A HYBRID DATA ASSOCIATION MODEL FOR EFFICIENT MULTI-TARGET MAXIMUM LIKELIHOOD ESTIMATION

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ABSTRACT

A key challenge in multi-target tracking is that the number of possible measurement-to-target associations grows exponentially with the number of targets. The popular PMHT approach bypasses this problem by using an arguably wrong assignment model that, however, allows evaluating the likelihood function with complexity linear both in numbers of targets and of measurements. Unfortunately, the resulting tracking quality may suffer due the wrong assignment model. In this paper, we propose a hybrid data association model that combines both the PMHT and original models. In this vein, the likelihood function can be evaluated efficiently in polynomial time while still providing tracking results close to the exact (but, in large scale cases, intractable) solution resulting from the original "correct" model. The feasibility of the new hybrid assignment model is demonstrated by means of maximum likelihood estimation of closely-spaced targets. Extension to marginalized probability calculation – that is, the joint probabilistic data association filter (JPDAF) [1] is in [2].

Index Terms— Data association, PMHT, JPDAF, Maximum Likelihood

1. INTRODUCTION

Multi-target tracking deals with the problem of estimating the states of several targets based on sensor measurements, where the association of measurements to targets is unknown [1]. For many sensors such as radar or sonar, the most realistic association model assumes that each target may give rise to at most one measurement per scan (in the following called the 1-2-1 model). Unfortunately, the number of feasible association hypotheses under this model grows exponentially with the number of targets, so that fast suboptimal methods are required. For example, in the context of the *Joint Probabilistic Data Association Filter (JPDAF)* [1], several fast ad-hoc formulas have been developed for calculating marginal association probabilities [3, 4, 5]. In [6], targets are decoupled by treating targets as clutter, and the *Nearest Neighbor Filter* [7] works with the most probable hypotheses, which comes with a significant loss of optimality.

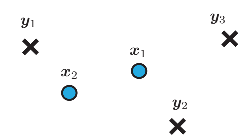

Fig. 1: Example: Measurements $\boldsymbol{y}_1, \boldsymbol{y}_2, \boldsymbol{y}_3$ and targets $\boldsymbol{x}_1, \boldsymbol{x}_2$.

A different line of research abandons the intractable 1-2-1 model in favor of a (intentionally wrong) so-called many-2-1 model [8] that models the measurement origins as independent discrete random variables. As the resulting likelihood function can be efficiently evaluated as a product, the many-2-1 model became a popular alternative to the 1-2-1 model. For example, the many-2-1 model is used within the *Probabilistic Multi-Hypothesis Tracker (PMHT)* [9] and particle filters in [10, 11]. In general, the modeling mismatch resulting from assuming the many-2-1 model can often be neglected. For example, [12] shows that similar results are obtained for both models when performing maximum likelihood estimation of a single target. Nevertheless, with an increasing number of closely-spaced targets the modeling mismatch more and more impacts the estimation quality.

This paper develops a novel assignment model that is closer to the 1-2-1 model than the many-2-1 model, while still preserving a non-exponential complexity. This so-called hybrid model systematically combines 1-2-1 assignments with many-2-1 assignments. The hybrid model is parameterized by a natural number n_s. If n_s equals the number of targets, the 1-2-1 model is obtained and for $n_s = 0$, it becomes the many-2-1 model. We show that for $n_s > 0$, the novel model provides a more accurate approximation to the (intractable) 1-2-1 model than the many-2-1 model. Further, for fixed n_s, the likelihood function can be evaluated in polynomial time with respect to the number of measurements and targets. All told, in contrast to existing (often heuristic) approximations to the 1-2-1 model, the hybrid model can be seen as a systematic approximation that is guaranteed to be

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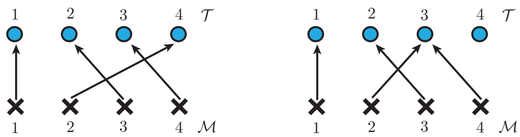
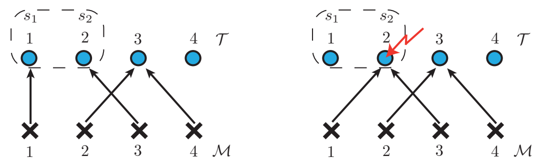


(a) 1-2-1 Model. (b) Many-2-1 Model.

Fig. 2: Assignment events (for the special case $m = n$): In the 1-2-1 model, each measurement is assigned to exactly one target. The many-2-1 model allows several measurements to be assigned to a single target (see target 3).



(a) Valid assignment. (b) Non-valid assignment.

Fig. 3: Hybrid measurement model for given s_1 and s_2, i.e., $n_s = 2$. Exactly one measurement has to be assigned to s_1 and s_2 (note that we assume detection probability of 1).

"lower bounded" by the many-2-1 model.

2. PROBLEM DESCRIPTION

The individual target state vectors (see Fig. 1) are denoted as $\boldsymbol{x}_{1,k}, \ldots, \boldsymbol{x}_{n,k}$, where k is the time and n the number of targets. The stacked state vector of all individual targets is defined as

$$\boldsymbol{x}_k = \left[\boldsymbol{x}_{1,k}^T, \ldots, \boldsymbol{x}_{n,k}^T\right]^T \tag{1}$$

and the set of target indices is defined as $\mathcal{T} = \{1, \ldots, n\}$. For the sake of simplicity, we assume a deterministic motion model for the states, i.e., $\boldsymbol{x}_{k+1} = F_k(\boldsymbol{x}_k)$, where F_k is the system function. At each time k, m measurements

$$\boldsymbol{y}_k = \left[\boldsymbol{y}_{1,k}^T, \ldots, \boldsymbol{y}_{m,k}^T\right]^T$$

with index set $\mathcal{M} = \{1, \ldots, m\}$ are available. The (unknown) measurement-to-target assignment

$$\boldsymbol{a}_k := [a_{1,k}, \ldots, a_{m,k}]^T \in \mathcal{T}_0^m \tag{2}$$

assigns each measurement $\boldsymbol{y}_{i,k}$ to a target $\boldsymbol{x}_{a_i,k}$, where $\mathcal{T}_0 := \mathcal{T} \cup \{0\}$ and $a_{i,k} = 0$ means that the measurement is a false measurement not originating from any target. For a given measurement-to-target assignment, the likelihood function for the stacked state is written as

$$p(\boldsymbol{y}_k \mid \boldsymbol{x}_k,\, \boldsymbol{a}_k) = \prod_{i \in \mathcal{M}} p(\boldsymbol{y}_{i,k} \mid \boldsymbol{x}_{a_i,k}) \ , \tag{3}$$

where $p(\boldsymbol{y}_{i,k} \mid \boldsymbol{x}_{a_i,k})$ is the likelihood for the $\boldsymbol{x}_{a_i,k}$ given that measurement $\boldsymbol{y}_{i,k}$ stems from target a_i.

The final likelihood function results from marginalizing over all association events

$$p(\boldsymbol{y}_k \mid \boldsymbol{x}_k) = \sum_{\boldsymbol{a}_k \in \mathcal{T}_0^M} p(\boldsymbol{y}_k \mid \boldsymbol{x}_k,\, \boldsymbol{a}_k) \cdot p(\boldsymbol{a}_k) \ , \tag{4}$$

where $p(\boldsymbol{a}_k)$ denotes the a priori probability of an association.

In the following sections, we omit the time index k as the focus lies on the measurement model.

3. USUAL ASSIGNMENT MODELS

3.1. 1-2-1 Assignment Model

The 1-2-1 model [1] assumes that each target may produce at most one single measurement per time k (see Fig. 2a).

In this paper, we make the pedagogical assumption that each target is assigned to exactly one measurement, i.e., the probability of detection is 1 (see Remark 1 on how to incorporate non-unity detection). False measurements may occur and a diffuse prior is used for their number. Under this assumption, $m \geq n$ and each target is assigned to exactly one measurement. In total, there are $\binom{m}{n} \cdot n!$ feasible assignments. If each assignment is a priori equally probable, the prior association probability in (4) becomes

$$p^{121}(\boldsymbol{a}) = \tfrac{1}{\binom{m}{n} \cdot n!} \tag{5}$$

for a valid assignment $\boldsymbol{a}$ and otherwise $p^{121}(\boldsymbol{a}) = 0$. Based upon this 1-2-1 assignment model, one has to enumerate an exponentially increasing number of association hypotheses in order to calculate the likelihood function (4).

3.2. Many-2-1 Assignment Model

The many-2-1 model [9] assumes that the association variable specifying the origin of a given measurement is independent of all others', i.e., the a priori assignment probability can be written as

$$p^{\mathrm{M21}}(\boldsymbol{a}) = \prod_{i \in \mathcal{M}} p^{\mathrm{M21}}(a_i) \ . \tag{6}$$

Due to this independence assumption, several measurements may be assigned to the same target (see Fig. 2b). However, this independence assumption also allows us to rewrite (4) as

$$p(\boldsymbol{y} \mid \boldsymbol{x}) = \prod_{i \in \mathcal{M}} \sum_{l \in \mathcal{T}_0} p(\boldsymbol{y}_i \mid \boldsymbol{x}_l) \cdot p^{\mathrm{M21}}(\boldsymbol{a}_i = l) \ , \tag{7}$$

which can be evaluated in linear time depending on the measurements and targets, i.e., $O(mn)$.

4. HYBRID ASSIGNMENT MODEL

In the following, we introduce an assignment model that does not require the enumeration of an exponential number of hypotheses, but that is still closer to the 1-2-1 model than the many-2-1 model.

First, assume that a specific many-2-1 model, a 1-2-1 model, and a parameter n_s with $0 \le n_s \le n$ is given. Then, we require that the 1-2-1 model holds for n_s targets, where these n_s targets are selected uniformly from the set of targets. For the remaining targets, the many-2-1 model is used, i.e., multiple assignments are allowed (see Fig. 3).

More formally, the selection of the n_s targets is modeled with a random vector

$$\boldsymbol{s} = [s_1, \ldots, s_{n_s}]^T \in \mathcal{T}^{n_s} \quad \text{, where} \tag{8}$$

$$p(\boldsymbol{s}) = \frac{1}{\binom{n}{n_s}} \tag{9}$$

if $s_l < s_{l+1}$ for all $1 \le l < n_s$, and else $p(\boldsymbol{s}) = 0$. In this manner, each subset of n_s targets is equally probable. Each target $\boldsymbol{x}_{s_i}$ is assigned to exactly one measurement m_i, i.e., we define the target-to-measurement assignment

$$\boldsymbol{m} = [m_1, \ldots, m_{n_s}]^T \quad \text{, with} \tag{10}$$

$$p(\boldsymbol{m}) = \frac{1}{\binom{m}{n_s} \cdot n_s!} \tag{11}$$

if all $m_i \in \mathcal{M}$ are distinct, and else $p(\boldsymbol{m}) = 0$. In the following, we use the abbreviations $\mathcal{T}^s := \{s_1, \ldots, s_{n_s}\}$ and $\mathcal{M}^s := \{m_1, \ldots, m_{n_s}\}$. Then, for the remaining targets $\mathcal{T} \setminus \mathcal{T}^s$ and measurements $\mathcal{M} \setminus \mathcal{M}^s$, the many-2-1 model is imposed, so that

$$p(\boldsymbol{a} \mid \boldsymbol{m}, \boldsymbol{s}) := \prod_{i \in \mathcal{M} \setminus \mathcal{M}^s} p^{\mathrm{M21}}(a_i \mid a_i \in \mathcal{T} \setminus \mathcal{T}^s) \tag{12}$$

if $a_{m_j} = s_j$ for all $1 \le j \le n_s$ and else $p^{\mathrm{M21}}(\boldsymbol{a} \mid \boldsymbol{m}, \boldsymbol{s}) = 0$.

Marginalizing out $\boldsymbol{m}$ and $\boldsymbol{s}$ gives the prior association probabilities, however, here we first calculate the likelihood given $\boldsymbol{s}, \boldsymbol{m}$ and then marginalize. In this manner, (4) becomes

$$p(\boldsymbol{y} \mid \boldsymbol{x}) = \sum_{\boldsymbol{s}} \sum_{\boldsymbol{m}} p(\boldsymbol{m}, \boldsymbol{s}) \prod_{j=1}^{n_s} p(y_{m_j} \mid x_{s_j}) \cdot \prod_{i \in \mathcal{M} \setminus \mathcal{M}^s} \left[\sum_{l \in \mathcal{T}_0 \setminus \mathcal{T}^s} p(\boldsymbol{y}_i \mid \boldsymbol{x}_l) \cdot p^{\mathrm{M21}}(\boldsymbol{a}_i = l) \right] \quad . \tag{13}$$

Remark 1. Detection probabilities in the 1-2-1 part of the hybrid model can be incorporated in (10) by allowing targets to be undetected and changing the a priori probabilities according to the detection probabilities. However, from a practical point of view, there is no need to explicitly consider detection probabilities as non-detections are captured by the many-2-1 part of the hybrid model.

Remark 2. Implicit to (13), via the summation over $\boldsymbol{s}$, is the summation over the selection of all sets of n_s measurements over which the 1-2-1 assignment model holds, see Table 1.

meas 1	1	1	1	1	2	2	3	3	2	2	3	3
meas 2	2	2	3	3	1	1	1	1	2	3	2	3
meas 3	2	3	2	3	2	3	2	3	1	1	1	1
meas 1	2	2	2	2	1	1	3	3	1	1	3	3
meas 2	1	1	3	3	2	2	2	2	1	3	1	3
meas 3	1	3	1	3	1	3	1	3	2	2	2	2
meas 1	3	3	3	3	1	1	2	2	1	1	2	2
meas 2	1	1	2	2	3	3	3	3	1	2	1	2
meas 3	1	2	1	2	1	2	1	2	3	3	3	3

Table 1: Example hybrid assignments for the case of 3 targets and measurements, and $n_s = 1$. The top table indicates the targets to which the measurements are assigned for $\boldsymbol{s} = [1]$, middle for $\boldsymbol{s} = [2]$ and lower for $\boldsymbol{s} = [3]$.

4.1. Special Cases

Obviously, for $n_s = n$, the 1-2-1 model is obtained and for $n_s = 0$, the many-2-1 model is obtained.

4.2. Computational Complexity

In order to compute (13) one has to consider all $\binom{n}{n_s}$ subsets $\mathcal{T}^s$ with cardinality n_s and all $\binom{m}{n_s} \cdot n_s!$ possible 1-2-1 assignments to measurements. In fact the many-2-1 part in the sum can be calculated in constant time once the full many-2-1 likelihood has been pre-calculated and the sums are enumerated in a proper order. As a consequence, the time complexity for (13) is

$$O\left(\binom{n}{n_s} \cdot \binom{m}{n_s} \cdot n_s! \right) \quad . \tag{14}$$

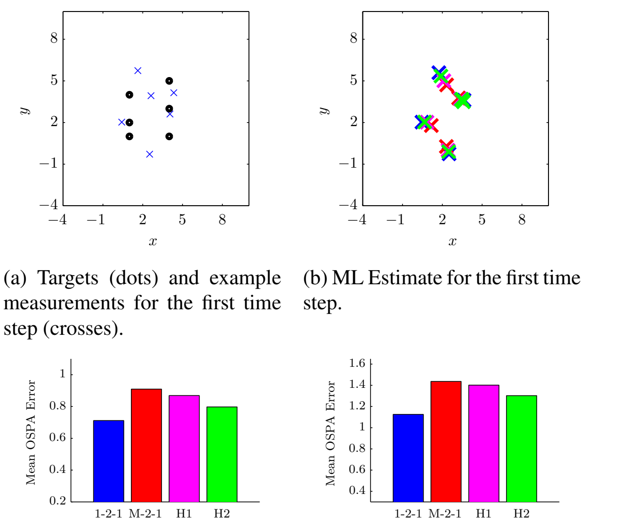
For example, for $n_s = 1$, the complexity is the same as for the plain many-2-1 model, i.e., $O(mn)$. For $n_s = 2$, the complexity is $O(m^2 n^2)$.

4.3. Approximation Quality

In the following, we show that the hybrid association model is a better approximation for the 1-2-1 model than the many-2-1 model in case $n_s > 0$ and the approximation quality increases with increasing n_s. We assume that in the many-2-1 model $p^{\mathrm{M21}}(a_i) = \frac{1}{n+1}$ for $i \in \mathcal{T}_0$. Let $u(\boldsymbol{a})$ denote the number of targets in the assignment $\boldsymbol{a}$ that are assigned to exactly one measurement. Then, for the hybrid model, the probability of an assignment is

$$\begin{aligned} p^H(\boldsymbol{a}) &= \sum_{\boldsymbol{m}} \sum_{\boldsymbol{s}} p(\boldsymbol{a} \mid \boldsymbol{m}, \boldsymbol{s}) \cdot p(\boldsymbol{m}, \boldsymbol{s}) \\ &= \frac{\binom{u(\boldsymbol{a})}{n_s}}{\binom{n}{n_s}} \cdot \frac{1}{\binom{m}{n_s} \cdot n_s!} \cdot \frac{1}{(n-s+1)^{(m-s)}} \quad , \end{aligned} \tag{15}$$

where $\binom{u(\boldsymbol{a})}{n_s} := 0$ for $u(\boldsymbol{a}) < n_s$. Hence, $p^H(\boldsymbol{a}) \sim \binom{u(\boldsymbol{a})}{n_s}$, which means that assignments with more "unique" targets



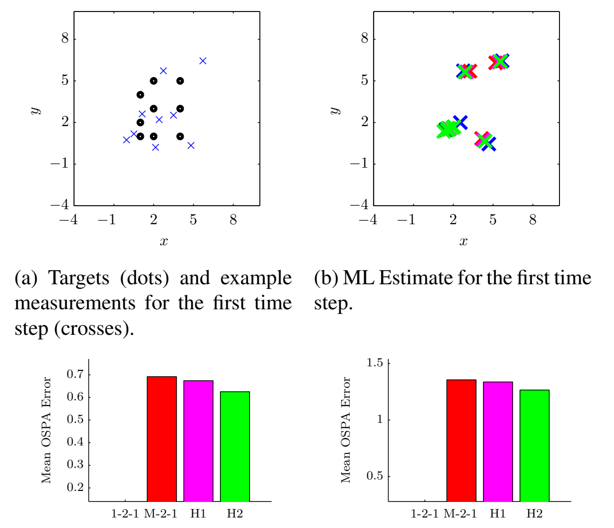
(a) Targets (dots) and example measurements for the first time step (crosses).

(b) ML Estimate for the first time step.

(c) MOSPA: Medium noise, 5th time step.

(d) MOSPA: High noise, 5th time step.

Fig. 4: Evaluation with 6 targets: 1-2-1 model (blue), many-2-1 model (red), hybrid model with $n_s = 1$ (magenta), and hybrid model with $n_s = 2$ (green).



(a) Targets (dots) and example measurements for the first time step (crosses).

(b) ML Estimate for the first time step.

(c) MOSPA: Medium noise, 5th time step.

(d) MOSPA: High noise, 5th time step.

Fig. 5: Evaluation with 9 targets: many-2-1 model (red), hybrid model with $n_s = 1$ (magenta), and hybrid model with $n_s = 2$ (green).

(meaning $u(\boldsymbol{a})$ is greater) are more probable, and the larger n_s becomes the more this probability increases. In the plain many-2-1 model, however, the probability of a 1-2-1 assignment is the same as for non 1-2-1 assignments, i.e., $p^{\text{M21}}(\boldsymbol{a}) = \frac{1}{(n+1)^m}$.

Note that a similar argument holds also for many-2-1 assignments with unequal association probabilities.

5. EVALUATION

The hybrid model is evaluated by means of maximum likelihood estimation of multiple targets similar to [12]. For the sake of simplicity, the targets are stationary, i.e., $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k$. The state of an individual target consists of the two-dimensional target position, i.e., $\boldsymbol{x}_{i,k} \in \mathbb{R}^2$. A sensor supplies noisy measurements of the target positions, i.e., $\boldsymbol{y}_{i,k} = \boldsymbol{x}_{a_i,k} + \boldsymbol{v}_{i,k}$, where $\boldsymbol{v}_{i,k}$ is zero-mean Gaussian noise. We consider two different covariance matrices for $\boldsymbol{v}_{i,k}$, i.e., medium noise $\mathbf{C}^v = \text{diag}(0.5, 0.5)$ and high noise $\mathbf{C}^v = \text{diag}(2, 2)$. The number of clutter measurements per scan is Poisson distributed with mean $\lambda = 0.5$.

We consider a scenario with 6 targets (see Fig. 4a) and a scenario with 9 targets (see Fig. 5a). For the purpose of illustration, the maximum likelihood estimates for the 1-2-1 model, the many-2-1 model, and the hybrid model are depicted (Fig. 4b and Fig. 5b) for the first time step $k = 1$ with the measurements given in Fig. 4a and Fig. 5a. It can be seen that the hybrid model yields estimates whose quality is between the 1-2-1 model and the many-2-1 model.

Fig. 4 also shows the estimation results after 5 time steps, where the overall likelihood function has been maximized. Since data and targets are unlabeled – and hence there is no natural MSE scoring – the *Optimal Sub-Pattern Assignment metric (OSPA)* distance [13] is used to compare performances (averaged over 20 runs). The simulations show that for the 6 target scenario, the 1-2-1 model is approximately 20% better than the many-2-1 model. With $n_s = 2$, the hybrid model with $n_s = 2$ is around 10% better than the many-2-1 model. For the 9 target scenario, the hybrid model with $n_s = 2$ is around $8 - 9\%$ better than the many-2-1 model.

The 1-2-1 model is not evaluated in the 9 target case as it takes an unreasonable time. For example, if there are 12 measurements, the complexity for the 1-2-1 model is $\binom{12}{9} \cdot 9! = 79833600$. For the many-2-1 and hybrid model with $n_s = 1$, the complexity is $9 \cdot 12 = 108$. For the hybrid model with $n_s = 2$, the time complexity is $\binom{12}{2} \cdot \binom{9}{2} \cdot 2! = 4752$.

In summary, the 1-2-1 model gives the best results as it is the correct model. However, the hybrid model allows for a significant faster calculation of the likelihood (for both 6 and 9 targets), while still providing better results than the many-2-1 model.

6. CONCLUSIONS AND FUTURE WORK

We have presented a hybrid data association model that serves for deriving systematic approximation to the original intractable 1-2-1 model, where the approximation quality is guaranteed not worse than assuming a many-2-1 model.

In the future, we will use this model within particle filters; for marginal associations in the JPDAF see [2].

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