ORDER STATISTICS BASED CDF SCHEDULING METHODS IN MULTIUSER HETEROGENEOUS SYSTEMS

Anh H. Nguyen, Yichao Huang, and Bhaskar D. Rao,*

University of California San Diego and Qualcomm Corporate R&D*.

ABSTRACT

In modern heterogeneous wireless networks, the task of supporting fairness along with user priorities and concurrently achieving the highest possible system throughput is desirable and challenging. Herein, a class of practical cumulative distribution function (CDF) scheduling algorithms are developed to achieve these goals. These algorithms are used when the channel fading model is unknown. The mapping from channel quality information (CQI) to the real CDF is unknown but is constructed exploiting the order statistics of the CQI sequence. The constructed CDF mapping methods are shown to converge to the actual CDF. Specifically, one algorithm uses the expected value of the ordered CDF scheduling while others called Non-parametric CDF scheduling (NPCS) algorithms reconstructs the CDF with an extra interpolation step. By collecting a moderate number of CQI data, the algorithms almost achieve the system throughput of CDF scheduling as if the CDF is known. Throughout the work, CDF scheduling algorithms, supported by simulations, are shown to be able to effectively support fairness and frequently outperform, and are potential alternatives to, the well known Proportional Fair (PF) scheduling method.

Index Terms— CDF scheduling, order statistics, feedback, multiuser, proportional fair.

1. INTRODUCTION

In wireless communications, exploiting multiuser diversity is critical to improving the overall system performance [1]. In general, mobile users can be at any location in a wireless network service area which leads to a diversity in pathloss, fading condition and channel statistics. The base station (BS) needs, on one hand to exploit multiuser diversity to maximize spectrum usage efficiency and on the other hand guarantees fairness and services to all the users to ensure that all the users are adequately served. Typically, these objectives conflict and so a compromise between system throughput and fairness has to be made. Among the existing scheduling methods, proportional fair scheduling (PF) [2] is widely used because it has a good balance between performance, fairness and simplicity. However, the resource allocation for users in PF is not easy to control and might need adjustments overtime to satisfy users's requirements in a long term manner. Also, even though it exploits multiuser diversity, the efficacy of the approach is unclear in an heterogeneous environment. This calls for better algorithms and we consider cumulative distribution function (CDF) based scheduling in this context. CDF scheduling, which is called CS in [3], can control resource allocation precisely and can exploit multiuser diversity effectively in a heterogeneous environment. However, its comparative performance is unknown and the general approach has received much less attention. As a result, there are no or limited practical implementations of CDF scheduling [4]. A principal challenge in implementation is that the scheduling method requires knowledge of the CDF of the CQI for each user which is typically unknown, changing frequently and location dependent. Motivated by the favorable properties of CDF scheduling, we develop non-parametric CDF scheduling methods to make CDF scheduling practical and along the way show its advantages over the widely used PF scheduling.

Literature review: Because guaranteeing services for all users is of utmost importance in wireless communications systems, various fair scheduling methods have been developed. Overview of scheduling algorithms are discussed in [5–7]. More specifically, commonly used fairness algorithms can be listed as temporal fairness [8], game theory based fairness [9], utilitarian fairness [10–12], Max-Min fairness [13, 14] which maximizes the minimum among rates of the users, round robin in [15] and max rate scheduling which are special cases of proportional fairness (PF) in [16–18]. Fairness can also be supported by using utility functions in [19], minimizing potential delay as in [5]. Though many algorithms have been developed, PF scheduling [2] is widely used because it offers a good trade-off between exploiting multiuser diversity [20–23] and maintaining fairness among users. Characteristics of the algorithm such as convergence behavior, instability as well as its applicability have also been studied in [24–26]. In general, PF is hard to analyze and the approach is unable to exactly control resource allocation for users. Furthermore, its effectiveness in a heterogeneous environment also leaves room for improvement.

A viable alternative is the CDF scheduling proposed in [3]. It is shown to be able to control precisely the probability of resource allocation for users, exploit multiuser diversity and deal with heterogeneous environments. The CDF scheduling is leveraged in a general multicell network in [27], in a partial feedback wideband relay system in [28] and in a random beamforming framework in [29]. All these works assume knowledge of the CDF of the channel, which is an idealistic and simplifying assumption. Acknowledging that the CDF is in general unknown and hard to learn [3], a nice practical scheme is proposed in [4] which can be applied to systems which support discrete rates. However, there are unanswered questions with respect to the optimality of the approach and also the convergence behavior, an important consideration in channel with short coherence time. Despite these concerns, the method developed in [4] is very interesting, motivational, and an important step in making CDF scheduling practical. Our approach is to examine CDF scheduling without being constrained by the finite number of discrete data rates and develop effective techniques. The hope is that the developed methods can provide insight, advance the state of the art, and lead to potential alternatives to the discrete data rate case, if desired.

The contributions of our work are as follows.

- Firstly, we propose to use historical data to capture more detailed information about the distribution, not just the mean as in PF scheduling, for scheduling purposes. Herein, a class of

This research was supported in part by the UCSD center for wireless communications and the NSF Grant CCF-1115645.

NPCS algorithms which are first of this type, are developed. These algorithms are used when the channel model such as pathloss, type of fading is unknown at the BS. In NPCS, the order statistics of the CQI sequence of each user are used directly for the scheduling process. These algorithms can be understood as, but not restricted to, exploiting empirical CDF. In particular, they can be cast in a framework to maximize an objective function of choice such as throughput, bit error rate, among others. The variables of optimization are the mapping functions which reflect how the CQI sequence is used for scheduling.

- Secondly, two specific algorithms NPCS-1 and NPCS-2, which aim at best exploiting the ordered CQI are proposed. Another advantage of NPCS algorithms is that each user memorizes and orders its own channel CQI and feeds back only the ordered index. This helps to offload some processing requirement to the users.
- Thirdly, the throughput of the proposed algorithms are verified and shown to be superior to the widely used PF scheduling frequently. Though the throughput of these algorithms are derived in Rayleigh fading, numerical results in Nakagami-m and log-normal fading are also observed with similar conclusions.
- Lastly, it is shown that the algorithms developed approach the performance of a CDF scheduler with perfect knowledge very rapidly. When compared with the performance achieved with exact CDF knowledge, the algorithms achieve $95 \rightarrow 97\%$ of the performance with only 10 i.i.d. channel samples, and achieve 99% the performance with only 30 i.i.d. samples.

2. SYSTEM MODEL

We consider a multiuser system where the base station (BS) has a single antenna and the K users are also each equipped with a single antenna. A downlink system is specifically assumed in the presentation and similar considerations apply to the uplink system. At a given time, the BS selects a user k and transmits a symbol s_k to this user. The received signal y_k is

$$y_k = h_k \sqrt{\rho} s_k + n_k, \tag{1}$$

where $h_k \in C^{1\times 1}$ is the channel from the BS to the selected user k which is assumed to be independent in time, $n_k \sim CN(0,1)$ is the additive noise at user k and ρ is the transmit SNR. The instantaneous CQI z_k and SNR x_k are given by $z_k = |h_k|^2$ and $x_k = \rho z_k$. Also, we denote the random variables associated with the CQI and SNR of user k by Z_k and X_k respectively. Throughout this work, we use upper case letters, e.g. Z, to denote a random variable and the corresponding lower case letters, e.g. z, to denote a certain value for that random variable.

To select a user to be served on the resource, CDF scheduling [3, 28] is used. Upon receiving the CQI z_k from users, fed back through an appropriate feedback channel, the BS utilizes the corresponding CDF of these CQI to evaluate a service metric and selects the user k^* with the highest value.

$$k^* = \arg\max_k F_{Z_k}(z_k)^{\frac{1}{w_k}}, \tag{2}$$

where $F_{Z_k}(.)$ is the CDF of the CQI of user k. Herein, $F_{Z_k}(.)$ is assumed unknown. The weight w_k represents the priority assigned to user k, which is equivalent to the proportion of the resource allocated to the user. The weight w_k is preassigned for all the users $k = 1, \ldots, K$ such that $\sum_{k=1}^{K} w_k = 1$.

3. NON-PARAMETRIC CDF SCHEDULING

We assume to possess N_k CQI samples from the current and previous channel uses for each user k and use these samples for the CDF scheduling algorithm. The number of CQI samples collected for each user is assumed different which depends on the user channel's coherence time and its activity levels. These N_k samples are sorted in an ascending order $z_{k(1)} \leq \cdots \leq z_{k(N_k)}$. We also assume that the current CQI, denoted by z_k^I,, is in the i_k-th position in the ordered set, i.e. $z_k^I = z_{k(i_k)}$. If the CDF was known, then the random CQI variable Z_k would be mapped to a random variable U_k, uniformly distributed in $[0,1]$, using the CDF, i.e. $U_k = F_{Z_k}(Z_k)$. Since the CDF is a nondecreasing function, the CDF mapping would map the ordered CQI to an ordered set of values in the interval $[0,1]$, i.e. the CDF mapping would result in $u_{k(i)} = F_{Z_k}(z_{k(i)})$ which are also ordered; $u_{k(1)} \leq \cdots \leq u_{k(i_k)} \leq \cdots \leq u_{k(N_k)}$. Hence, the current CQI would map to $u_{k(i_k)}$, the i_k-th position and we use the indicator function $1_{u_{k(i_k)}}$ to denote that the most recent CQI has position i_k in the order statistics. Though we do not know the exact values of each $u_{k(i_k)}$, we know their statistics [30], and can make use of them to develop a practical scheduler. One option is to use the average value $u_{k(i_k)}$ for the CDF scheduler leading to a scheme, that we call ECS, or using the expected value of the ordered CDF.

- **Initialization:** For each user k, collect N_k CQI including the current instantaneous CQI z_k^I and the past $(N_k - 1)$ ones.
- Sort the CQI in an ascending order. Identify the position of the current CQI z_k^I, say i_k.
- Calculate the expected value[1] of the variable obtained by the CDF mapping of the ordered variable. This is given by $q_k = E\{U_{k(i_k)}\} = \frac{i_k}{N_k+1},\ k = 1,..,K.$ [30].
- Select[2] a user $k^* = \arg\max_k q_k^{\frac{1}{w_k}}$.

Using the expected value of the ordered CDF is simple and can be an option for CDF scheduling. When we investigate the portion of resource allocated to the users, this method does not guarantee the desired resource allocation. Because the mismatch between the desired and the actual allocation can be significant, we are interested in algorithms which can provide better control over resource allocation.

For this purpose, we consider the construction of a mapping from $z_{k(i)}$ to the i-th element $\tilde{u}_{k(i)}$ preserving some statistical properties of $u_{k(i)}$, and ensure that the resultant random variable $\tilde{U}_k$ is a uniform random variable on the interval $[0,1]$, mimicking the statistics of U_k. A minimum requirement on $f_{\tilde{u}_{k(i)}}(x)$, which denotes the density function of $\tilde{u}_{k(i)}$, is that it satisfy two following conditions

$$\begin{aligned} &\int_0^1 f_{\tilde{U}_{k(i)}}(x)dx = 1, && \forall i = 1,\ldots,N_k,\ \forall k \quad (a) \\ &\frac{1}{N_k}\sum_{i=1}^{N_k} f_{\tilde{U}_{k(i)}}(x) = 1, && \forall x,\ \forall k \quad (b) \end{aligned} \tag{3}$$

where (a) comes from the properties of a density function and (b) comes from the fact that N_k equally likely ordered random variables $\tilde{U}_{k(i)}$ constitute an uniform random variable $\tilde{U}_k$ on the interval$[0,1]$. Any mapping that satisfies this properties results in a mapping from the CQI to a uniform random variable on the interval $[0,1]$ and can potentially be the basis of the NPCS algorithm as described below.

[1] Instead of the mean, the median or any specifically manipulated value e.g. $E\left\{U_{k(i_k)}^{\frac{1}{w_k}}\right\}$ can be used.

[2] If two users have the same q_k, a tie breaking rule has to be implemented.

- Each user generates a sample value for resource allocation
  - **Initialization:** User k collects N_k CQI samples.
  - Sort the CQI in an ascending order and identify the position of the instantaneous CQI, say i_k.
  - Generate the corresponding sample value $\tilde{u}_k$ for user k to be used for resource allocation.
    * Generate $\tilde{u}_{k(i_k)}$ a sample using the pdf $f_{\tilde{U}_{k(i_k)}}(\cdot)$ and set $\tilde{u}_k = \tilde{u}_{k(i_k)}$.
- Select a user $k^* = \arg\max_k \tilde{u}_k^{\frac{1}{w_k}}$.

The achievable system throughput can be shown to be

$$R_{NPCS} = \sum_{k=1}^{K}\sum_{i=1}^{N_k}\frac{1}{N_k}Pr\{k^* = k|1_{u_{k(i_k)}}\}R_{k(i_k)}, \quad (4)$$

where $Pr\{1_{u_{k(i_k)}}\} = \frac{1}{N_k}$. The rate $R_{k(i_k)} = \int_0^\infty \log_2(1+x)f_{X_{k(i_k)}}(x)dx$ depends only on the distribution of $X_{k(i_k)}$ and does not depend on the mapping technique.

3.1. NPCS-1

From the position i_k-th of the recent CQI on the ordered CQI sequence, the algorithm generates $\tilde{U}_{k(i_k)}$ with the same distribution as $U_{k(i_k)}$. Note that the density function of $U_{k(i_k)}$ is known because it is the i_k-th order statistic of N_k i.i.d random variables uniform on the interval $[0,1]$. This mapping satisfies the constraints in (3).

Proposition 1. *The NPCS-1 mapping has the following properties*

1. *The expectation of the i-th variable $\tilde{U}_{k(i_k)}$ in the constructed ordered sequence as described in NPCS-1 algorithm is given*

$$E\{\tilde{U}_{k(i_k)}\} = \frac{i_k}{N_k+1}, \quad (5)$$

2. *The mapping from the CQI to a variable $\tilde{U}_{k(i_k)}$ is unbiased, i.e. $E(U_{k(i_k)}) = E(\tilde{U}_{k(i_k)})$.*
3. *The variance of the difference between $U_{k(i_k)}$ and $\tilde{U}_{k(i_k)}$ is*

$$\sigma^2_{U_{k(i_k)}-\tilde{U}_{k(i_k)}} = 2\frac{i_k(N_k+1-i_k)}{(N_k+1)^2(N_k+2)} \xrightarrow[N_k\to\infty]{} 0. \quad (6)$$

Proof. The detail proof is provided in Appendix 6. □

Given $\tilde{u}_k = \tilde{u}_{k(i_k)}$, user k is selected if $\tilde{u}_j^{\frac{1}{w_j}} < \tilde{u}_{k(i_k)}^{\frac{1}{w_k}}$, $\forall j \neq k$. With a value of $\tilde{u}_{k(i_k)}$, probability user k is selected is $Pr\{k^* = k|\tilde{u}_{k(i_k)}; 1_{u_{k(i_k)}}\} = \prod_{j\neq k}\tilde{u}_{k(i_k)}^{\frac{w_j}{w_k}} = \tilde{u}_{k(i_k)}^{\frac{1}{w_k}-1}$. The probability user k is selected, given its channel's instantaneous CQI has position i_k-th, is calculated by taking expectation over value of $\tilde{U}_{k(i_k)}$,

$$\begin{aligned} Pr\{k^* = k|1_{u_{k(i_k)}}\} &= \int_0^1 x^{\frac{1}{w_k}-1}f_{\tilde{U}_{k(i_k)}}(x)dx \\ &= N_k\binom{N_k-1}{i_k-1}B\left(i_k+\frac{1}{w_k}-1, N_k-i_k+1\right), \end{aligned} \quad (7)$$

where $f_{\tilde{U}_{k(i_k)}}(x)$ is the distribution of the i_k-th order statistics [30] of N_k i.i.d random variables uniform on the interval $[0,1]$ and the last equation follows from the definition of Beta function [31, 8.380].

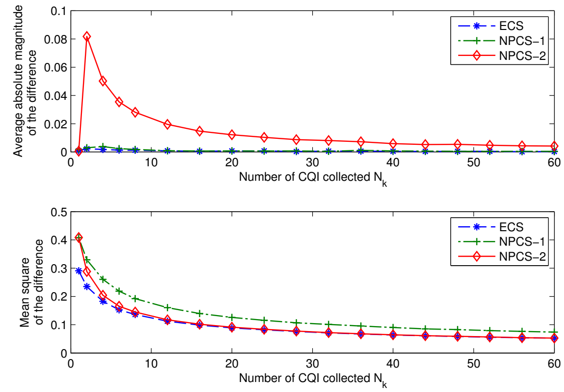

Fig. 1. The precision of the mapping from ordered CQI to the quantized and interpolated CDF - the top figure is the average of the magnitude difference and the lower figure is the variance of the difference between the real CDF value with the constructed one.

To provide an illustration of system performance, we consider all the links to the users to be under Rayleigh fading. Then, the distribution of the SNR can be represented by $F_{X_k}(x) = 1-e^{-x/\rho_k}, x > 0$, with $\rho_k = \rho c_k$ is the received SNR of the user k with $c_k = E\{Z_k\}$ is the pathloss from the BS to user k.

Theorem 1. *In a multiuser system under Rayleigh fading with K users, the overall system throughput for NPCS-1 is*

$$\begin{aligned} R_1 = \frac{N_k}{\ln 2}\sum_{k=1}^{K}\sum_{i_k=1}^{N_k}\binom{N_k-1}{i_k-1}B\left(i_k+\frac{1}{w_k}-1, N_k-i_k+1\right) \\ \times\sum_{l=0}^{i_k-1}\binom{i_k-1}{l}\frac{(-1)^{l+1}e^{\frac{N_k+l-i_k+1}{\rho_k}}}{N_k+l-i_k+1}Ei\left(-\frac{N_k+l-i_k+1}{\rho_k}\right), \end{aligned} \quad (8)$$

where $B(\cdot,\cdot)$ is beta function, $Ei(.)$ is the exponential integral function [31], and $Q_{k,i_k} = \frac{i_k}{N_k}$; w_k and ρ_k are correspondingly the assigned weight and the received SNR of user k.

Proof. The result is obtained by combining (4) and (7). □

3.2. NPCS-2

In this method, the random variable $\tilde{U}_k$ is created from $\tilde{u}_{k(i_k)}$ which is uniformly distributed in $[Q_{k,i_k-1}, Q_{k,i_k}]$. The boundary points Q_{k,i_k} are set equally spaced as in an uniform quantizer [32]. $Q_{k,i_k} = i_k\Delta_k$, for $i_k = 0,\ldots,N_k$ and $\Delta_k = \frac{1}{N_k}$. The values for $\tilde{u}_{k(i_k)}$ is generated employing a random variable uniformly distributed in $[Q_{k,i_k-1}, Q_{k,i_k}]$. This mapping also satisfies the constraints in (3). The performance of NPCS-2 can be found similarly and can be referred to [33].

4. SIMULATION RESULTS

To evaluate the analytical results and the performance of the proposed approaches, we consider a multiuser system with $K = 10$ users. Each user k is assigned weights $w_k = (k+1)a_k$, with a_k is the normalization parameter to guarantee that $\sum_{k=1}^{K} w_k = 1$. The link from the BS to the users are under Rayleigh fading. The pathloss of each user k is $c_k = be^{-\lambda k}$ with $\lambda = 0.1$ and b is a constant so $\sum_{k=1}^{K} c_k = K$. The transmit SNR at the BS is set $\rho = 10$dB.

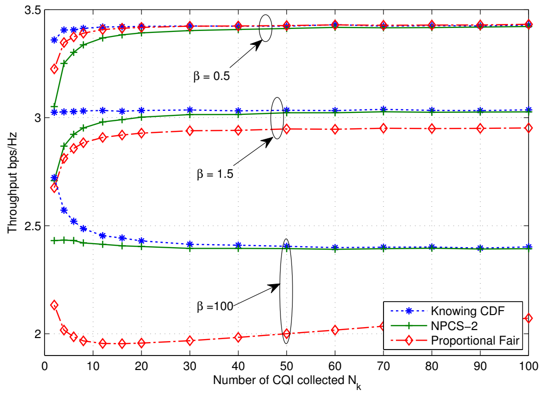

Fig. 2. Comparison between CDF and PF scheduling as a function of collected channel data N_k under Rayleigh fading.

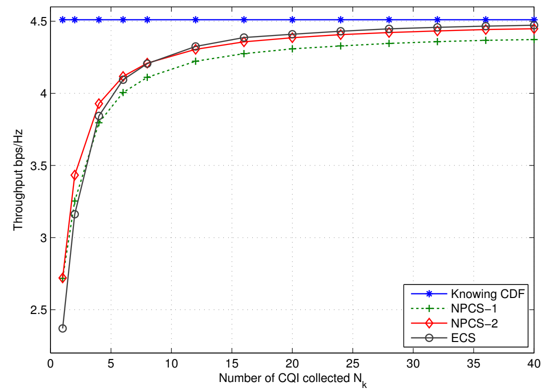

Fig. 3. Performance of practical CDF scheduling as a function of collected channel data N_k under Rayleigh fading.

In Fig. 1, the precision of the mappings developed is evaluated. The metric used for the evaluation are $|E(\tilde{U}-U)|$ and var$(\tilde{U}-U)$, where $\tilde{U}$ and U are the estimated and actual random variables. The first metric is a measure of unbiasedness and the second is a measure of the mean squared error. Both ECS and $NPCS-1$ are unbiased estimates and so have small $|E(\tilde{U}-U)|$. In terms of mean squared error (MSE), NPCS-2 has smaller MSE than NPCS-1. However, both have higher MSE than ECS because of the randomization introduced in these methods in order to generate a uniform random variable. Both the average error and the MSE decrease as the number of samples N_k increase and go to zero when $N_k \to \infty$.

The performance of CDF and PF scheduling are compared in Fig. 2. To ease the comparison, we consider a system with two users with the difference in the average received SNR being 10dB. The performance using PF scheduling with its parameter β [34] is investigated first. For each value of β, the allocation probability for the users, which is collected by averaging over 10^6 experiments, is used to set the corresponding weights in CDF scheduling, e.g. weight of user 1 is set to equal $Pr\{k^*=1\}$ in PF scheduling. We note that though the CDF scheduling with perfect channel knowledge does not depend on N_k, its performance is not a constant in Fig. 2 because weights for users which is taken based on the performance of PF scheduling method, change slightly. When $\beta \lesssim 0.6$, the PF scheduling is close to Opportunistic scheduling which favors and almost always (with probability $> 90\%$ in this experiment) allocates resource to the user with higher SNR. In this case, PF scheduling is better than CDF scheduling. When priorities of users are stressed more, it is observed that CDF scheduling outperforms PF scheduling. This situation happens frequently when we want to allocate comparable amounts of resource to the users. Similar results are observed with other combinations of fading types.

The performance of the proposed scheduling methods as a function of number of samples N_k is shown in Fig. 3. From the figure, the performance of ECS is better than NPCS-2 for large N_k. However, this small throughput incentive is achieved with a tradeoff in the inaccuracy in resource allocation for users. In our experiments, such inaccuracy can easily exceed 10% for many users. It can be seen that NPCS-2 obtains higher system throughput than NPCS-1 does. When N_k increases, the achieved system throughput increases as expected since we obtain more accurate information about the channels of the users. With N_k large enough, e.g. $N_k > 30$ samples, the loss in throughput is under 1% in comparison to the case when the channel distribution is known perfectly. This bodes well for CDF based methods as this requirement is typically easy to obtain in real networks where the fast fading is of the order of milliseconds and the channel model does not change significantly in seconds. For example, if a CQI sample is collected in every 1-millisecond LTE frame, a thousand samples will be collected in 1 second which easily enables NPCS-2 to approach the performance of knowing perfectly the CDF of the channel.

5. CONCLUSION

We have proposed practical approaches to enable the application of CDF scheduling technique in heterogeneous multiuser systems. The proposed NPCS algorithms are shown to precisely control resource allocation for users, simple enough to be employed in real systems, and frequently have better performance than the existing PF scheduling. In the comparison with PF scheduling, the CDF scheduling is better when the fairness among users is of major concern. For the methods developed, the achievable throughput of each user increases and quickly approaches the throughput achievable with perfect knowledge of the CDF.

6. APPENDIX - THE PRECISION OF THE MAPPINGS

The expectation of the i_k-th variable in the constructed ordered sequence of CDF as described in NPCS algorithm, is

$$E\{U_{k(i_k)}\} = \int_0^1 N_k \binom{N_k-1}{i_k-1} x^{i_k-1}[1-x]^{N_k-i_k} x dx$$
$$= N_k \binom{N_k-1}{i_k-1} B(i_k+1, N_k-i_k+1) = \frac{i_k}{N_k+1}, \qquad (9)$$

where we get 1st equation from the PDF of the variable and by utilizing the fact the CDF of a variable uniformly distributed in $[0,1]$ is $F_U(x)=x$. The end result comes from utilizing [31, 8.380] and the definition of Beta function [31, 8.384.1]. Similarly, the variance of $U_{k(i_k)}$ can be calculated

$$\sigma^2_{U_{k(i_k)}} = E\{U^2_{k(i_k)}\} - E\{U_{k(i_k)}\}^2 = \int_0^1 N_k \binom{N_k-1}{i_k-1} x^{i_k+1}$$
$$\times [1-x]^{N_k-1} dx - \frac{i_k^2}{(N_k+1)^2} = \frac{i_k(N_k+1-i_k)}{(N_k+1)^2(N_k+2)}, \quad (10)$$

where $E\{U^2_{k(i_k)}\} = \frac{i_k(i_k+1)}{(N_k+1)(N_k+2)}$ which is similar to the calculation in deriving the mean. The variance of $U_{k(i_k)} - \tilde{U}_{k(i_k)}$ is twice the variance of $U_{k(i_k)}$.

7. REFERENCES

[1] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *IEEE Int. Conf. on Commun. (ICC)*, vol. 1, Jun. 1995.

[2] F. P. Kelly, A. K. Maulloo, and D. K. Tan, "Rate control for communication networks: shadow prices, proportional fairness and stability," *J. of the Operational Research society*, vol. 49, no. 3, pp. 237–252, 1998.

[3] D. Park, H. Seo, H. Kwon, and B. G. Lee, "Wireless packet scheduling based on the cumulative distribution function of user transmission rates," *IEEE Trans. on Commun.*, vol. 53, no. 11, pp. 1919 – 1929, Nov. 2005.

[4] D. Park and B. G. Lee, "Qos support by using cdf-based wireless packet scheduling in fading channels," *IEEE Trans. on Commun.*, vol. 54, no. 11, pp. 2051–2061, Nov. 2006.

[5] L. Massoulie and J. Roberts, "Bandwidth sharing: objectives and algorithms," *IEEE/ACM Trans. on Networking*, vol. 10, no. 3, pp. 320 –328, Jun. 2002.

[6] H. Fattah and C. Leung, "An overview of scheduling algorithms in wireless multimedia networks," *IEEE Wireless Commun.*, vol. 9, no. 5, pp. 76–83, Dec. 2002.

[7] A. Asadi and V. Mancuso, "A survey on opportunistic scheduling in wireless communications," *IEEE Commun. Surveys Tutorials*, pp. 1–18, Jan. 2013.

[8] X. Liu, E. K. P. Chong, and N. Shroff, "Opportunistic transmission scheduling with resource-sharing constraints in wireless networks," *IEEE J. on Sel. Areas in Commun.*, vol. 19, no. 10, pp. 2053–2064, Oct. 2001.

[9] Z. Fang and B. Bensaou, "Fair bandwidth sharing algorithms based on game theory frameworks for wireless ad-hoc networks," in *INFOCOM*, vol. 2. IEEE, 2004, pp. 1284–1295.

[10] Z. Jiang, Y. Ge, and Y. Li, "Max-utility wireless resource management for best-effort traffic," *IEEE Trans. on Wireless Commun.*, vol. 4, no. 1, pp. 100–111, Jan. 2005.

[11] S. Stanczak, M. Wiczanowski, and H. Boche, "Distributed utility-based power control: Objectives and algorithms," *IEEE Trans. on Signal Proc.*, vol. 55, no. 10, pp. 5058–5068, Oct. 2007.

[12] Z. Shen, J. Andrews, and B. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints," *IEEE Trans. on Wireless Commun.*, vol. 4, no. 6, pp. 2726–2737, Nov. 2005.

[13] D. P. Bertsekas and R. G. Gallager, *Data Networks*, 2nd ed. Prentice Hall, 1992.

[14] L. Tassiulas and S. Sarkar, "Maxmin fair scheduling in wireless networks," in *INFOCOM*, vol. 2, 2002, pp. 763–772.

[15] E. Hahne, "Round-robin scheduling for max-min fairness in data networks," *IEEE J. on Sel. Areas in Commun.*, vol. 9, no. 7, pp. 1024–1039, Sep. 1991.

[16] A. Jalali, R. Padovani, and R. Pankaj, "Data throughput of CDMA-HDR a high efficiency-high data rate personal communication wireless system," in *IEEE Veh. Tech. Conf.*, vol. 3, 2000, pp. 1854–1858.

[17] C. Wengerter, J. Ohlhorst, and A. von Elbwart, "Fairness and throughput analysis for generalized proportional fair frequency scheduling in ofdma," in *IEEE Veh. Tech. Conf.*, vol. 3, 2005, pp. 1903–1907.

[18] D. Wu and R. Negi, "Downlink scheduling in a cellular network for quality-of-service assurance," *IEEE Trans. on Veh. Tech.*, vol. 53, no. 5, pp. 1547 – 1557, Sep. 2004.

[19] Z. Shen, J. G. Andrews, and B. L. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints," *IEEE Trans. on Wireless Commun.*, vol. 4, no. 6, pp. 2726–2737, Nov. 2005.

[20] J.-G. Choi and S. Bahk, "Cell-throughput analysis of the proportional fair scheduler in the single-cell environment," *IEEE Trans. on Veh. Tech.*, vol. 56, no. 2, pp. 766 –778, Mar. 2007.

[21] N. Sharma and L. Ozarow, "A study of opportunism for multiple-antenna systems," *IEEE Trans. on Inf. Theory*, vol. 51, no. 5, pp. 1804 – 1814, May. 2005.

[22] H. Kwon, Y. Kim, J.-K. Han, D. Kim, H. W. Lee, and Y. K. Kim, "Performance evaluation of high-speed packet enhancement on CDMA2000 1xEV-DV," *IEEE Comm. Mag.*, vol. 43, no. 4, pp. 67–73, Apr. 2005.

[23] P. Svedman, S. K. Wilson, L. J. Cimini, and B. Ottersten, "Opportunistic beamforming and scheduling for OFDMA systems," *IEEE Trans. on Commun.*, vol. 55, no. 5, pp. 941 –952, May. 2007.

[24] S. Sigdel and W. Krzymien, "Simplified fair scheduling and antenna selection algorithms for multiuser MIMO orthogonal space-division multiplexing downlink," *IEEE Trans. on Veh. Tech.*, vol. 58, no. 3, pp. 1329–1344, Mar. 2009.

[25] H. J. Kushner and P. A. Whiting, "Convergence of proportional-fair sharing algorithms under general conditions," *IEEE Trans. on Wireless Commun.*, vol. 3, no. 4, pp. 1250–1259, Jul. 2004.

[26] M. Andrews, "Instability of the proportional fair scheduling algorithm for HDR," *IEEE Trans. on Wireless Commun.*, vol. 3, no. 5, pp. 1422–1426, Sep. 2004.

[27] Y. Huang and B. D. Rao, "An analytical framework for heterogeneous partial feedback design in heterogeneous multicell OFDMA networks," *IEEE Trans. on Signal Proc.*, vol. 61, no. 3, pp. 753–769, Feb. 2013.

[28] A. Nguyen, Y. Huang, and B. D. Rao, "Weighted CDF-based scheduling for an OFDMA relay downlink with partial feedback," in *IEEE Asilomar Conf. on Signals, Systems and Computers*, Nov. 2012.

[29] Y. Huang and B. Rao, "Random beamforming with heterogeneous users and selective feedback: Individual sum rate and individual scaling laws," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 5, pp. 2080–2090, May. 2013.

[30] H. A. David and H. N. Nagaraja, *Order Statistics, Third Edition*. Wiley, 2003.

[31] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*, 7th ed., 2007.

[32] A. Gersho and R. M. Gray, *Vector quantization and signal compression*. Kluwer Academic Publishers, 1991.

[33] A. Nguyen, Y. Huang, and B. Rao, "Learning methods for CDF scheduling in multiuser heterogeneous systems," *IEEE Trans. on Signal Proc.*, 2013, submitted.

[34] E. Dahlman, S. Parkvall, and J. Skold, *4G: LTE/LTE-Advanced for Mobile Broadband*, 1st ed. Academic Press, 2011.