RSS-BASED LOCALIZATION IN NON-HOMOGENEOUS ENVIRONMENTS

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ABSTRACT

In this paper, we deal with the problem of RSS-based self-localization of a wireless blind node using a statistical path loss model for the measurements. The considered environment is non-homogeneous, i.e., the attenuation factors of the various links are different. We propose a two-stage procedure: the first stage exploits measurements between anchors to estimate transmitted powers and attenuation factors. Then, a ML localization algorithm, fed by the measurements at the blind node only, is used to estimate the unknown position. In this second stage, the attenuation factors between the blind node and the anchors are modeled as IID RVs ruled by a Gaussian distribution with mean and variance to be computed based on the estimated attenuation factors of the first stage. The performance assessment shows that the proposed approach could be a viable means to handle localization in non-homogeneous environments.

***Index Terms*—** Received signal strength (RSS), localization, maximum likelihood (ML) estimation.

1. INTRODUCTION

With the widespread of telecommunication systems, RF communication signals from different sources and technologies are found in almost every environment of daily life and can be exploited for localization purposes. Several localization approaches have been proposed over the years [1, 2, 3, 4, 5, 6]. Techniques based on the received signal strength (RSS) are the preferred option when simplicity, low cost, and technology obliviousness are the main requirements. In addition, RSS is readily available from any radio interface through a simple energy detector and can be modeled by the well-known path loss model (PLM) [7] regardless of the particular communication scheme. Based on that, RSS can be exploited to implement "opportunistic" localization for different wireless technologies, like e.g. Wi-Fi [8], cellular networks [9, 10], DECT [11] or even FM [12]. In some standards, like e.g. IEEE 802.15.4, a RSS indicator (RSSI) is encoded directly into the protocol stack, hence localization features can be implemented with a minimum additional cost [13].

Localization approaches can be grouped in two main categories: range-based and range-free. Range-based are simpler, but suboptimal, techniques which first measure distances from known locations (typically called anchors or beacons) and then combine these data to estimate the unknown position. The drawback is that the accuracy is usually limited, especially when the standard lateration algorithm is adopted [14], and also some bias issues arise in the ranging phase [15]. Range-free techniques, conversely, aim at directly estimating the target position, e.g. via maximum likelihood (ML) estimation [14, 16]: they typically outperform range-based techniques in terms of accuracy at the price of an additional computational complexity. In both cases a complication is that the RF propagation must be considered, i.e., the channel characteristics vary due to multipath (fading) and non-negligible modifications occur also due to mid-to-long term changes in the environment, leading to non-stationary channel parameters [17]. Although many existing works assume that the latter are known or can be measured off-line, in real applications the problem of positioning implicitly requires that the sensing of the environment, i.e., channel estimation (commonly called "calibration") must be performed continuously [14, 17, 18, 19] with no human assistance. A viable solution is to exploit nodes in known positions (anchors) to derive estimation procedures (self-calibration) with reduced complexity [14, 17, 18, 20].

A few papers have considered the possibility that the transmission powers from each node are not known, due either to lack of information about the sources (cognitive approach [21]) or to the uncertainty on the height and orientation of the node, tolerances in the transmitter components, losses in RF connectors, and power supply voltage variations [22]. To the best of our knowledge, however, no previous work has addressed the problem of RSS-based localization in a fully non-homogenous environment where all parameters (i.e., transmission powers and attenuation factors) are unknown. We propose a two-stage procedure: the first stage exploits measurements between anchors to estimate transmitted powers and attenuation factors (so generalizing the approach in [14, 17, 18]). Then, a ML localization algorithm,

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fed by the measurements at the blind node only, is proposed. In this second stage, the attenuation factors between the blind node and the anchors are modeled as independent and identically-distributed (IID) random variables (RVs) ruled by a Gaussian distribution with mean and variance to be computed based on the attenuation factors estimated at the first stage (namely, corresponding to links between anchors). Simulation results show that the proposed approach outperforms a conventional ML algorithm derived assuming a homogeneous environment.

The paper is organized as follows: next section is devoted to the problem formulation while Section 3 contains the design of the proposed localizer; Section 4 is devoted to the performance assessment while Section 5 contains some concluding remarks.

2. PROBLEM FORMULATION

Assume N nodes moving over known trajectories (anchors). Nodes average the instantaneous power for each received signal, over sufficiently short time intervals (corresponding to an average over a few wavelengths), in order to filter out the rapid variations of the received power due to multipath. We assume though that the trajectories of the anchors are of limited extension so that the attenuation factor of each link can be considered approximately constant over the overall observation time interval. As a consequence, using a statistical path loss law with lognormal shadowing [7], the kth measurement of the average power collected from the jth anchor and transmitted by the ith one, $P_{i,j}(k)$ say, is given by

$$P_{i,j}(k) = P_{0,i} - 10\alpha_{i,j}\log_{10}\|\boldsymbol{r}_i(k) - \boldsymbol{r}_j(k)\| + w_{i,j}(k) \qquad k = 1,\ldots,K \quad (1)$$

where $P_{0,i}$ is the power transmitted by the ith anchor and received at the reference distance of 1 m, $\alpha_{i,j}$ is the path loss exponent for the the link between the ith and the jth node (typical values in between 2 and 4), $\|\cdot\|$ denotes the norm of a vector, $\boldsymbol{r}_i(k)$ and $\boldsymbol{r}_j(k)$ denote the average position of nodes i and j (according to a given Cartesian reference system) over the (short) time interval used to compute the kth measurement of the average received power, $w_{i,j}(k)$ the corresponding shadowing term, and K the number of measurements made between anchors. Blind nodes with unknown position might also be present. Obviously, the kth measurement of the power received from a blind node can be modeled as

$$P_{i,b}(k) = P_{0,i} - 10\alpha_{i,b}\log_{10}\|\boldsymbol{r}_i(k) - \boldsymbol{r}_b\| + w_{i,b}(k) \qquad k = 1,\ldots,K \quad (2)$$

where now the pair (i,b) indicates the link from anchor i to the blind node b, and $\boldsymbol{r}_b$ the unknown position of the blind node. Based on the $P_{i,j}(k)$s and possibly the $P_{i,b}(k)$s several estimation problems can be conceived. In particular, we propose and assess a two-stage procedure to localize the blind node. As a first stage, the power values $P_{0,i}$s and the $\alpha_{i,j}$s are estimated from the $P_{i,j}(k)$s. As a second stage, a ML estimator of the blind node position $\boldsymbol{r}_b$ is derived; it relies on the assumption that the attenuation factors $\alpha_{i,b}$ are IID RVs ruled by a Gaussian model whose parameters are the sample mean and the sample variance of the estimated $\alpha_{i,j}$s.

3. ALGORITHM DESIGN

Starting from (1) we let

$$\boldsymbol{y}(k) = [P_{1,2}(k)\ \cdots\ P_{1,N}(k)\cdots \cdots\ P_{N,1}(k)\ \cdots\ P_{N,N-1}(k)]^T \in \mathbb{R}^{N(N-1)\times 1}$$

and $\boldsymbol{y} = [\boldsymbol{y}^T(1)\cdots\boldsymbol{y}^T(K)]^T \in \mathbb{R}^{KN(N-1)\times 1}$, with ^{T} denoting transpose. Moreover, we let $\alpha_{i,j} = \alpha_{j,i}$ and define the following vectors of unknowns $\boldsymbol{p} = [P_{0,1}\cdots P_{0,N}]^T \in \mathbb{R}^{N\times 1}$ and

$$\boldsymbol{a} = [\alpha_{1,2}\ \cdots\ \alpha_{1,N}\ \alpha_{2,3}\ \cdots\ \alpha_{2,N}\ \alpha_{3,4}\cdots \cdots\ \alpha_{3,N}\ \cdots\ \alpha_{N-1,N}]^T \in \mathbb{R}^{\frac{N(N-1)}{2}\times 1}.$$

It follows that $\boldsymbol{y}$ can be represented in terms of a linear model, namely as

$$\boldsymbol{y} = \boldsymbol{H}_1\boldsymbol{p} + \boldsymbol{H}_2\boldsymbol{a} + \boldsymbol{w} = \boldsymbol{Q}\boldsymbol{x} + \boldsymbol{w} \quad (3)$$

where $\boldsymbol{x} = [\boldsymbol{p}^T\ \boldsymbol{a}^T]^T$, $\boldsymbol{Q} = [\boldsymbol{H}_1\ \boldsymbol{H}_2]$,

$$\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{1}_{N-1} & & \boldsymbol{0} \\ \vdots & \cdots & \vdots \\ \boldsymbol{0} & & \boldsymbol{1}_{N-1} \\ \vdots & \cdots & \vdots \\ \boldsymbol{1}_{N-1} & & \boldsymbol{0} \\ \vdots & \cdots & \vdots \\ \boldsymbol{0} & & \boldsymbol{1}_{N-1} \end{bmatrix} \in \mathbb{R}^{KN(N-1)\times N}$$

$$\boldsymbol{H}_2 = \begin{bmatrix} \boldsymbol{H}_2(1) \\ \vdots \\ \boldsymbol{H}_2(K) \end{bmatrix} \in \mathbb{R}^{KN(N-1)\times\frac{N(N-1)}{2}}$$

with $\boldsymbol{H}_2(k) \in \mathbb{R}^{N(N-1)\times\frac{N(N-1)}{2}}$ a matrix in which each column contains two non-zero terms. As to $\boldsymbol{1}_n \in \mathbb{R}^{n\times 1}$, it is an n-dimensional column vector of ones. Finally, we have

$$\boldsymbol{w}(k) = [w_{1,2}(k)\cdots w_{1,N}(k)\cdots \cdots w_{N,1}(k)\cdots w_{N,N-1}(k)]^T \in \mathbb{R}^{N(N-1)\times 1}$$

and, hence, $\boldsymbol{w} = [\boldsymbol{w}^T(1)\cdots\boldsymbol{w}^T(K)]^T \in \mathbb{R}^{KN(N-1)\times 1}$.

Assuming that $\boldsymbol{w} \sim \mathcal{N}\left(\boldsymbol{0}, \sigma^2\boldsymbol{I}_{KN(N-1)}\right)$ and modeling $\boldsymbol{x}$ and σ^2 as deterministic unknown parameters, it follows that the ML estimators (MLEs) of $\boldsymbol{x}$ and σ^2 can be obtained as

$$\hat{\boldsymbol{x}} = \begin{bmatrix} \hat{\boldsymbol{p}} \\ \hat{\boldsymbol{a}} \end{bmatrix} = \arg\min_{\boldsymbol{x}}\|\boldsymbol{y} - \boldsymbol{Q}\boldsymbol{x}\|^2 = \left(\boldsymbol{Q}^T\boldsymbol{Q}\right)^{-1}\boldsymbol{Q}^T\boldsymbol{y} \quad (4)$$

$$\hat{\sigma}^2 = \frac{1}{KN(N-1)} \boldsymbol{y}^T \boldsymbol{P}_{\boldsymbol{Q}}^{\perp} \boldsymbol{y} \tag{5}$$

where $\boldsymbol{P}_{\boldsymbol{Q}}^{\perp}$ is the projector onto the orthogonal complement of the space spanned by the columns of $\boldsymbol{Q}$. Observe that we have implicitly assumed that matrix $\boldsymbol{Q}$ is full-column-rank; this is generally true if anchors are moving and provided that $K > 1$ and $N \geq 2$. Conversely, with stationary anchors, matrix $\boldsymbol{Q}$ becomes rank deficient.

The localization of a blind node exploits the estimated values $\hat{\boldsymbol{x}}$ and $\hat{\sigma}^2$, given by (4) and (5), respectively, together with the KN measurements of the power received from the blind node. More precisely, we assume that the $P_{0,i}$s are known and model the set of the $\alpha_{i,b}$s and the $w_{i,b}(k)$s as an overall set of independent Gaussian RVs; in addition, we suppose that $\alpha_{i,b} \sim \mathcal{N}\left(\mu_\alpha, \sigma_\alpha^2\right)$ with μ_α and σ_α^2 estimated from $\hat{\boldsymbol{a}}$ as follows

$$\hat{\mu}_\alpha = \frac{2}{N(N-1)} \sum_{i,j} \hat{\alpha}_{i,j}$$

and

$$\hat{\sigma}_\alpha^2 = \frac{2}{N(N-1)} \sum_{i,j} \left(\hat{\alpha}_{i,j} - \hat{\mu}_\alpha\right)^2$$

and $w_{i,b}(k) \sim \mathcal{N}\left(0, \sigma^2\right)$ with σ^2 replaced with (5). Thus, the vector of observations at the blind node can be written as

$$\boldsymbol{y}_b = \boldsymbol{H}_{1b}\boldsymbol{p} + \boldsymbol{H}_{2b}\boldsymbol{a}_b + \boldsymbol{w}_b \tag{6}$$

where $\boldsymbol{y}_b = [\boldsymbol{y}_b^T(1) \cdots \boldsymbol{y}_b^T(K)]^T \in \mathbb{R}^{KN\times 1}$, with $\boldsymbol{y}_b(k) = [P_{1,b}(k) \ \cdots \ P_{N,b}(k)]^T \in \mathbb{R}^{N\times 1}$,

$$\boldsymbol{H}_{1b} = \begin{bmatrix} \mathbf{1}_N & & \mathbf{0} \\ \vdots & \cdots & \vdots \\ \mathbf{0} & & \mathbf{1}_N \\ \vdots & \cdots & \vdots \\ \mathbf{1}_N & & \mathbf{0} \\ \vdots & \cdots & \vdots \\ \mathbf{0} & & \mathbf{1}_N \end{bmatrix} \in \mathbb{R}^{KN\times N}$$

$$\boldsymbol{H}_{2b} = \begin{bmatrix} \boldsymbol{H}_{2b}(1) \\ \vdots \\ \boldsymbol{H}_{2b}(K) \end{bmatrix} \in \mathbb{R}^{KN\times N}$$

$\boldsymbol{H}_{2b}(k)$ being an $N \times N$ diagonal matrix whose diagonal elements are $-10\log_{10}\|\boldsymbol{r}_i(k) - \boldsymbol{r}_b\|$, $i = 1,\ldots,N$, $\boldsymbol{a}_b = [\alpha_{1,b} \cdots \alpha_{N,b}]^T \in \mathbb{R}^{N\times 1}$, $\boldsymbol{w}_b = [\boldsymbol{w}_b^T(1) \cdots \boldsymbol{w}_b^T(K)]^T \in \mathbb{R}^{KN\times 1}$, and $\boldsymbol{w}_b(k) = [w_{1,b}(k) \cdots w_{N,b}(k)]^T \in \mathbb{R}^{N\times 1}$. It is also a simple matter to show that vector $\boldsymbol{y}_b$ is a normal random vector with mean

$$E[\boldsymbol{y}_b] = \boldsymbol{H}_{1b}\boldsymbol{p} + \mu_\alpha \boldsymbol{H}_{2b}\mathbf{1}_N \triangleq \boldsymbol{\mu} \tag{7}$$

and covariance matrix $\boldsymbol{R} \in \mathbb{R}^{KN\times KN}$ whose elements can be determined according to

$$\begin{aligned} &E\left[(P_{i,b}(k) - E[P_{i,b}(k)])\,(P_{j,b}(h) - E[P_{j,b}(h)])\right] \\ &\quad = \begin{cases} 0, & i \neq j \\ \sigma_\alpha^2 B_{i,b}(k) B_{i,b}(h), & i = j, k \neq h \\ \sigma^2 + \sigma_\alpha^2 B_{i,b}^2(k), & i = j, k = h \end{cases} \end{aligned} \tag{8}$$

where $B_{i,b}(k) = -10\log_{10}\|\boldsymbol{r}_i(k) - \boldsymbol{r}_b\|$, $i = 1,\ldots,N$, $k = 1,\ldots,K$. Finally, the MLE of the position of the blind node can be obtained as (remember that the unknown values of $\boldsymbol{p}$, μ_α, σ_α^2, and σ^2 are replaced with the available estimates)

$$\begin{aligned} \hat{\boldsymbol{r}}_b = \arg\max_{\boldsymbol{r}_b} \Bigg\{ & \frac{1}{(2\pi)^{KN/2}\det(\boldsymbol{R})^{1/2}} \\ & \times \exp\left[-\frac{1}{2}\left(\boldsymbol{y}_b - \boldsymbol{\mu}\right)^T \boldsymbol{R}^{-1}\left(\boldsymbol{y}_b - \boldsymbol{\mu}\right)\right]\Bigg\}. \end{aligned} \tag{9}$$

Observe now that the cost function is highly nonlinear in the the variable $\boldsymbol{r}_b$ since it is included into both $\boldsymbol{\mu}$ and $\boldsymbol{R}$ (through (7) and (8), respectively); we thus resort to numerical optimization techniques to solve the problem. The corresponding localization algorithm will be referred to in the following as non-homogeneous ML (NH-ML) localizer.

4. PERFORMANCE ASSESSMENT

In this section, we use the standard Monte Carlo simulation to evaluate the performance of the proposed algorithm, also in comparison to a more conventional approach designed for a homogeneous environment. Such a competitor algorithm assumes, at the design stage, that all links share a common and unknown attenuation factor α (homogeneous environment). It first estimates (through the maximum likelihood technique) α and $\boldsymbol{p}$ using the measurements between anchors; subsequently, it uses such estimates to perform the localization of the blind node (again through a maximum likelihood technique), such an algorithm will be referred to in the following as homogeneous ML (H-ML) localizer.

The simulated scenario is the following. In a square coverage area of 100 m$\times$100 m there are N anchors. Each anchor moves on a circular trajectory of radius R (whose center is independent of the Monte Carlo run) with a constant angular velocity such that it makes a complete revolution during the time required to collect K observations. The initial angle for each anchor is chosen at random uniformly in $[0, 2\pi]$. One stationary blind node is also present in the coverage area and its position is a RV uniformly distributed in the area. The path loss exponents $\alpha_{i,j}$s and $\alpha_{i,b}$s are independent RVs uniformly distributed in $[2, 4]$, the powers $P_{0,i}$s are RVs uniformly distributed in $[-30, 0]$ dBm, and the noise samples $w_{i,j}(k)$s and $w_{i,b}(k)$s are normally distributed with zero mean and variance $\sigma^2 = 36$. It is worth highlighting that the simulated scenario does not fit the design assumption that the attenuation

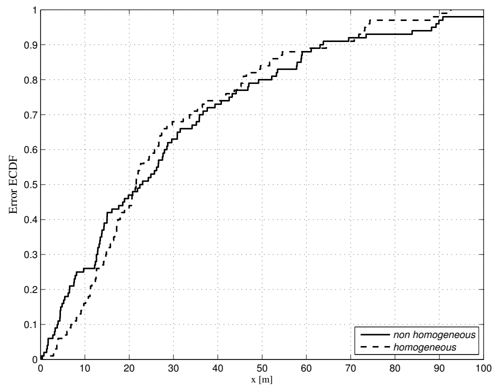

Fig. 1. ECDF of the localization error, $N = 7$, $R = 1$ m.

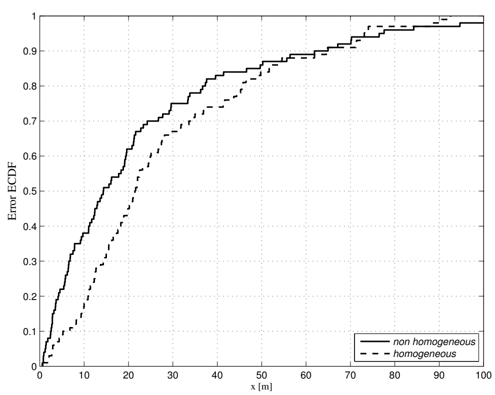

Fig. 2. ECDF of the localization error, $N = 7$, $R = 2$ m.

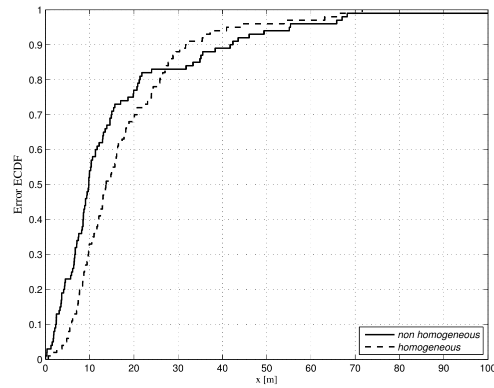

Fig. 3. ECDF of the localization error, $N = 10$, $R = 1$ m.

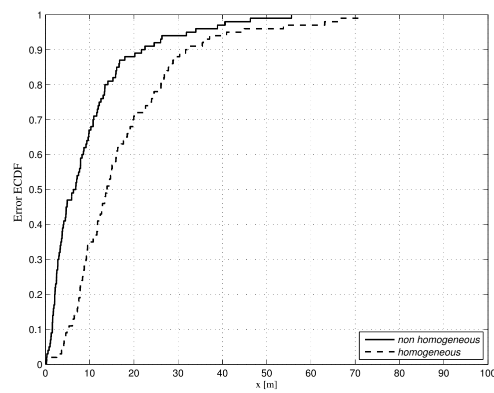

Fig. 4. ECDF of the localization error, $N = 10$, $R = 2$ m.

factors are normally distributed. Figs. 1-4 show the empirical cumulative distribution function (ECDF) of the localization error, i.e., an estimate of $P\{\|\hat{\boldsymbol{r}}_b - \boldsymbol{r}_b\| \leqslant x\}$. Moreover, we set $K = 50$ and use 100 iterations to compute the ECDF. In Fig. 1 we plot the ECDFs of the localization error as functions of x (in meters) for $N = 7$ and $R = 1$ m. As we can see, both the H-ML and NH-ML localizers provide basically the same performance. From Fig. 2 we can observe that an increase of the radius to $R = 2$ m produces a better performance for the NH-ML algorithm. Figs. 3 and 4 show the results for $N = 10$ and radius $R = 1$ m and $R = 2$ m, respectively. From these figures we see that the NH-ML outperforms the H-ML even for small values of R; again, as R increases the performance gap between NH-ML and H-ML increases; this can be easily understood by recalling that matrix $\boldsymbol{Q}$ tends to become ill-conditioned when anchors tends to be stationary. In particular, for $N = 10$ and $R = 2$ m (Fig. 4), in half of the cases, the localization error of the NH-ML algorithm is well below 10 m while that of the H-ML one is twice larger.

5. CONCLUSIONS

In this paper, we have dealt with the problem of RSS-based self-localization of a blind node based upon the statistical path loss model and assuming a non-homogeneous scenario. The attenuation factors of the anchor-to-blind links have been modeled as random quantities with unknown distributional parameters. Within this framework, a ML localization algorithm has been designed, where the unknown channel parameters are firstly estimated resorting to a set of anchor-to-anchor measurements. Some simulation studies have been presented to illustrate the performance of the proposed approach. The analysis indicates that the Bayesian framework could be a viable means to deal with non-homogeneous environments.

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