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ROBUST BLIND CALIBRATION VIA TOTAL LEAST SQUARES

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ABSTRACT

This paper considers the problem of blindly calibrating large sensor networks to account for unknown gain and offset in each sensor. Under the assumption that the true signals measured by the sensors lie in a *known* lower dimensional subspace, previous work has shown that blind calibration is possible. In practical scenarios, perfect signal subspace knowledge is difficult to obtain. In this paper, we show that a solution robust to misspecification of the signal subspace can be obtained using total least squares (TLS) estimation. This formulation provides significant performance benefits over the standard least squares approach, as we show. Next, we extend this TLS algorithm for incorporating exact knowledge of a few sensor gains, termed partially-blind total least squares.

***Index Terms*—** Blind calibration, sensor networks, total least squares.

1. INTRODUCTION

Sensor systems are used in various types of monitoring systems, from environmental to manufacturing to surveillance. As sensor manufacturers focus on low-cost sensors that can be deployed in large sensing systems, organizations are beginning to plan deployments of hundreds or more sensors to monitor a phenomenon of interest. A major barrier to the feasibility of such large sensing systems is that of maintenance of data quality. Sensor measurements are known to drift over time, requiring a periodic validation known as calibration. When sensors are deployed on a relatively small scale, each sensor can be calibrated individually by hand using a known stimulus. However, as the number of sensors increases, as well as the area over which the sensors are deployed, visiting each sensor individually becomes infeasible. In such cases, one would like to be able to calibrate the sensors using only routine measurements – a process known as *blind calibration*.

Recent results on blind calibration employ a variety of methods [2, 3, 4, 5]. In [2], the authors provide a method of detecting sensor faults and correcting for sensor drift using spatial kriging and Kalman filtering. The methods in [3, 4] assume dense deployments of sensors. In [3], a collaborative approach is used, where neighboring sensors exchange information to reach a global consensus on the true signal value; all sensors are assumed to be measuring a spatially stationary signal. A similar approach is taken in [4]; the signal is assumed to have negligible differences over space and time, and the authors use expectation-maximization to compensate for noise and gossip-based approaches to calibrate the network. In this paper, we wish to move away from the uniform phenomenon model. Most similar to our formulation is work in calibration for compressed sensing measurements [5], where the true signal is only assumed to be sparse in a known basis. This work also uses an optimization framework for calibration, but has no theoretical guarantees and is only for measurements that are a compressed linear combination of true signal values.

We build on [1], in which the authors proved that blind calibration is possible when the signals of interest lie in a subspace of the Euclidean space (i.e., the true signals are correlated). Using the modeled subspace, the gains can be found through a simple minimization using either the singular value decomposition (SVD) or least squares (LS) methods. Using the estimated gains, the offsets can be at least partially recovered. In the case where the number of known offsets is at least the rank of the signal subspace, it was shown that the offsets can be fully recovered. These results hold in the case where the subspace is perfectly known, an assumption which is difficult to achieve in practice. Thus, solutions that are robust to subspace error are a topic of interest if these methods are to be employed in practical scenarios.

This paper involves two contributions. First, we show that the SVD solution from [1] is equivalent to solving a total least squares (TLS) problem, which solves systems of equations when there is misspecification in the measurement matrix. This reformulation clarifies why the SVD solution in [1] is robust to measurement noise as well as errors in the subspace model. Next, we describe a new algorithm, termed partially-blind total least squares (PB-TLS), which incorporates knowledge of the gains at several sensors to provide improved estimation performance. Simulation results show that the TLS estimator performs significantly better under model mismatch.

2. PROBLEM FORMULATION & PREVIOUS WORK

Suppose we observe signals from n sensors, and these sensors measure the phenomenon of interest with an unknown gain and offset. Let the measured signal vector be $\mathbf{y} \in \mathbb{R}^n$, and let $\mathbf{Y} = \mathrm{diag}(\mathbf{y})$. The true signal measured by the sensors can then be written as

$$\mathbf{x} = \mathbf{Y}\alpha + \beta, \tag{1}$$

where $\alpha \in \mathbb{R}^n$ and $\beta \in \mathbb{R}^n$ are the gain and offset vectors, respectively, the i^{th} component of each vector corresponding to the i^{th} sensor. Assuming the true signals $\mathbf{x}$ lie in a known low-dimensional subspace, it was shown in [1] that blind calibration is possible. Let $\mathcal{S}$ be the be the rank-r subspace where the signals lie and $\mathbf{P}$ be the orthogonal projection matrix onto the orthogonal complement of $\mathcal{S}$ such that $\mathbf{P}\mathbf{x} = 0$ for all $\mathbf{x} \in \mathcal{S}$. For k snapshots taken at discrete time points, this results in the following system of kn equations

$$\mathbf{P}(\mathbf{Y}_i\alpha + \beta) = 0,\ i = 1, \ldots, k, \tag{2}$$

where $\mathbf{Y}_i = \mathrm{diag}(\mathbf{y}_i)$ denotes the i^{th} snapshot, and at most $k(n-r)$ equations are independent. It can be seen that $\mathbf{P}\beta = -\mathbf{P}\bar{\mathbf{Y}}\alpha$, where $\bar{\mathbf{Y}} = \frac{1}{k}\sum_{i=1}^{k}\mathbf{Y}_i$. Thus, (2) can be rewritten as

$$\mathbf{P}(\mathbf{Y}_i - \bar{\mathbf{Y}})\alpha = 0,\ i = 1, \ldots, k. \tag{3}$$

For a given gain estimate $\hat{\alpha}$, the offset can be estimated as $\hat{\beta} = -\bar{\mathbf{Y}}\hat{\alpha}$. One important observation to draw from this estimate of β is that the component of the offsets in the signal subspace cannot be identified without further information.

Define the matrix

$$\mathbf{C}_0 = \begin{bmatrix} \mathbf{P}(\mathbf{Y}_1 - \bar{\mathbf{Y}}) \\ \vdots \\ \mathbf{P}(\mathbf{Y}_k - \bar{\mathbf{Y}}) \end{bmatrix}. \tag{4}$$

In the case where there is no measurement noise and the signal subspace is perfectly known, [1] shows that when $k \geq r$, $\mathbf{C}$ has rank $n-1$; therefore, the calibration gains can be found as the non-trivial solution to the system

$$\mathbf{C}_0\alpha = 0. \tag{5}$$

Note that the α can only be recovered up to a scaling factor, and therefore we must at least have further information if we wish to recover α exactly, for example knowing a single calibration gain would suffice. However, this represents a small cost in comparison to manual calibration of all sensors.

In [1] it was suggested to formulate Equation (5) as an optimization problem in order to make the solution more robust to noise. In order to avoid the trivial solution $\alpha = 0$, we must impose additional constraints, such as constraining the norm of α as follows:

$$\begin{aligned} \hat{\alpha} = \arg\min_{\alpha} \quad & \|\mathbf{C}_0\alpha\|_2^2 \\ \text{subject to} \quad & \|\alpha\|_2^2 = 1, \end{aligned} \tag{6}$$

The solution of this problem is the right singular vector of $\mathbf{C}_0$ associated with the smallest singular value.

Another optimization approach that avoids the trivial solution would be to assume a single calibration gain is known. We assume $\alpha(1) = 1$ without loss of generality. Denote the columns of $\mathbf{C}_0$ by $\mathbf{c}_1, \ldots, \mathbf{c}_n$ and let $\mathbf{b}_0 = -\mathbf{c}_1 \in \mathbb{R}^{kn\times 1}$ and $\mathbf{A}_0 = [\mathbf{c}_2 \ldots \mathbf{c}_n] \in \mathbb{R}^{kn\times(n-1)}$. Then the problem can also be solved via the least squares minimization

$$\tilde{\alpha}_{LS} = \arg\min_{\mathbf{x}} \|\mathbf{A}_0\mathbf{x} - \mathbf{b}_0\|_2^2, \tag{7}$$

where $\tilde{\alpha}_{LS} = [\alpha(2) \ldots \alpha(n)]^T$. This minimization may be solved using QR decomposition or any other least squares solution techniques.

One benefit of this least-squares approach is that when the gains at multiple sensors are known, the extra knowledge can be incorporated into this formulation as follows. Let $\tilde{\mathbf{b}}_0$ be the sum of $-\alpha(i)\mathbf{c}_i$ for the known sensor gains, and let $\tilde{\mathbf{A}}_0$ be the matrix of columns corresponding to unknown sensor gains. Then we may solve the following optimization, which we refer to as partially-blind least squares (PB-LS).

$$\tilde{\alpha}_{PB-LS} = \arg\min_{\mathbf{x}} \|\tilde{\mathbf{A}}_0\mathbf{x} - \tilde{\mathbf{b}}_0\|_2^2. \tag{8}$$

The simulation results from [1] show that as the number of known calibration gains increases, the PB-LS method results in lower estimation error, as expected.

3. BLIND CALIBRATION WITH MODEL MISMATCH

In practical scenarios, the subspace $\mathcal{S}$ is not perfectly known, resulting in model error in $\mathbf{P}$, denoted by $\Delta\mathbf{P}$. Define

$$\Delta\mathbf{C} = \begin{bmatrix} \Delta\mathbf{P}(\mathbf{Y}_1 - \bar{\mathbf{Y}}) \\ \vdots \\ \Delta\mathbf{P}(\mathbf{Y}_k - \bar{\mathbf{Y}}) \end{bmatrix}. \tag{9}$$

Incorporating this model error into Equation (4), we see that the resulting matrix from the sensor readings is $\mathbf{C} = \mathbf{C}_0 + \Delta\mathbf{C}$. Clearly this model error will affect all optimization problems described in the previous section, (6), (7), (8), through $\mathbf{C}_0$ as well as $\mathbf{A}_0$ and $\mathbf{b}_0$. Therefore we may reformulate the optimization problems by considering this perturbation.

Let $\Delta\mathbf{A}$ and $\Delta\mathbf{b}$ denote the errors in $\mathbf{A}$ and $\mathbf{b}$ respectively, resulting from a perturbation in $\mathbf{P}$. Define $\mathbf{A} = \mathbf{A}_0 + \Delta\mathbf{A}$ and $\mathbf{b} = \mathbf{b}_0 + \Delta\mathbf{b}$. The LS solution is founded on the assumption that there are errors in $\mathbf{b}$, but that $\mathbf{A}$ is perfectly known. A solution to (7) that is robust to errors in both $\mathbf{A}$ and $\mathbf{b}$ exists and is known as the total least squares (TLS) solution [6, 7].

The solution to the TLS problem can be found as follows. Let $\mathbf{A} = [\mathbf{a}_1\ \ldots\ \mathbf{a}_{n-1}]$ where $\mathbf{a}_i$ is the i^{th} column of $\mathbf{A}$. Form the matrix

$$[\mathbf{A}|\mathbf{b}] = [\mathbf{a}_1\ \ldots\ \mathbf{a}_{n-1}\ \mathbf{b}]\ ,$$

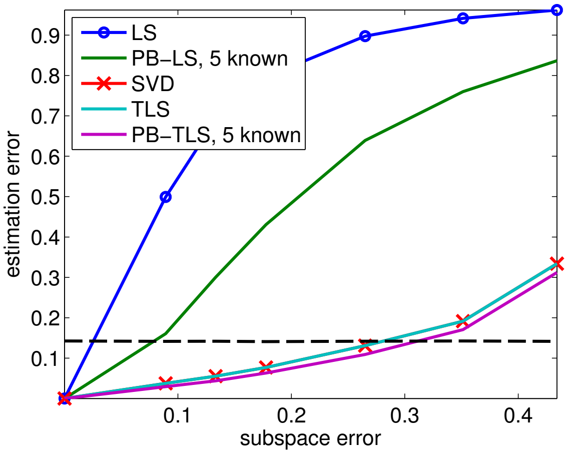


Fig. 1. Error in gain estimation as a function of subspace error, where $n = 100$ and $r = 20$. The dashed line represents the uncalibrated error.

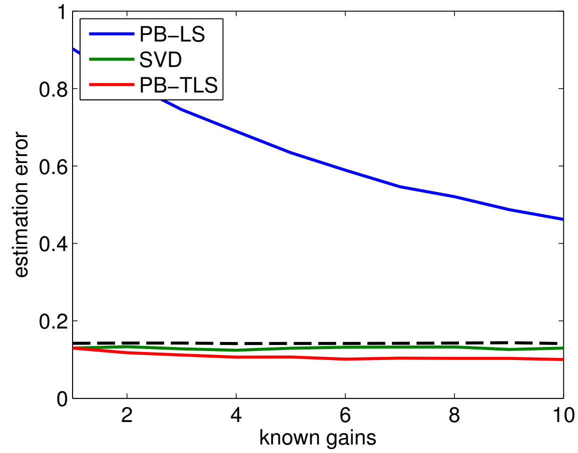


Fig. 2. Error in gain estimation as a function of the number of known gains, where $n = 100$ and $r = 20$. The dashed line represents the uncalibrated error.

and let $\mathbf{U\Sigma V}^T$ be the SVD of $[\mathbf{A}|\mathbf{b}]$. Let $\mathbf{v}_1$ be the first $n-1$ elements of the last column of $\mathbf{V}$, and let v_2 be the last element of the same column.

The optimal TLS solution then seeks an $\mathbf{x}$ such that $\mathbf{Ax} \approx \mathbf{b}$ when there is error in both $\mathbf{A}$ and $\mathbf{b}$. The optimal solution is achieved by [6, 7]

$$\breve{\alpha} = -\frac{\mathbf{v}_1}{v_2}, \tag{10}$$

where $\breve{\alpha}$ is an estimate of $[\alpha(2)\ldots\alpha(n)]^T$.

We may also incorporate a diagonal weighting matrix $\mathbf{D}$ if not all columns of $[\mathbf{A}|\mathbf{b}]$ are equally perturbed. Now let $\mathbf{U\Sigma V}^T$ be the SVD of $[\mathbf{A}|\mathbf{b}]\mathbf{D}$, and let $\mathbf{D}_1$ be submatrix of the first $(n-1)\times(n-1)$ elements of $\mathbf{D}$, and d_2 be the last or (n,n) element of $\mathbf{D}$. Then the optimal solution to this weighted total least squares problem is given by [6, 7]

$$\tilde{\alpha}_{TLS} = -\frac{\mathbf{D}_1\mathbf{v}_1}{d_2 v_2}. \tag{11}$$

This is the TLS estimator we use in this paper.

It is straightforward to see that (11) is equivalent to solving (6) when $\mathbf{D}$ is the identity matrix and $\alpha(1) = 1$. Therefore, it is natural to expect the optimization as described using SVD to outperform the LS optimization under subspace modeling errors.

One interesting fact is that in the case of perfect subspace knowledge but noisy measurements $\mathbf{y}$, both $\mathbf{A}$ and $\mathbf{b}$ will contain errors. As a result, it is natural to assume that the TLS solution outperforms LS regardless of the error type–a fact that is observed through the simulation results in [1].

The TLS solution given above (and likewise, the SVD solution from [1]) assumes that a single sensor gain is known. Using the reformulation described for TLS, it is also possible to incorporate the knowledge of several sensor gains. We refer to this process as partially-blind total least squares. Similar to the case of (8), let $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{b}}$ be the perturbed versions of $\tilde{\mathbf{A}}_0$ and $\tilde{\mathbf{b}}_0$. Define $\tilde{\mathbf{U}}\tilde{\mathbf{S}}\tilde{\mathbf{V}}^T = [\tilde{\mathbf{A}}|\tilde{\mathbf{b}}]\mathbf{D}$. Then the unknown gains can be found as

$$\tilde{\alpha}_{PB-TLS} = -\frac{\mathbf{D}_1\tilde{\mathbf{v}}_1}{d_2\tilde{v}_2}, \tag{12}$$

where $\tilde{\mathbf{v}}_1$, and $\tilde{v}_2$ are the respective partitions of $\tilde{\mathbf{v}}$.

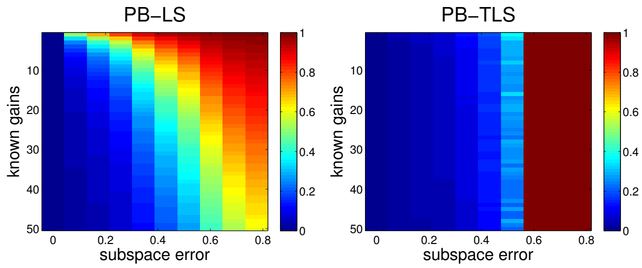
The TLS solution provides a consistent estimator when the errors in $\mathbf{A}$ and $\mathbf{b}$ are independent and identically distributed (*iid*), regardless of the distribution [7]. For blind calibration, we assume the $\mathbf{Y}_i$ are independent across snapshots. In the case where the errors $\Delta\mathbf{P}$ are *iid* and independent of $\mathbf{Y}_i$, $i = 1, \ldots, k$, it is easy to see that the values of $\Delta\mathbf{C}$ are uncorrelated, though not necessarily independent. However, under some special cases, these errors may be *iid*, in which case the TLS estimator will be consistent. A consistent estimator for blind calibration is a topic for future exploration. In the case of PB-TLS, the errors in $\tilde{\mathbf{b}}$ will be a summation of the errors in the columns of $\tilde{\mathbf{A}}$, resulting in non-uniform errors in the matrix $[\tilde{\mathbf{A}}|\tilde{\mathbf{b}}]$. This issue can be overcome by using $\mathbf{D}$ to scale each column of the matrix.

4. SIMULATIONS

To demonstrate the performance characteristics of the TLS and PB-TLS methods, we perform blind calibration on simulated data. We use the relative error metric defined as

$$\epsilon_\alpha = \frac{\|\alpha - \hat{\alpha}\|_2}{\|\alpha\|_2}. \tag{13}$$

The relative error is averaged over 100 Monte Carlo simulations for all figures except fig. 3, where 50 simulations were



(a) (b)

Fig. 3. Estimation error as a function of increasing number of known sensor gains and subspace error for 3(a) PB-LS and 3(b) PB-TLS methods. Parameters are $n = 100$, $r = 20$.

performed to reduce computation time. The number of snapshots used is $k = \lceil 3rlog(n) \rceil$, where $\lceil \gamma \rceil$ denotes the smallest integer not less than γ. The offsets are distributed uniformly on $[-0.5, 0.5]$. The gains are distributed uniformly on $[0.75, 1.25]$, where we have set $\alpha(1) = 1$. For the TLS and PB-TLS estimates, the weight matrix is chosen so that the last column of the matrix $[\mathbf{A}|\mathbf{b}]\mathbf{D}$ (resp. $[\tilde{\mathbf{A}}|\tilde{\mathbf{b}}]\mathbf{D}$) is scaled by a factor of $1/\sqrt{\sum_{i=1}^{p} \alpha(p)}$, where p is the number of known gains. This scaling is chosen so that the noise variance is uniform across columns.

Consider a randomly generated subspace from which we draw random samples. Subspace error is induced by adding random Gaussian perturbations to the actual subspace with zero-mean and increasing variance. The error metric is defined as

$$\epsilon_{\mathcal{S}} = \frac{1}{k} \sum_{i=1}^{k} \frac{\|\mathbf{P}\mathbf{x}_i\|_2}{\|\mathbf{x}_i\|_2}. \tag{14}$$

Let the number of sensors $n = 100$ and the rank of the subspace $r = 20$. Fig. 1 shows the relative error as a function of subspace error for each of the described methods, where partial-blind methods use 5 known gains. The dashed line represents the uncalibrated case where it is assumed that α is a vector of ones. Note that in this case, the LS estimate and PB-LS with one known gain yield the same performance. Similarly, SVD and TLS result in the same performance, confirming the previously stated result that SVD minimization of (6) is equivalent to performing TLS minimization. The figure shows that the TLS solutions are robust to subspace error as predicted. Moreover, by calibrating a few sensors, the extra knowledge can be incorporated to further reduce estimation error, as seen by the PB-TLS performance.

Fig. 2 shows the estimation error as a function of increasing number of known gains with an average subspace error of 0.27. Due to the subspace error, the PB-LS method performs poorly even when 20 gains are known. SVD/TLS performs slightly better than the uncalibrated case, with an average estimation error of 0.13. With 5 known calibration gains, PB-TLS results in an estimation error of 0.11, while

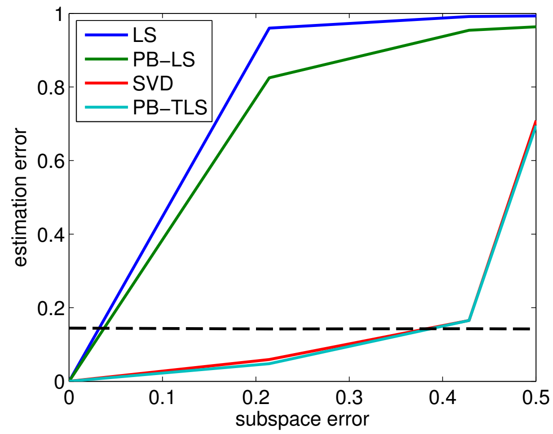


Fig. 4. Error in gain estimation as a function of subspace error, where $n = 500$ and $r = 40$. Partial-blind methods use 5 known gains. The dashed line represents uncalibrated error.

with 10 known gains the error is reduced to 0.1 – an improvement of approximately 23% over the case of a single known gain. Thus, PB-TLS successfully incorporates the additional knowledge while also being robust to subspace errors.

Fig. 3 shows plots of the estimation error (ranging from 0 to 1) as a function of both the subspace error and the number of known gains for both the PB-LS and PB-TLS methods. An important feature to note is that PB-TLS performs significantly better for a subspace error up to about 0.5. After this point, there is an apparent phase transition, where PB-TLS performs poorly. Characterizing this phase transition is a topic of our future research.

As a final demonstration of the robustness of TLS-based methods, we perform blind calibration on a system with $n = 500$ sensors and an underlying subspace with rank $r = 40$. Fig. 4 shows the resulting error, where the partial-blind methods use 5 known gains. The figure shows that TLS-based blind calibration outperforms uncalibrated gains when the subspace error is as high as 0.38, while LS-based methods fail under subspace error of less then 0.05. This suggests that when n is large – the regime where blind calibration is most necessary – TLS methods are considerably more robust than standard LS, even for relatively large subspace errors.

5. CONCLUSION

We have demonstrated a robust method for blind calibration of sensor networks using total least squares minimization. In the case where a single sensor gain is known, we have shown that the TLS minimization is equivalent to the SVD solution from [1]. To incorporate knowledge of multiple sensor gains, we have presented the PB-TLS method. Simulation results demonstrate the performance improvement of the proposed method over the PB-LS minimization.

6. REFERENCES

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