QUASI-MAXIMUM LIKELIHOOD ESTIMATOR OF MULTIPLE POLYNOMIAL-PHASE SIGNALS

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ABSTRACT

This paper addresses the parameter estimation of multicomponent polynomial-phase signals (mc-PPSs). Recently proposed quasi-maximum likelihood (QML) method based on the short-time Fourier transform (STFT) has been extended to deal with multiple PPSs. The proposed method outperforms state-of-the-art parametric methods developed to deal with multiple PPSs in terms of robustness against noise, while attaining the Cramér-Rao lower bound.

Index Terms— Polynomial-phase signal, parameter estimation, non-parametric estimation, short-time Fourier transform, quasi-ML

1. INTRODUCTION

Estimation of polynomial-phase signals (PPSs) has been extensively studied in the last two decades [1–9]. These techniques are based on traditional phase differentiation (PD) approach, where the phase order of the underlying PPS is decreased until a complex sinusoid is obtained [1–3]. The sinusoid frequency is proportional to the highest order PPS coefficient. Therefore, estimation of this coefficient boils down to the sinusoid frequency estimation. Lower order coefficients are obtained by repeating the same procedure on the original PPS dechirped by the previously estimated higher order coefficients [1, Section III]. However, the PD-based approach suffers from several drawbacks such as increased signal-to-noise-ratio (SNR) threshold, spurious cross-terms in case of multicomponent PPSs (mc-PPS), which can cover the desirable spectral components, and the propagation of estimation error from higher to lower order PPS coefficients [6].

The influence of noise and cross-terms is mitigated in the product high-order ambiguity function (PHAF) [4]. However, due to exponential rise of the number of cross-terms in the PD [6], the PHAF is suitable for lower order PPSs. From the standpoint of reducing the number of cross-terms, it is fundamental to reduce the number of PDs. In addition, each PD increases the SNR threshold by approximately 6 dB [3]. The transform that exploits this fact is the cubic phase function (CPF) proposed in [5] for the estimation of third-order PPSs (cubic phase signals). The CPF is further generalized to higher order PPSs in [6], and this approach is known as the hybrid CPF-HAF.

In [8, 9], an efficient method for high order PPS estimation, based on the short-time Fourier transform (STFT), has been proposed. It is referred to as the quasi-maximum likelihood (QML) method. In order to avoid the use of PD as the main source of inaccuracy in methods that use the PD, the QML method estimates the PPS coefficients from the instantaneous frequency (IF) of the PPS, which is extracted from the STFT. However, since the STFT is a biased estimator, refinement strategy proposed in [10] is used to achieve the Cramér-Rao lower bound (CRLB) for the SNR above the threshold. The QML method has significantly lower SNR threshold than the PD-based approaches. In this paper, we extend the QML method to deal with mc-PPSs. To that end, components have been (a) separated in the time-frequency (TF) plane, (b) estimated coarsely from the corresponding IFs and (c) refined by the mc-PPS refinement strategy proposed in [7].

The paper is organized as follows. The signal model and the QML method are described in Section 2. The proposed estimator is introduced in Section 3. Simulation results are presented in Section 4, whereas conclusions are drawn in Section 5.

2. SIGNAL MODEL AND QML METHOD

2.1. Signal model

The mc-PPS can be described by the following model:

$$\begin{aligned} s(n) &= \sum_{m=1}^{M} s_m(n) \\ &= \sum_{m=1}^{M} A_m e^{j\sum_{p=0}^{P} a_{m,p}\frac{(n\Delta)^p}{p}}, \quad |n| \le N/2, \end{aligned} \tag{1}$$

where A_m and $a_{m,p}$, $p = 0, ..., P$ are the amplitude and phase coefficients of the m-th component, respectively, M is the number of components, $N + 1$ is the number of samples, and

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Δ is the sampling interval. Without loss of generality, we assume that N is even. Our aim is to estimate parameters of $s(n)$ from noisy observations

$$x(n) = s(n) + \nu(n),$$

where $\nu(n)$ is zero-mean white complex Gaussian noise with variance σ^2.

2.2. QML method

The QML method [9] has been proposed for the parameter estimation of monocomponent PPSs ($M = 1$). This method estimates the vector of parameters $\mathbf{a} = [a_{1,1}, ..., a_{1,P}]^T$, and can be described by the following steps:

Step 1. Evaluate the STFT for various window lengths h, $h \in H$

$$\text{STFT}_h(n,\omega) = \sum_k x(n+k) w_h(k) e^{-j\omega k\Delta},$$

where $w_h(k)$ is window function such that $w_h(k) \neq 0$ for $|k| \leq h/2$ and $w_h(k) = 0$ elsewhere.

Step 2. Estimate the IF of $s(n)$ from each $\text{STFT}_h(n,\omega)$ calculated in Step 1:

$$\begin{aligned} &\hat{\omega}_h(n) = \arg\max_\omega |\text{STFT}_h(n,\omega)| \\ &n \in [-N/2 + h/2, \cdots, N/2 - h/2], \quad h \in H. \end{aligned} \tag{2}$$

Step 3. Perform polynomial fitting to estimate phase parameters $\hat{\mathbf{a}}_h = \left[\hat{a}^h_{1,1}, ..., \hat{a}^h_{1,P}\right]^T$, $h \in H$, from the estimated IFs:

$$\hat{\mathbf{a}}_h = (\mathbf{X}_h^T \mathbf{X}_h)^{-1} \mathbf{X}_h^T \mathbf{y}_h, \tag{3}$$

where

$$\mathbf{X}_h = \begin{bmatrix} 1 & -N_h\Delta & \cdots & [-N_h\Delta]^{P-1} \\ 1 & (-N_h+1)\Delta & \cdots & [(-N_h+1)\Delta]^{P-1} \\ \cdots & \cdots & \cdots & \cdots \\ 1 & (N_h-1)\Delta & \cdots & [(N_h-1)\Delta]^{P-1} \\ 1 & N_h\Delta & \cdots & [N_h\Delta]^{P-1} \end{bmatrix}$$

$$\mathbf{y}_h = [\hat{\omega}_h(-N_h), \hat{\omega}_h(-N_h+1), \cdots, \hat{\omega}_h(N_h)]^T,$$
$$N_h = \frac{N-h}{2}.$$

Step 4. Refine estimates $\hat{\mathbf{a}}_h$ using the approach proposed in [10] to obtain $\hat{\mathbf{a}}^r_h = \left[\hat{a}^{r,h}_{1,1}, ..., \hat{a}^{r,h}_{1,P}\right]^T$, $h \in H$. This step is necessary since the STFT is biased IF estimator.

Step 5. The final estimate is a vector $\hat{\mathbf{a}}^f_h = [\hat{a}_{1,1}, ..., \hat{a}_{1,P}]^T$ that satisfies

$$\hat{\mathbf{a}}^f_h = \arg\max_{\hat{\mathbf{a}}^r_h} |\text{ML}(\hat{\mathbf{a}}^r_h)|, \tag{4}$$

$$\text{ML}(\hat{\mathbf{a}}^r_h) = \left| \sum_n x(n) e^{-j\sum_{p=1}^P \hat{a}^{r,h}_{1,p} \frac{(n\Delta)^p}{p}} \right|. \tag{5}$$

The key step in this algorithm is Step 5, where the optimal window length in the STFT calculation is chosen. The optimality criterion is maximization of the ML function $\text{ML}(\hat{\mathbf{a}}^r_h)$. In this way, instead of direct search over all parameters in the ML function, which is computationally exhaustive for higher order PPSs, the ML function is calculated for estimates provided by the STFT.

3. QML FOR MC-PPS

The QML proposed in [8] cannot be directly used for the mc-PPS estimation ($M > 1$). In order to extend the QML method to deal with mc-PPSs, steps described in Section 2.2 should be modified. For example, the STFT now contains several components which should be separated in the TF plane. Further, the refinement strategy proposed in [10] should be modified to take into account the existence of several components. Finally, the ML function $\text{ML}(\hat{\mathbf{a}}^r_h)$ cannot be used as defined by (5) in Section 2.2.

In the sequel, we propose an extension of the QML algorithm for mc-PPSs which can be summarized by the following steps:

Step 1. For each h, $h \in H$, calculate the STFT of $x(n)$ as

$$\text{STFT}_h(n,\omega) = \sum_k x(n+k) w_h(k) e^{-j\omega k\Delta}.$$

Step 2. For each h, separate signal components $s_m(n)$, $m = 1, \cdots, M$, i.e., estimate the corresponding IFs [11]

$$\hat{\omega}_{m,h}(n) = \arg\max_\omega |\text{STFT}^m_h(n,\omega)|, \tag{6}$$

where $\text{STFT}^m_h(n,\omega)$ is the STFT region occupied by the m-th component for window length h.

Step 3. For each m, $m = 1, \cdots, M$, perform polynomial fitting to estimate phase parameters $\hat{\mathbf{a}}_{m,h} = \left[\hat{a}^h_{m,1}, ..., \hat{a}^h_{m,P}\right]^T$, $h \in H$, from the corresponding estimated IFs, using relation (3).

Step 4. Refine estimates $\hat{\mathbf{a}}_{m,h}$ using the approach proposed in [7] to obtain $\hat{\mathbf{a}}^r_{m,h} = \left[\hat{a}^{r,h}_{m,1}, ..., \hat{a}^{r,h}_{m,P}\right]^T$, $h \in H$.

Step 5. The final estimate sets $\hat{\mathbf{a}}^f_{m,h}$, $m = 1, \cdots, M$ are obtained for window length h that satisfies [12, eq. (35)]

$$h_{opt} = \arg\min_h \left\{ -\mathbf{x}^\dagger \mathbf{F}_h \left(\mathbf{F}^\dagger_h \mathbf{F}_h\right)^{-1} \mathbf{F}^\dagger_h \mathbf{x} \right\}, \tag{7}$$

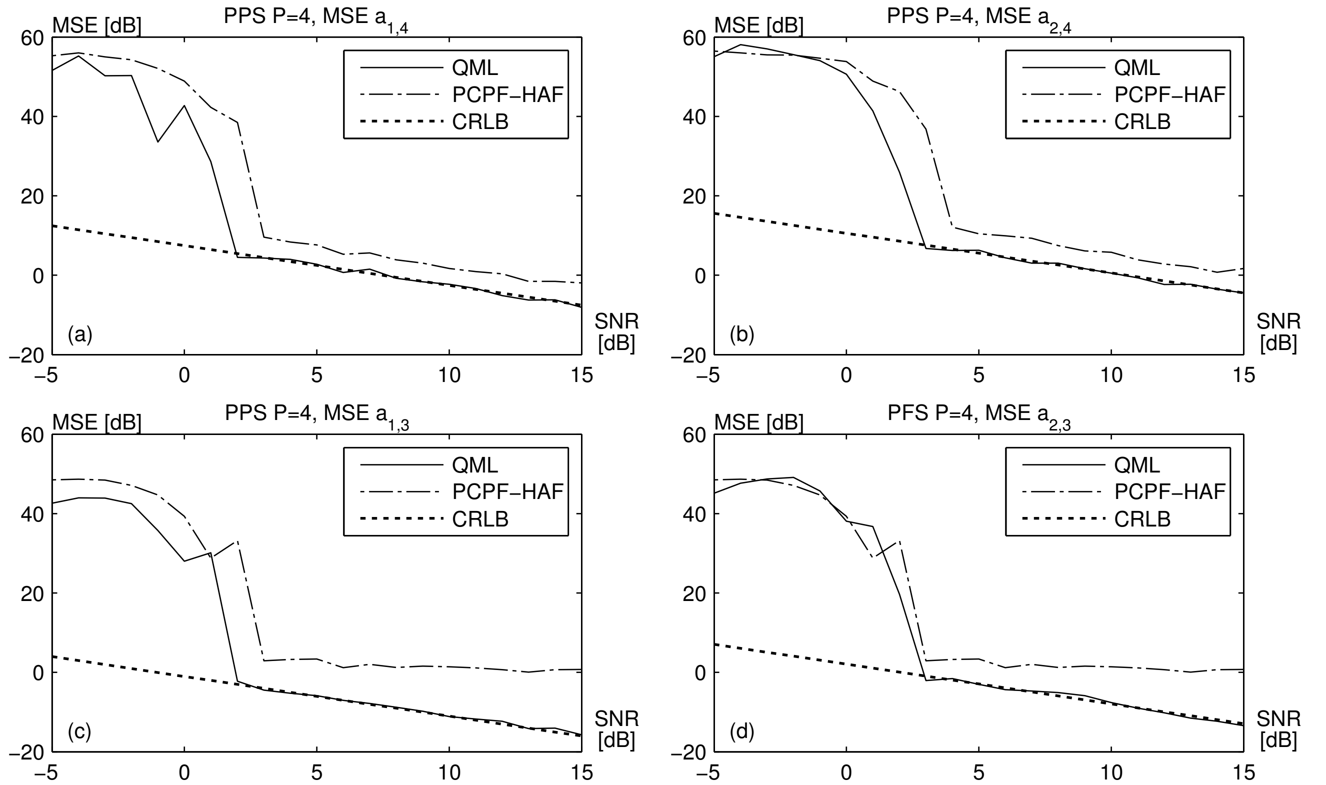

Fig. 1. MSE of the highest two phase coefficients of two signal components obtained by the proposed algorithm and the PCPF-HAF. *Left column*: MSE of $a_{1,4}$ and $a_{1,3}$. *Right column*: MSE of $a_{2,4}$ and $a_{2,3}$.

where

$$\mathbf{x} = [x(-N/2), \cdots, x(N/2)]^T,$$

$$\mathbf{F}_h =$$

$$\begin{bmatrix} G_1(-\frac{N}{2}) & G_2(-\frac{N}{2}) & \cdots & G_M(-\frac{N}{2}) \\ G_1(-\frac{N}{2}+1) & G_2(-\frac{N}{2}+1) & \cdots & G_M(-\frac{N}{2}+1) \\ \cdots & \cdots & \ddots & \vdots \\ G_1(\frac{N}{2}) & G_2(\frac{N}{2}) & \cdots & G_M(\frac{N}{2}) \end{bmatrix}$$

$$G_m(n) = e^{j\sum_{p=1}^{P} a_{m,p}(n\Delta)^p/p},$$

and † represents the Hermitian operator.

Although the STFT has been used to separate signal components in the TF plane, i.e., to provide their coarse IF estimation, some other TF tool can also be used. A good candidate for this purpose could be the S-method [13], which provides both good concentration of components in the TF plane and suppression of cross-terms.

The proposed method assumes that the polynomial order P is known in advance. However, the method can be readily extended to deal with unknown order, as well as with signals with non-polynomial phase laws, as considered in [9].

4. SIMULATIONS

In this section, we evaluate the proposed mc-PPS estimation method on the sum of two fourth-order PPSs ($M = 2$ and $P = 4$ in (1)) embedded in Gaussian noise. The parameters of the first PPS component are $A_1 = 1$, $a_{1,0} = 0$, $a_{1,1} = -58\pi$, $a_{1,2} = 30\pi$, $a_{1,3} = 24\pi$ and $a_{1,4} = -28\pi$, whereas of the second component are $A_2 = 0.7$, $a_{2,0} = 0$, $a_{2,1} = 54\pi$, $a_{2,2} = -22\pi$, $a_{2,3} = 24\pi$ and $a_{2,4} = 26\pi$. Also, $N = 256$ and $\Delta = 2/N$.

In the QML method, the STFT is calculated using the Hann window with lengths from set $H=\{8, 12, \cdots, 124, 128\}$.

The proposed QML method is compared to the product version of the CPF-HAF (PCPF-HAF) [6], known to outperform the PHAF in terms of accuracy and the SNR threshold. The PCPF-HAF is calculated following the guidelines given in [6].

Performance has been evaluated through the mean squared error (MSE) defined as

$$\text{MSE} = 10\log_{10}\left[\frac{1}{N_{MC}}\sum_{k=1}^{N_{MC}}\left(a_{true} - a_{est}^k\right)^2\right],$$

where a_{true} represents the true coefficient value, a_{est}^k the estimated value in the k-th simulation, and N_{MC} is the number of Monte-Carlo simulations. Here, $N_{MC} = 200$. The MSE of the highest two coefficients of both components is shown in Fig. 1, along with the corresponding CRLBs, where the SNR is varied from -5 to 15 dB with increment of 1 dB.

Figure 1 clearly shows that the QML mc-PPS estimation method is characterized by both lower SNR threshold and lower MSE compared to the PCPF-HAF. Also, above the SNR threshold, the QML approach attains the CRLB, which is not the case with the PCPF-HAF.

The PHAF-based results have not been included in Fig. 1 since the PHAF fails to estimate PPS coefficients in the considered SNR range due to large number of cross-terms caused by the PD operation [6].

5. CONCLUSIONS

Parameter estimation of mc-PPSs is considered. The core of the proposed estimator can be summarized by three steps: (a) use the STFT to provide non-parametric IF estimates of each component, (b) perform polynomial fitting on non-parametric IF estimates to obtain PPS coefficients, and (c) refine the obtained coefficients. These three steps are repeated for all considered window lengths in the STFT and the optimal window length (i.e., optimal PPS coefficients) is chosen to optimize the QML cost function.

Simulation results prove the validity of the method, showing that it outperforms the PCPF-HAF in terms of both the MSE and the SNR threshold. Finally, the method attains the CRLB for the considered signal.

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