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ROBUST SPECTRUM MANAGEMENT WITH INCOMPLETE INFORMATION

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ABSTRACT

This paper studies the problem of competitive spectrum management in the presence of channel estimation errors. In particular, we study the effect of the channel estimation error on the Bayesian Interference Game (BIG), in which two selfish wireless systems (players) share the same frequency band, where each player knows its own channel gains but does not know the other players channel gains. In the case where the channel is estimated perfectly, the BIG is known to have a spectrally efficient equilibrium point, which produces a higher payoff to both players than the trivial equilibrium, in which both players always interfere with each other. However, the assumption that each player knows its own channel gains impeccably is not practical due to estimation error. The latter leads to payoff perturbations, which can reduce spectral efficiency by driving the spectrally efficient equilibrium point unstable. In this paper, we show that the spectral efficiency is robust to small estimation errors; i.e., the BIGs spectrally efficient equilibrium point preserves its properties in the presence of estimation errors.

1. INTRODUCTION

Consider independent wireless communication systems, which share the same frequency band. These systems create interference, which results in major performance loss that decreases the overall spectrum utilization. Thus, efficient spectrum management is an important proliferate field of research [1–8, e.g.]. An important type of spectrum management is the competitive one where each system shapes its spectrum to maximize its utility, in this paper it is the information rate. Each system's action affects other systems' utility and vice versa. Thus, a natural tool to analyze such interactions is game theory. Particularly useful is the notion of Nash Equilibrium (NE) [9, see e.g.] which describes a stable operating point; i.e., a strategy profile that each player (system) can only lose if it unilaterally deviates from it. For a game where players share a flat fading interference channel with complete information[1], it is known [5] that the Full Spread (FS) strategy, where both players spread their powers equally over the entire band, is a NE point. It is also known [5, 10] that in many cases, the FS NE point is spectrally inefficient. This happens when joint Frequency Division Multiplexing (FDM) is better for both players than mutual FS but the system operates in a mutual FS because the players are subject to the prisoner's dilemma [4, 10].

In practice, however, communication systems operate with some estimation error. The only aspect of this problem that has been studied is where players fail to meet some operating constraints, in particular, interference constraints. In this case, the perturbed game (the game under estimation error) can be seen as a robust game, in which players maximize their own payoff under a constraint, on their strategy profile, which limits the interference. This game was first formulated and analysed [11] for spectrum sharing between cognitive radios which are constrained not to interfere with the primary user [12–16]. A similar problem also appears in DSL [17] for non-competitive spectrum management; i.e., an optimization of a joint utility rather than a game in which each player selfishly optimizes its own utility.

We consider a different aspect of the competitive spectrum management problem under estimation error, in which the error affects players' utility, rather than strategy profile. We consider a two-player game with two types of channel uncertainty. The first is where each player knows its own channel gains imperfectly due to estimation error. This estimation error leads to payoff perturbations; i.e., each player has some uncertainty on its own payoff. The second uncertainty is where each player does not know the opponent's channel gains. Rather, it knows the opponent's channel statistics[2]. In the case of flat fading channels and no estimation error, the above game, known as the Bayesian Interference Game (BIG) [19], has a non pure-FS equilibrium point; that is, an equilibrium point where players may chose FDM, depending on the their channel gains. This non-FS equilibrium point pareto dominates the FS NE point; i.e., an equilibrium point which improves both players' performance with respect to the FS NE point, and therefore, leads to a better spectrum utilization. However, in the case of estimation error, there are payoff perturbations, which may have a devastating effect on equilibrium points and drive the system out of stability; i.e., a player can gain by unilaterally deviating from the equilibrium point. Such a phenomenon makes equilibrium points, which are derived under the assumption of no estimation error, unuseful in practice.

In this paper, we show that the BIG is robust to small estimation errors; that is, it is shown that BIG's equilibrium points which are functions of the true channel gains are still useful when the players use the estimated gains.

The paper is organized as follows. Section 2 presents the system model and reviews the BIG. Section 3 discusses the BIG and its properties under estimation errors. Sections 4 and 5 provide simulations and conclusions, respectively.

2. THE BAYESIAN INTERFERENCE GAME(BIG)

Consider a two-user flat-fading interference channel (see [19] Fig. 1(a)) without interference cancelation; i.e., each user treats the other user's signal as noise. During the channel coherence time, player i's

[1] By complete information, we mean that every user knows all direct and cross channel gains of all users in the network.

[2] This statistics vary slowly compared to the instantaneous channel gains and therefore, can be easily communicated [18].

signal is given by

$$W_i(t) = H_{ii}V_i(t) + H_{ij}V_j(t) + N_i(t), \tag{1}$$

where $i, j \in \{1, 2\},\ i \neq j$, $V_i(t), V_j(t)$ are user i's and j's transmit signals, respectively, $N_i(t)$ is a white Gaussian noise with variance σ_N^2 and $H_{iq}, i, q \in \{1, 2\}$ are random fading-channel gains. Throughout this paper, the index j is never equal to i. Both players have a total power constraint of $\bar{p}$. We denote user i's Signal to Noise Ratio (SNR) and Interference to Noise Ratio (INR) as $X_i = \gamma|H_{ii}|^2$ and $Y_i = \gamma|H_{ij}|^2$, respectively, where $\gamma = \bar{p}/\sigma_N^2$. The realizations (sample points) of X_i, Y_i are denoted by x_i, y_i, respectively.

In the BIG, user i's channel state information (CSI) at the transmitter side is x_i, y_i. It does not observe Y_j and X_j but only knows its distributions. The channel is divided into two equal sub-bands and player 1's and 2's actions are given by

$$\begin{aligned}\mathbf{p}_1(\theta_1) &= \bar{p}[\theta_1,\ 1-\theta_1]^T \\ \mathbf{p}_2(\theta_2) &= \bar{p}[1-\theta_2,\ \theta_2]^T\end{aligned} \tag{2}$$

respectively. The first and second entries of each vector represent the power invested in the first and second sub-bands, respectively (see also [19] Fig. 1(b)), where $\theta_i \in \Theta_i = \{1, 1/2\}$. The actions $\theta_i = 1$ and $\theta_i = 1/2$ correspond to FDM and FS, respectively. This formalism implies that players coordinate in advance to use disjoint subbands in the case of FDM. We assume that during a single coherence period, players manage their spectrum only once, based on their knowledge. Therefore, if the interaction is repeated it will be with different and independent channel realizations. This represents a case where the channel varies fast or a case where simplicity requirements enable a single spectrum shaping every coherence period.

Definition 1 *The Bayesian Interference Game (BIG) is defined by the following:*

1. *Set of players* $\{1, 2\}$.
2. *Action set* $\Theta = \{1, 1/2\}$. *Let* $\theta_i \in \Theta$ *be the action chosen by player* i*, then according to (2),* $\theta_i = 1$ *corresponds to FDM and* $\theta_i = 1/2$ *corresponds to FS.*
3. *A set of positive and independent random variables* $X_1,\ Y_1,\ X_2,\ Y_2$ *whose distributions are common knowledge. Each player* i *observes the realized values of* X_i, Y_i *but does not observe* X_j, Y_j.
4. *A utility function* $u_i(\theta_i, \theta_j, x_i, y_i)$ *given in [19, Table 1].*
5. *A set of pure strategies* $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2$ *where every* $S_i \in \mathcal{S}_i$ *is a function that maps values of* x_i, y_i *to an action in* Θ_i*; i.e.,* $S_i : \mathcal{X}_i \times \mathcal{Y}_i \longrightarrow \Theta_i$*, where* $\mathcal{X}_i = \mathrm{Range}(X_i)$ *and* $\mathcal{Y}_i = \mathrm{Range}(Y_i)$.

Player i's objective is to maximize his conditional expected payoff given his private information x_i, y_i*; i.e.,*

$$\pi_i(S_i, S_j, x_i, y_i) \triangleq \mathrm{E}\left\{u_i(S_i, S_j, X_i, Y_i) | X_i = x_i, Y_i = y_i\right\} \tag{3}$$

3. THE BIG UNDER ESTIMATION ERROR

In [19, Theorem 4] it is shown that the BIG has an ϵ-NE [19, Definition 5] point in which both players have higher payoff than in the FS NE point. An ϵ-NE is defined as follows:

Definition 2 *For* $\epsilon > 0$*, an* ϵ*-NE point is a strategy profile* $(\hat{S}_1, \hat{S}_2)$ *such that*

$$\pi_i\left(\hat{S}_i, \hat{S}_j, x_i, y_i\right) \geq \sup_{S_i \in \mathcal{S}_i} \pi_i\left(S_i, \hat{S}_j, x_i, y_i\right) - \epsilon,\ \forall x_i, y_i \tag{4}$$

The idea behind ϵ-NE points is that if one of the players deviates from it, he can gain no more than ϵ additional payoff. From a practical point of view, for sufficiently small ϵ, ϵ-NE points are as stable as ordinary NE points.

However, if the channels are not estimated perfectly, the estimation error affects the utility $u_i(x_i, y_i, \theta_i, \theta_j)$, which is the mutual information between the received signal W_i and the transmitted signal V_i, assuming Gaussian signaling[3]. The new utility is

$$\grave{u}_i(\hat{x}_i, \hat{y}_i, \theta_i, \theta_j) = I(W_i; V_i | S_i = \theta_i, S_j = \theta_j, \hat{X}_i = \hat{x}_i, \hat{Y}_i = \hat{y}_i) \tag{5}$$

An important question arises: will a small estimation error drive the system out of stability? The following definition extends the well-known notion of robustness of NE points [9, Definition 12.1] to ϵ-NE point.

Definition 3 *An* ϵ*-NE point* $(\hat{s}_1, \hat{s}_2)$ *[19, Definition 5] with payoff* u_1, u_2 *is said to be robust if for every* $\delta > 0$ *there exists* $\eta > 0$ *such that for every* $\grave{u}_1, \grave{u}_2$ *which satisfies* $\max_{\theta_i, \theta_j} |u_i - \grave{u}_i| < \eta, i, j \in \{1, 2\}, i \neq j$*, the point* $(\hat{s}_1, \hat{s}_2)$ *is a* $(\epsilon + \delta)$*-NE in the perturbed game; i.e., the same game with perturbed payoff* $\grave{u}_1, \grave{u}_2$.

In simple words, an ϵ-NE point is robust if small payoff perturbations makes it an $(\epsilon + \delta)$-NE. Because ϵ and δ are very small, the ϵ-NE and the $(\epsilon + \delta)$- are essentially the same. This is an important property since if an equilibrium point moves drastically (that is, the point's strategy profile varies drastically) due to small payoff perturbations it is completely useless in practice, since players could gain significantly by deviating from that original point. In what follows we show that the ϵ-NE point given in [19, Theorem 4] is robust to perturbations resulting from estimation errors. The first step towards this goal is to analyze the estimation error effect on the utility. The following lemma shows that this utility is "continuous" with respect to the channel estimates; i.e., that the perturbed utility converges to the true utility as the estimation error "approaches" (to be defined in the lemma) zero.

Lemma 1 *Assume that the channels gains have finite first and second moments and that the channels are estimated by a sequence of estimates* $\{\hat{H}_{iq}^l\}_{l=1}^{\infty}, 1 \leq i, q \leq 2$*, all defined on the same probability space* $(\Omega, \mathcal{F}, P)$*. Assume further that* $\mathcal{F}_l \subseteq \mathcal{F}_{l+1}$ *where*[4] $\mathcal{F}_l = \sigma\left(\hat{H}_{iq}^l\right)$*,* $\sigma(X) = \{A \in \mathcal{F} : A = X^{-1}(B), B \in \mathcal{B}(\mathbb{R})\}$*, and* $\mathcal{B}(\mathbb{R})$ *is the Borel sigma filed on* $\mathbb{R}$*. Then, for every* γ*, the utility in* (5) *satisfies*

$$\begin{aligned}\grave{u}\left(\hat{X}_i^l, \hat{Y}_i^l, \theta_i, \theta_j\right) - u\left(\hat{X}_i^l, \hat{Y}_i^l, \theta_i, \theta_j\right) \xrightarrow{l \to \infty} 0 \quad a.s., \\ \forall \theta_i, \theta_j \in \{1/2, 1\},\end{aligned} \tag{6}$$

where $\hat{X}_i^l = \gamma|\hat{H}_{ii}^l|^2$ *and* $\hat{Y}_i^l = \gamma|\hat{H}_{ij}^l|^2$*. Furthermore, the result is not limited to a game in which the players are restricted to Gaussian signaling.*

Proof: [20].

[3] See [5], for further discussion of the rational for choosing this utility.

[4] Intuitively, this condition implies that $\hat{H}_{iq}^{l+1}$ exploits the measurements used by its predecessor $\hat{H}_{iq}^l$ and additional measurements.

In the next theorem it is shown that BIG's spectrally efficient equilibrium point is robust to estimation error.

Theorem 2 *Let $\hat{S}_i^l$ be the strategy profile in [19, Theorem 4] with $\hat{X}_i^l, \hat{Y}_i^l$ substituted for X_i, Y_i. Then, under the conditions of Lemma 1, the BIG's non pure-FS ϵ-NE point [19, Theorem 4] is robust to estimation error. That is, for every ϵ, there exists γ_0, such that for every $\gamma > \gamma_0$ there exists L such that for every $l > L$, $(\hat{S}_1^l, \hat{S}_2^l)$ is an ϵ-NE of the perturb BIG; i.e., the BIG but with information $\hat{X}_i^l, \hat{Y}_i^l$ and utility $\grave{u}_i$ instead of X_i, Y_i and u_i, respectively.*

Proof: [20].

Note that the robustness indicated by Theorem 2 is different than Definition 3 in that it is restricted to perturbation due to estimation error.

4. SIMULATION RESULTS

While Theorem 2 shows robustness to estimation errors, it does not indicate how small the error must be. We now address this problem via simulation. Consider the BIG in a flat Rayleigh fading channel, where in every coherence period, user i obtains unbiased estimates $|\hat{H}_{iq}|^2,\ q = 1, 2$. The estimates are then used for spectrum shaping, according to the non-FS ϵ-NE strategy profile in [19, Theorem 4], instead of the perfectly known $|H_{iq}|^2$. We assume that during the estimation phase players coordinate to transmit their training signals in disjoint sub-bands. Thus, each player observes

$$\mathbf{W}_{iq}^t = H_{iq}^t \mathbf{d}_t + \mathbf{N}_t,\ \ t = 1, ..., T,\ q = 1, 2 \qquad (7)$$

where t is the channel coherence-interval index, which consists M time slots. The vector $\mathbf{W}_{ii}^t \in \mathbb{C}^{M\times 1}$ is used for the direct channel estimation and $\mathbf{W}_{ij}^t$ is used for the interference estimation. Also, $\mathbf{d}_t = [d_1^t, ..., d_M^t]^T$ is a known training signal[5] and $\mathbf{N}_t$ is a white circularly Gaussian noise vector with covariance $\sigma_N^2\mathbf{I}$. The channels $H_{iq}^t, q = 1, 2$ distribute as $H_{iq}^t \backsim CN\left(0, \sigma_{iq}^2\right)$.

In the simulation we use the following channel model

$$H_t = \sigma\Phi_t \qquad (8)$$

where $\Phi_t,\ t = 1, ..., T$ are i.i.d. $CN(0, \sigma^2)$ and σ is a deterministic that remains constant for $t = 1, ..., T$ [21].

4.1. Known Channel Statistics

The first simulation studies the BIG in the case where σ is known and the unknown Φ_t (of equivalently H_t) is estimated at each coherence time form $\mathbf{W}_t$ (see (7)). The MMSE estimate of H_t is

$$\hat{H}_t = \frac{\mathbf{d}^H\mathbf{W}_t}{\sigma_N^2\left((\sigma^2)^{-1} + ||\mathbf{d}||^2/\sigma_N^2\right)} \qquad (9)$$

The simulation is carried out as follows. We draw a sample of flat-fading channels with pathloss $\sigma_{11}^2 = \sigma_{22}^2 = 105$ dB, and $ISR_1 = ISR_2 = -2$ dB, where $ISR_i = \sigma_{ij}^2/\sigma_{ii}^2$. The training signal $\mathbf{d}_t$ is chosen as a constant vector of ones. The noise floor is -121 dBm and $\bar{p} = 0$ dBm. Each player estimates its own channel and interference; i.e., player i estimates $\hat{H}_{iq},\ q = 1, 2$. In the presence of estimation error, there is no closed form expression for the BIG payoff. We therefore use the lower bound in Table 1, proven in Appendix **??**, to evaluate the performance gain; i.e., the lower bound on the performance gain is the ratio between the bound in Table 1 to the payoff obtained when both players choose FS without estimation error. The conditional variance used in Table 1 is given by

[5]This assumption is made to simplify the analysis. In general, since the BIG has no interference cancellation, only the interference power is required and thus, players can use energy detector without knowing their opponent's training sequence.

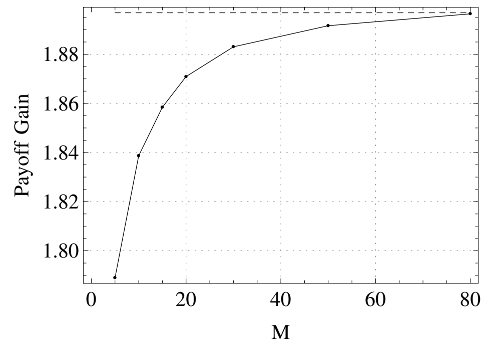

Fig. 1. Payoff gain (ratio), with estimation error (solid lines) and without estimation error (dashed lines), of the ϵ-NE point's payoff with respect to the FS point's payoff as a function of the training size. The results were averaged over 10^4 Monte Carlo trials.

$$\sigma^2_{H_{iq}|\hat{H}_{iq}} = \text{var}\left(H_{iq}|\hat{H}_{iq}\right) = \frac{\sigma_{iq}^2\sigma_n^2}{\|\mathbf{d}\|^2\sigma_{iq}^2 + \sigma_n^2} = \frac{\sigma_{iq}^2}{\sigma_{iq}^2\|\mathbf{d}\|^2/\sigma_n^2 + 1} \qquad (10)$$

In [20] it is shown that the bound in Table 1 approaches the non perturbed payoff [19, Table 1] as the estimation error approaches zero. Figure 1 depicts the payoff gain as a function of the training size M. The result shows that the BIG is robust to estimation errors even for short training length.

4.2. Unknown channel statistics

In the previous simulation we assumed that the channel distribution is perfectly known. In many practical cases, the channel distribution is only known up some to unknown parameters. To demonstrate the problem we return to the channel model (8) $H_t = \sigma\Phi_t$, where now σ, which represents the channel statistic, is a deterministic unknown, in addition to the random unknown $\Phi_t,\ t = 1, ..., T$. This is a hybrid estimation problem; i.e. an, estimation problem where some of the unknown parameters are deterministic while the others are random.

The problem was studied in [21] which suggested estimating σ using the asymptomatically optimal ML estimator [see e.g. 22, Chapter 8.5]

$$\hat{\sigma}^2_{\text{ML}} = \frac{1}{\|\mathbf{d}\|^4}\mathbf{d}^H\left(\mathbf{R} - \mathbf{I}\sigma_v^2\right)\mathbf{d}, \qquad (11)$$

where $\mathbf{R}_t = \frac{1}{t}\sum_{q=1}^t \mathbf{W}_q\mathbf{W}_q^H$, and estimating $\hat{H}_T$ as

$$\hat{H}_T = \frac{\mathbf{d}^H\mathbf{W}_T}{\sigma_n^2\left((\hat{\sigma}^2_{\text{ML}})^{-1} + ||\mathbf{d}||^2/\sigma_n^2\right)} \qquad (12)$$

To justify the use of $\hat{H}_T$, [21] derived the Hybrid Cramèr-Rao lower Bound (HCRB) on the estimation error of H_T

$$\sigma^2_{H,\text{HCRB}} = \frac{1}{\sigma^{-2} + \|\mathbf{d}\|^2/\sigma_n^2} \qquad (13)$$

and showed in simulations that the estimator in (12) achieves the HCRB as $T \longrightarrow \infty$. In the following lemma we show that the HCRB is indeed an asymptotically tight bound and is achieved by

Table 1. Lower bound on user i's payoff $u_i(\hat{X}_i, \hat{Y}_i, \theta_i, \theta_j)$

	$\theta_j = 1$	$\theta_j = 1/2$
$\theta_i = 1$ FDM	$\frac{1}{2}\log_2\left(1 + \frac{\mathrm{E}\left\{X\|\hat{X}_i\right\}}{\sigma^2_{H\|\hat{H}}\gamma+1}\right)$	$\frac{1}{2}\log_2\left(1 + \frac{\mathrm{E}\left\{X\|\hat{X}_i\right\}_i}{1+3\sigma^2_{H\|\hat{H}}\gamma/2+\mathrm{E}\left\{Y\|\hat{Y}_i\right\}/2}\right)$
$\theta_i = \frac{1}{2}$ FS	$\frac{1}{2}\log_2\left(1 + \frac{\mathrm{E}\left\{X\|\hat{X}_i\right\}/2}{\sigma^2_{H\|\hat{H}}\gamma/2+1}\right) + \frac{1}{2}\log_2\left(1 + \frac{\mathrm{E}\left\{X\|\hat{X}_i\right\}/2}{1+3\sigma^2_{H\|\hat{H}}\gamma/2+\mathrm{E}\left\{Y\|\hat{Y}_i\right\}}\right)$	$\log_2\left(1 + \frac{\mathrm{E}\left\{X\|\hat{X}_i\right\}/2}{1+\sigma^2_{H\|\hat{H}}\gamma+\mathrm{E}\left\{Y\|\hat{Y}_i\right\}/2}\right)$

the estimator in (12) as $T, M \longrightarrow \infty$.

Lemma 3 *The estimator $\hat{H}_T$ in* (12) *satisfies*

$$\frac{(\hat{H}_T - H)}{\sigma_{H,HCRB}} \xrightarrow{d} CN\,(0,1) \tag{14}$$

as $M, T \longrightarrow \infty$, where $\xrightarrow{d}$ denotes convergence in distribution.

proof: [20].

Now that we have established that the estimator in (12) is asymptotically optimal, we repeat the previous simulation (Sec. 4) in the case where the channel statistic is also unknown. The difference is that in this simulation each player estimates the channel gains (12), which requires estimating the channel statistics, via (11); i.e., player i estimates H_{iq}, and σ_{iq} for $q = 1, 2$. Each player chose an action using [19, Theorem 4] where the estimated channels and estimated channel statistics are substituted for the true values. At each Monte Carlo trail, the game is repeated with different channel realizations drawn from an independent and identically distributed random variables. The only thing that is accumulated from stage to stage is the estimation of the channel statistic; i.e., at game t, $\mathbf{R}(t)$ in (11) is calculated as $\mathbf{R}(t) = (t-1)/t\mathbf{R}(t-1) + \mathbf{r}_t\mathbf{r}_t^H$. Fig. 2 depicts the payoff gain for $M = 5$, averaged over 10000 Monte-Carlo trails. It shows that the equilibrium point is still robust even for small values of T.

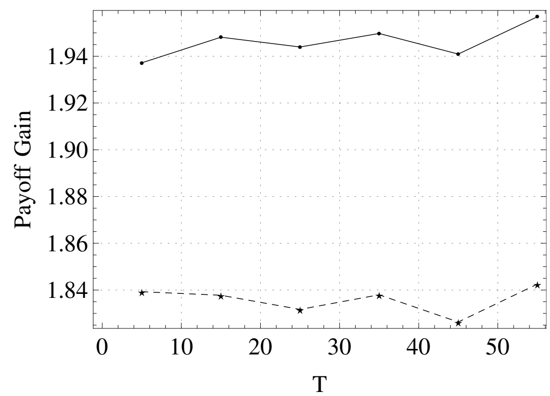

Fig. 2. Payoff gain (ratio) of using the ϵ-NE point's payoff to the FS point's payoff as a function of the number of the channel's coherence periods T.

5. SUMMARY

In this paper we studied the robustness of the competitive spectrum management problem to estimation errors by analysing BIG. In the case where there are no estimation errors, the BIG is known to have a spectrally efficient equilibrium point. We have shown that this spectral efficiency is not affected by small estimation errors.

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