data and transmitted to the channel (notice that in the MAC the users do not have access to each other information). A schematic of the proposed encoder is presented in Fig. 1.

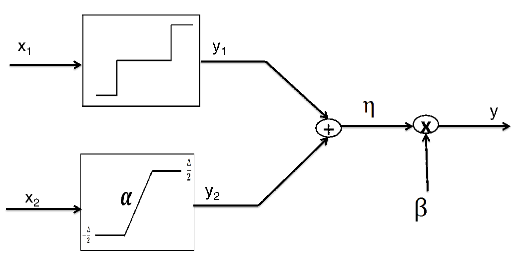

Fig. 1. Block diagram showing the SQLC encoder. x_1 is passed to a uniform scalar quantizer of step Δ and x_2 is scaled and clipped to force it to lie within $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$.

The decoder receives $z = y + n$ and performs MMSE decoding on z to obtain the MMSE estimate of η as [1]

$$\hat{\eta} = \frac{\beta}{\sigma_n^2 + \beta^2} z. \quad (8)$$

Afterwards the decoder quantizes the obtained $\hat{\eta}$ according to (3) to obtain an estimate of the quantized first symbol, $\hat{y_1}$, as $\hat{y_1} = \Delta \lceil \frac{\hat{\eta}}{\Delta} \rfloor$. The second symbol estimate is obtained by subtracting $\hat{y_1}$ from $\hat{\eta}$, that is $\hat{y_2} = \hat{\eta} - \hat{y_1}$. The transmitted symbol pair is estimated from $(\hat{y_1}, \hat{y_1})$ as

$$\hat{x_1} = \hat{y_1} \quad (9)$$
$$\hat{x_2} = \frac{1}{\alpha} \hat{y_2} \quad (10)$$

The decoder structure of the SQLC system is shown in Fig. 2.

[1] Note that this derivation is valid when the distribution of z is Gaussian. We have corroborated through simulation that indeed the distribution of z is close to a Gaussian (due to space constraints we do not elaborate on this).

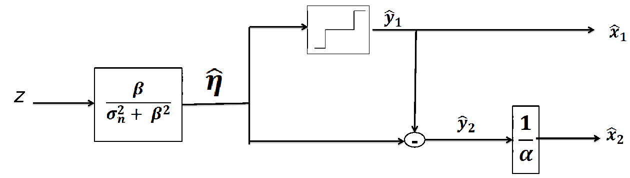

Fig. 2. Decoder of the SQLC scheme. Notice that the quantizer is the same as the one used in the encoder.

The proposed scheme can be thought as equivalent to projecting the source pair (x_1, x_2) onto a space filling curve of dimension 1 to produce (y_1, y_2), as shown in Fig. 3.

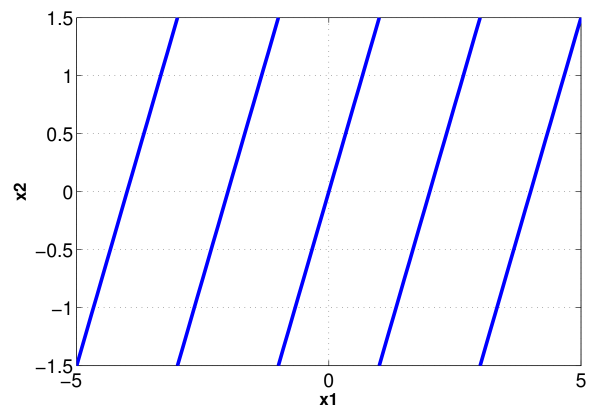

Fig. 3. Space filing curve utilized in the proposed system. Source symbols (x_1, x_2) are projected onto the curve to produce (y_1, y_2).

Observe the discontinuity of the curve shown in Fig. 3. If y_2 is very close to the edge of the lines shown in the figure and if the noise value is large so that the decoder moves the received pair from one branch to the next, then the distortion for x_2 will be severe. This is the well known threshold effect in analog joint source channel coding, which results from the discontinuity of the curve. To alleviate this problem we propose a modified version of SQLC that we call *Alternating Sign SQLC* (AS-SQLC). The proposed scheme better suits the broadcast channel, taking advantage of the fact that in the broadcast channel the transmitter has access to *both* users data and can encode them jointly.

3.1. Alternating sign SQLC

AS-SQLC works as SQLC with the difference in how y_2 is generated. Rather than using (4), in AS-SQLC the second user symbol is generated according to

$$y_2 = \kappa\, \mathcal{L}_{\pm\frac{\Delta}{2}}(\alpha x_2) \quad (11)$$

where

$$\kappa = \begin{cases} +1 & \text{when } \lceil \frac{y_1}{\Delta} \rfloor \text{ is even} \\ -1 & \text{when } \lceil \frac{y_1}{\Delta} \rfloor \text{ is odd .} \end{cases} \quad (12)$$

Notice that in order to generate the second user symbol, we need knowledge of the first user symbol. This is not possible for the MAC.

At the decoder, we use the same procedure as SQLC with the difference on how $\hat{y_2}$ is obtained. Specifically, $\hat{y_2}$ is obtained by multiplying the estimate by the factor κ as follows

$$\hat{y_2} = \kappa\left(\hat{\eta} - \hat{y_1}\right) \tag{13}$$

where

$$\kappa = \begin{cases} +1 & \text{when } \lceil \frac{\hat{y_1}}{\Delta} \rfloor \text{ is even} \\ -1 & \text{when } \lceil \frac{\hat{y_1}}{\Delta} \rfloor \text{ is odd.} \end{cases} \tag{14}$$

This is equivalent to projecting the source pair (x_1, x_2) onto the space filling curve shown below in Fig. 4. Note that the space filling curve is *continuous* and x_2 does not experience any threshold effect.

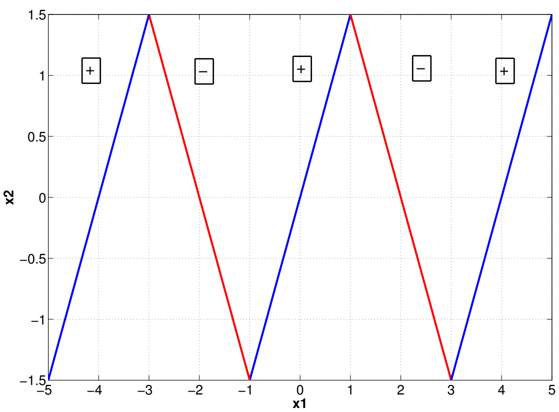

Fig. 4. Space filling curve for AS-SQLC. The two sources (x_1, x_2) are projected onto the curve and the resulting pair is transmitted as indicated in the text. Notice the *continuity* of the curve eliminating the threshold effect.

It is interesting to remark the difference between the space filling curve in Fig. 4 and standard space filling curves designed for point to point communication, such as Shannon-Kotelnikov mappings (see [7] and [8]). Standard space filling curves are designed for a *symmetric* distortion case. That is, the distortion incurred by user 1, D_1, is usually the same as the distortion incurred by user 2, D_2. Moreover, the distortion measure used in standard designs is the *average* distortion of both sources defined as $D = \frac{D_1+D_2}{2}$. However, the proposed space filling curve for the broadcast channel shown in Fig. 4 has the advantage of being able to control the distortions incurred by each user. This is achieved by changing the quantization step, Δ. If Δ is small, source 1, x_1, will be finely quantized and the resulting distortion, D_1, will be small. At the same time, since Δ is small, source 2, x_2 will be "squeezed" and fit within the small interval $[-\frac{\Delta}{2}, \frac{\Delta}{2}]$ so that D_2 will be large. On the other hand, if Δ is large source 1 is quantized coarsely and incurs in a large distortion. At the same time, x_2 is spread over a longer interval and D_2 will be smaller.

3.2. Proposed Communication System

In this section, we present the complete communication system. We use AS-SQLC to encode the data of user 1, x_1 (the user with larger noise variance) and the data of the second user, x_2 (the user with lower noise variance). We then transmit the resulting y through the channel. We employ the AS-SQLC decoding technique at the decoders of both users. At the second user's decoder (the one with less noise), we are able to extract *both* the discrete (quantized) component and the analog component. Hence, we will be able to recover both x_2 and x_1. Note that user 2 data is only x_2, however it needs to know x_1 in order to invert the modulation, enabling the recovery of x_2. At the first user's decoder (the user with more noise), we apply the same decoding procedure. However, only the discrete component of the signal (user 1 data) is recovered in this case.

The system diagram of the complete system is shown in Fig. 5.

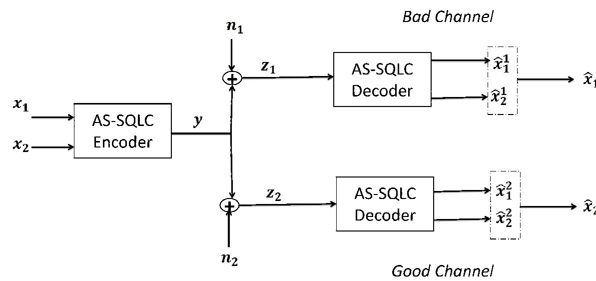

Fig. 5. Complete system diagram.

4. CORRELATED SOURCES

So far we have focused on two independent Gaussian sources. In this section we extend that framework to the case in which the two sources are correlated. If the two Gaussian sources are zero mean with Covariance Matrix $\boldsymbol{C}$ given by

$$\boldsymbol{C} = \begin{pmatrix} 1 & \rho^2 \\ \rho^2 & 1 \end{pmatrix}, \tag{15}$$

then the theoretical limit for the distortion region is provided by the equations given in section III of [6].

In order to transmit correlated sources, we use the scheme described in section 3 with only two modifications at the transmitter and the two receivers (the rest of the system is exactly as described above). First, equations (3), (4) at the encoder are modified to become:

$$y_1 = \Delta\lceil \frac{ax_1 + bx_2}{\Delta} \rfloor = \Delta\lceil \frac{w_1}{\Delta} \rfloor \tag{16}$$

$$y_2 = \mathcal{L}_{\pm\frac{\Delta}{2}}(\alpha(cx_1 + dx_2)) = \mathcal{L}_{\pm\frac{\Delta}{2}}(\alpha(w_2)) \tag{17}$$

Note that we have introduced a linear transformation on the users data $\mathbf{x} = [x_1 x_2]$ to produce $\mathbf{w} = [w_1 w_2]$. That is

$$\mathbf{w} = \mathbf{H}\mathbf{x}, \qquad \text{where} \qquad \mathbf{H} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{18}$$

Second, the equations describing the decoding in (9), (10) at the two receivers are also modified to become

$$\hat{\mathbf{w}} = \mathbf{H}^{-1}\hat{\mathbf{y}} \tag{19}$$

$$\hat{x_1} = \hat{w_1} \tag{20}$$

$$\hat{x_2} = \frac{1}{\alpha}\hat{w_2} \tag{21}$$

Note that the scheme used for the transmission of uncorrelated sources can be considered a special case of the scheme just described with $a = 1,\ b = 0,\ c = 0,\ d = 1$.

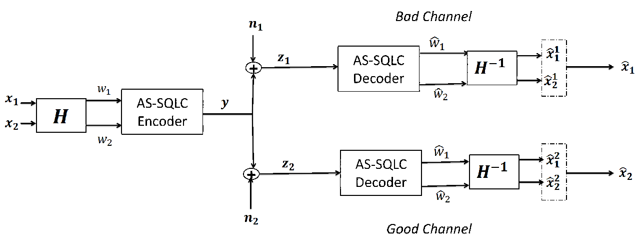

Fig. 6. Complete system diagram for the correlated case.

5. SIMULATION RESULTS

In this section we present the simulation results for the proposed scheme for several cases of interest. First, we present in Fig. 7 the results for the transmission of independent Gaussian sources when $P = 1,\ N_2 = 0.002$ and $N_1 = 10N_2$, and compare them with a time-sharing strategy. The results are obtained through Monte-Carlo optimization of the parameters Δ and α in (3) and (4) so that the Signal to Distortion Ratio, SDR, is maximized. The optimum value of Δ for different values of D_2 is shown in Fig. 8. The SDR upperbound shown in Fig. 7 is obtained from (1) and (2). The power, P, is given by $P = \mathbb{E}(y^2)$.

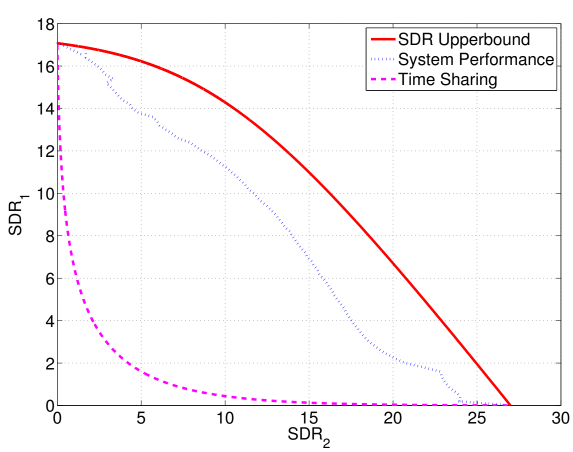

Fig. 7. System performance for $P = 1,\ N_2 = 0.002$ and $N_1 = 10N_2$ for the optimal values of parameters Δ and α.

The time sharing performance is obtained by utilizing the channel ζ of the time to transmit user 1 data and $1 - \zeta$ of the time to transmit user 2 data, where $0 \leq \zeta \leq 1$.

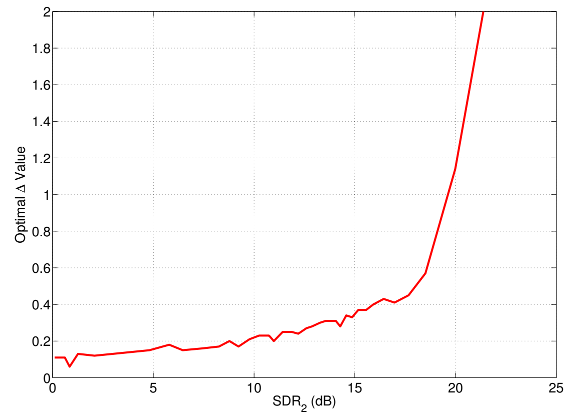

Fig. 8. Optimal Δ value to achieve the performance shown in Fig. 7 for different values of D_2.

We now give the performance of the system when transmitting correlated sources. We perform the simulation of the system explained in Section 4 for source correlation value of $\rho = 0.95$. We perform Monte Carlo optimization of the parameters Δ, α as well as the matrix $\mathbf{H}$. Fig. 9 shows the resulting performance for the correlated case when $P = 1,\ N_2 = 0.002,\ N_1 = 5N_2$ and source correlation $\rho = 0.95$. The SDR upperbound in Fig. 9 is described by the equations given in section III of [6].

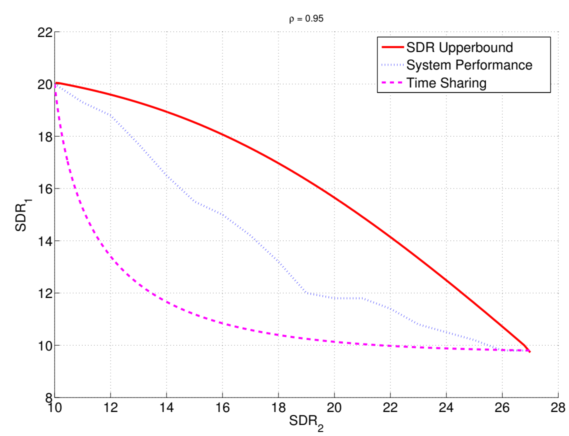

Fig. 9. System performance for $P = 1,\ N_2 = 0.002$ and $N_1 = 5N_2$ with correlation $\rho = 0.95$.

6. CONCLUSION

We have demonstrated a new technique for analog joint source channel coding over the Gaussian Broadcast Channel, extending the framework of Nested Quantization. The proposed technique, AS-SQLC, utilizes a novel space-filling curve that is well suited to the Broadcast Channel. Simulation results show that the resulting performance when independent and correlated Gaussian data is transmitted to each user is close to the theoretical limits for a wide range of distortion pairs.

7. REFERENCES

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