TIME-VARYING FILTERING AND SEPARATION OF NONSTATIONARY FM SIGNALS IN STRONG NOISE ENVIRONMENTS

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ABSTRACT

Motivated by the existing time-frequency peak filtering (TFPF) algorithm, herein a robust time-varying filtering (RTVF) algorithm is proposed for filtering and separating multicomponent frequency modulation (FM) signals. The performance of the TFPF based on windowed Wigner-Ville distribution is limited by the linear constraint on the waveform of the received signal. The proposed RTVF significantly improves the filtering performance with low complexity by applying a sinusoidal time-frequency distribution, which allows a sinusoidal constraint on the signal's waveform. The RTVF can successfully decompose a multicomponent signal into individual components based on an initial instantaneous frequency (IF) estimate of each component. Unlike existing time-varying filters, the RTVF is much less sensitive to the accuracy of the IF estimate, which can be gradually refined by performing an iterative RTVF procedure.

Index Terms— Time-varying filtering, signal separation.

1. INTRODUCTION

There has been a great deal of research on instantaneous frequency (IF) estimation of frequency modulation (FM) signals [1–4]. However, most reported estimation methods are under the high SNR assumption. There are strong interests in the development of IF estimation of FM signals in low SNR environments. It has been a common practice that filtering operations are used before applying estimation methods.

In the literature, the available time-frequency distributions (TFDs) [5,6] of signals offer the possibility of performing filtering in time-frequency (TF) domain. In [7], a windowed WVD-based time-frequency peak filtering (WWVD-TFPF) was reported. The principle of the TFPF is to encode the noisy signal as the IF of an analytic signal, and then estimate the peak in the WWVD of the encoded signal for obtaining the underlying signal. The WWVD-TFPF is signal independent and can achieve promising filtering performance even though the IF information of the signal is unrecognized in the TF domain [8, 9]. However, the WWVD-TFPF is biased for encoded signals whose variation order of frequency contents is higher than linear, and a limited window size is used to control the bias quantity. Therefore, high sampling rates are required to reduce the noise variance when using short window

sizes, which becomes impractical due to the requirements of expensive computation and data memory resources. Furthermore, separating components from a multicomponent signal has been impossible by using the WWVD-TFPF algorithm.

The limitation of the WWVD-TFPF lies in its short window length due to the nonlinearity of received signal waveform. Theoretically speaking, most signals modulated on a sine-wave carrier can be approximated as a generalized sinusoidal waveform within a time interval. This motivates us to pay the attention to the TFDs that present a continuum of delta functions along a sinusoidal IF law, so that the window size requirement could be substantially relaxed. This paper proposes a robust time-varying filtering (RTVF) algorithm for filtering and separating multicomponent FM signals with heavy noise. The RTVF algorithm is based on a sinusoidal timefrequency distribution (STFD) [10] instead of on the WWVD. The STFD is designed for optimally tracking spectral contents of encoded signals with sinusoidal IF variations, which overcomes the linear restriction of the WWVD-TFPF.

This paper is organized as follows. Section 2 presents the details of the RTVF algorithm. The bias analysis of the RTVF algorithm is conducted in Section 3. Section 4 and Section 5 give numerical results and conclusions, respectively.

2. THE PROPOSED RTVF ALGORITHM

Let us consider a multicomponent FM signal model as

$$s(t) = x(t) + n(t) = \sum_{k=1}^{K} x_k(t) + n(t),$$
(1)

where K is the number of components and n(t) is the zeromean additive noise. Assume a sinusoidal FM (SFM) signal with a sinusoidal IF $f_i(t) = \mu \rho \cos(2\pi f_m t + \theta)$

$$z(t) = e^{j2\pi\mu \int_0^t \rho \cos(2\pi f_m \lambda + \theta) d\lambda} = e^{j\frac{\mu}{f_m}\rho \sin(2\pi f_m t + \theta)}, \quad (2)$$

where μ is the modulation index, f_m is the modulation frequency, ρ and θ are constant amplitude and phase. The STFD of z(t) in (2) is based on a kernel adapted to the SFM signal

$$\mathrm{STFD}_{z}(t,f) = \mathcal{FT}_{\tau} \big\{ h(\tau) K_{z}(t,\tau) \big\}, \tag{3}$$

where \mathcal{FT}_{τ} denotes the Fourier transform on τ , $h(\tau)$ is a lag window function, and $K_z(t, \tau)$ is the kernel function

$$K_z(t,\tau) = \prod_{i=1}^p \left\{ z(t+\tau+a_i)^{b_i} z^* (t-\tau-a_i)^{b_i} \right\}.$$
 (4)

It is expected the STFD optimally concentrates signal energy along the sinusoidal IF

$$\mathcal{FT}_{\tau}\big\{h(\tau)K_{z}(t,\tau)\big\} = \delta\big(f - \mu\rho\cos(2\pi f_{m}t + \theta)\big).$$
(5)

One of the solutions to (5) can be resolved by using simple trigonometric relations when p = 2, which gives

$$a_1 = 0, \quad a_2 = \frac{1}{4f_m},$$
 (6)
 $b_1 = \pi f_m \tau \sin(2\pi f_m \tau), \quad b_2 = \pi f_m \tau \cos(2\pi f_m \tau).$

Therefore the kernel function in (4) is transformed into

$$K_{z}(t,\tau) = \left[z(t+\tau)z^{*}(t-\tau)\right]^{f_{m}\tau\pi\sin(2\pi f_{m}\tau)}$$
(7)

$$\cdot \left[z(t+\tau+\frac{1}{4f_{m}})z^{*}(t-\tau-\frac{1}{4f_{m}})\right]^{f_{m}\tau\pi\cos(2\pi f_{m}\tau)}.$$

The idea of the RTVF algorithm is to firstly encode the signal s(t) in (1) into the IF of a unit amplitude analytic signal

$$z_s(t) = e^{j2\pi\mu \int_0^t x(\lambda)d\lambda} e^{j2\pi\mu \int_0^t n(\lambda)d\lambda} = z_x(t) \cdot z_n(t), \quad (8)$$

and then the underlying signal $\hat{x}(t)$ is obtained by estimating the IF of the encoded signal $z_s(t)$, which can be conducted by detecting the peak of the STFD of $z_s(t)$. However, the implementation of $\text{STFD}_{z_s}(t, f)$ is expensive due to the fast Fourier transform (FFT) on the kernel function. Alternatively, the IF estimation of $z_s(t)$ can be realized by a simple phase operation without the implementation of the STFD. Specifically, the exponent of the kernel function in (7) is dependent on the time lag τ , which permits a direct estimation of the underlying signal by performing an operation on the phase of the kernel by replacing b_i in (4) with $\frac{b_i}{2\pi\tau}$. This operation allows an unbiased recovery of a noisy sine signal $s(t) = \rho \cos(2\pi f_m t + \theta) + n(t)$, expressed as

$$\mathbb{E}\left\{\Theta_{z_s}(t,\tau)\right\} = \mathbb{E}\left\{\arg\left\{K_{z_s}(t,\tau)\right\} \cdot \frac{1}{2\pi\tau\mu}\right\}$$
$$= \arg\left\{\left\{z_x(t+\tau)z_x^*(t-\tau)\right\}^{\frac{f_m\sin(2\pi f_m\tau)}{2}} \cdot \left\{z_x(t+\tau+\frac{1}{4f_m})z_x^*(t-\tau-\frac{1}{4f_m})\right\}^{\frac{f_m\cos(2\pi f_m\tau)}{2}}\right\}/\mu$$
$$= \rho\cos(2\pi f_m t + \theta). \tag{9}$$

where $\arg\{\cdot\}$ and $\mathbb{E}\{\cdot\}$ denote the phase of a complex value and the expectation operator on random variable, respectively.

It is observed that the phase term $\Theta_{z_x}(t,\tau)$ is a constant over the time lag τ , which gives a precise estimation of the sine signal. In presence of noise, $\Theta_{z_s}(t,\tau)$ changes with the variation of τ . We propose to denoise the noisy signal s(t) modeled in (1) by taking the mean operation on the fluctuated phase term of $z_s(t)$ over a real lag window $h(\tau)$

$$\widehat{x}(t) = \sum_{k=1}^{K} E_{\tau} \bigg\{ h(\tau) \Theta_{z_s}^k(t,\tau) \bigg\},$$
(10)

where $\Theta^k_{z_s}$ is the phase of the k^{th} sub-kernel function on $z_s(t)$

$$\Theta_{z_s}^k(t,\tau) = \arg\left\{K_{z_s}^k(t,\tau)\right\} / 2\pi\tau\mu$$
(11)
= $\arg\left\{\prod_{i=1}^2 \left\{z_s(t+\tau+a_{ik})\right\}^{\frac{b_{ik}}{2\pi\tau}} \left\{z_s^*(t-\tau-a_{ik})\right\}^{\frac{b_{ik}}{2\pi\tau}}\right\} / \mu_s$

where $a_{1k} = 0$, $a_{2k} = \frac{1}{4f_{ik}(t)}$, $b_{1k} = \pi f_{ik}(t)\tau \sin(2\pi f_{ik}(t)\tau)$, $b_{2k} = \pi f_{ik}(t)\tau \cos(2\pi f_{ik}(t)\tau)$, and $f_{ik}(t)$ is the IF of the k^{th} component. Note that from (10) each component $x_k(t)$ can be separated and filtered from the signal s(t) by

$$\widehat{x}_k(t) = E_\tau \bigg\{ h(\tau) \Theta_{z_s}^k(t,\tau) \bigg\}.$$
(12)

The filtering based on (10) is named as the robust timevarying filtering (RTVF) since the noise variance can be greatly reduced by using a long window $h(\tau)$. The difference of the RTVF from the WWVD-TFPT is that the signal is filtered by performing the operation in (10) instead of detecting the peak of the WWVD of $z_s(t)$. From the viewpoint of implementation, the operation in (10) is less complex by avoiding the use of FFT operation. To implement the RTVF, the IF information $f_{ik}(t)$ in (11) should be known or estimated in advance. It will be demonstrated that one iteration is sufficient for a desirable filtering performance. However, the filtered component using the initial IF estimate offers a new IF estimate, which means that the RTVF can be formulated in an iterative manner, i.e., the RTVF is repeated by updating the IF information until the satisfied performance is achieved.

3. BIAS ANALYSIS OF THE RTVF ALGORITHM

The RTVF algorithm is proved to be unbiased in (9) for noisy stationary signals with known f_m . In practice, signals with varying frequency as well as estimated IFs are often encountered, which makes the RTVF algorithm be biased. The zeromean white noise does not introduce stochastic bias to signal estimate, and the bias only comes from the deterministic signal. In the following, the bias of monocomponent signal filtering and the interference of multicomponent separation are analyzed. Afterwards, the optimal lag window length of $h(\tau)$ is determined by analyzing the derived bias expressions.

3.1. Bias analysis of RTVF for monocomponent signals

Defining $\tau \in \left[-\frac{L}{2f_s}, \frac{L}{2f_s}\right]$, where f_s is the sampling rate and L is the lag window length. According to (10) (where K = 1), the bias of the RTVF for monocomponent signals can

be computed by $B(t) = x(t) - \mathbb{E}\{\hat{x}(t)\}\)$, where $x(t) = \rho \cos\left(2\pi \int_0^t f_i(\lambda)d\lambda + \theta\right)$. Let us consider the IF estimation error, $\Delta_f(t) = \hat{f}_i(t) - f_i(t)$. Without specially referring to any particular IF estimator, the IF error is modeled as a zeromean Gaussian noise process with a power variance σ_e^2 . According to the Weierstrass approximation theorem [11], we assume the received signal with an arbitrary frequency variation is approximated as a linear frequency modulated (LFM) signal $x(t) = \rho \cos(2\pi f_0 t + \pi r_m t^2 + \theta)$ within the lag window duration, where f_0 is the initial frequency and r_m is the chirp rate. The estimated signal by the RTVF is derived as

$$\hat{x}(t) = E_{\tau} \left\{ h(\tau) \Theta_{z_{x}}(t,\tau) \right\} = E_{\tau} \left\{ \sum_{k=1}^{2} \rho b_{k} \right\}$$
(13)
$$\cdot \left(\frac{\sin \left[2\pi f_{0}(t+\tau+a_{k}) + \pi r_{m}(t+\tau+a_{k})^{2} + \theta \right]}{f_{0} + r_{m}(t+\tau+a_{k})} - \frac{\sin \left[2\pi f_{0}(t-\tau-a_{k}) + \pi r_{m}(t-\tau-a_{k})^{2} + \theta \right]}{f_{0} + r_{m}(t-\tau-a_{k})} \right\},$$

where $a_1 = 0$, $a_2 = \frac{1}{4\hat{f}_i(t)}$, $b_1 = \hat{f}_i(t) \sin(2\pi \hat{f}_i(t)\tau)/2$, $b_2 = \hat{f}_i(t) \cos(2\pi \hat{f}_i(t)\tau)/2$, and $\hat{f}_i(t) = f_0 + r_m t + \Delta_f(t)$.

It is difficult to deduce a closed-form bias expression from (13) for a signal with time-varying frequency and IF estimate error. The biases resulting from varying frequency and estimated IF error, give different effects on the signal amplitude and signal frequency. Thus, we independently analyze the biases due to estimated IF error and signal's nonstationarity.

In the case of constant $f_i(t)$ with estimate errors, the estimated IF is $\hat{f}_i(t) = f_0 + \Delta_f(t)$. The bias of the RTVF is derived from (13)

$$B_1(t) \approx x(t) \cdot \mathbb{E}\left\{1 - \frac{\left(f_0 + \Delta_f(t)\right)}{f_0} E_\tau\left\{\cos[2\pi\Delta_f(t)\tau]\right\}\right\},\tag{14}$$

which shows that the value of the bias depends on Δf , L and x(t). The maximum deviation occurs at a peak or a valley of the sinusoidal waveform, whereas the deviation is zero when x(t) = 0, which means that the effect of the inaccurate IF estimation is to reduce the signal amplitude and does not affect the signal frequency.

In the case of time-varying $f_i(t)$ with $\Delta_f(t) = 0$, the bias for time-varying $f_i(t)$ is simplified into

$$B_{2}(t) \approx \rho \cos(2\pi f_{0}t + \pi r_{m}t^{2} + \theta)$$
(15)
- $E_{\tau} \{ \cos(2\pi f_{0}t + \pi r_{m}t^{2} + \pi r_{m}\tau^{2} + \theta) \}.$

3.2. Interference of RTVF for component separation

The RTVF algorithm can be used for component separation according to (12). Next, we analyze the cross-interference between different components, and derive a sufficient condition under which the component separation in (12) can be successfully implemented. Due to space limitation, we assume a multicomponent signal, where each component has constant amplitude and frequency. Defining the encoded multicomponent signal $z_x(t) = e^{j2\pi\mu} \int_0^t \sum_{k=1}^K \rho_k \cos(2\pi f_{mk}\lambda + \theta_k) d\lambda$, we derive that the k^{th} component can be successfully separated from others provided that

$$\sum_{k'\neq k}^{K} \rho_{k'} \cos(2\pi f_{mk'}t + \theta_{k'})$$

$$\cdot E_{\tau} \left\{ \frac{f_{mk}}{f_{mk'}} \sin(2\pi f_{mk}\tau) \sin\left(2\pi f_{mk'}(\tau)\right) \qquad (16)$$

$$+ \frac{f_{mk}}{f_{mk'}} \cos(2\pi f_{mk}\tau) \sin\left(2\pi f_{mk'}(\tau + \frac{1}{4f_{mk}})\right) \right\} \approx 0.$$

It is seen that the interference value is dependent on the window length as well as the frequency difference between the component of interest and other ones.

3.3. Determination of optimal window length

We are more concerned with the frequency information rather than amplitude information. It is noted from (14) and (15) the frequency information is slightly impacted by estimated IF errors and significantly affected by signal's nonstationarity. Therefore, the optimal window size for spectral estimation can be chosen by constraining the maximum of the elementary part $\pi r_m \tau^2$ in (15)

$$\max_{\tau} \left\{ \left| \pi r_m \tau^2 \right| \right\} = \pi r_m (\frac{L}{2f_s})^2 < \frac{\pi}{N},$$
(17)

where N is a constant coefficient used to control the quantity of bias. As a result, the expression of the window length as a function of f_s , r_m , and N, is expressed by

$$1 \le L \le \frac{2f_s}{\sqrt{Nr_m}}.\tag{18}$$

Although the above analysis is based on LFM signal model, the derived result in (18) is also suitable for other nonstationary signals. The reason is that the segment of a random nonstationary signal within the lag window can be approximately viewed as an LFM signal.

4. NUMERICAL RESULTS

Monocomponent filtering: The initial IF estimation for the RTVF is conducted by detecting the peaks based on the WWVD of the filtered signal by the WWVD-TFPF. We consider an LFM signal and an SFM signal. The sampling rates of LFM and SFM signal are f_s =2560 Hz and 2048 Hz, respectively. The window lengths of the WWVD-TFPF and the RTVF for LFM signal are set as 9 and 301 samples according to [7] and (18), whereas the lengths for SFM signal are 7 and 151, respectively. The simulation results of filtering and IF



Fig. 1. (a) Filtering results in time (samples) and IF estimation for an LFM signal with SNR=-10 dB (left) and an SFM signal with SNR=-4 dB (right) ((i) filtered signal by WWVD-TFPF. (ii) IF estimation of (i). (iii) filtered signal by RTVF. (iv) IF estimation of (iii)). (b) NMSEs of IF estimation of filtered LFM signal (top) and SFM signal (bottom). (c) IF estimation of LFM signal with SNR=-12 dB (left) and SFM signal with SNR=-5 dB (right) by the iterative RTVF ((i) WWVD-TFPF. (ii) RTVF with 1 iteration. (iii) RTVF with 2 iterations. (iv) RTVF with 3 iterations).



Fig. 2. (i) WWVD of unfiltered signal. (ii) BD of unfiltered signal. (iii-iv) BDs of the two separated SFM components by RTVF (1 iteration). (v) BD of clean multicomponent signal.

estimation are shown in Fig. 1 (a). The RTVF can better restore the corrupted signal compared to the WWVD-TFPF. Moreover, the RTVF is more effective in minimizing the noise effects so that IF estimation is more accurate. The normalized mean square errors (NMSEs) of IF estimation of the filtered signals in Fig. 1 (a) are given in Fig. 1 (b). For performance assessment, the RTVF with perfect IF information is also simulated. Two issues are to be stressed. Firstly, compared to the case with known IF, the RTVF with estimated IF obtains desirable performance when SNR>-10 dB (LFM) and SNR>-4 dB (SFM). Secondly, the RTVF outperforms the WWVD-TFPF by a large margin. It should be mentioned that the performance gap between the WWVD-TFPF and the RTVF in Fig. 1 (a) and (b) can be further enlarged by applying the iterative RTVF. Fig. 1 (c) shows the IF estimation results using the WWVD-TFPF and the iterative RTVF. We note that desired IF estimation performance is achieved after two or three iterations.

Multicomponent separation: The B-distribution (BD) [12] is chosen to handle multicomponent signals because it has a desirable capability of cross-term suppression. The initial IF estimation of multicomponent signals for the RTVF is

obtained based on the BD of the noisy signal by using the method in [13]. A noisy multicomponent signal with two overlapped SFM components is simulated at SNR=-2 dB. The sampling rate and the window length are 1024 Hz and 65 samples, respectively. The BDs of the multicomponent signal with and without noise is shown in Fig. 2 (ii) and (v). Based on the estimated IFs of the two SFM components, the RTVF is used for separating the multicomponent signal into individual components. The BDs of two separated SFM components are shown in Fig. 2 (iii-iv). Note that the SFM components are well filtered and separated with a much better TF resolution.

5. CONCLUSION & RELATION TO PRIOR WORK

The proposed RTVF algorithm is an extension of the WWVD-TFPF algorithm in [7]. The RTVF can substantially improve the filtering performance, and it is accomplished with linear complexity $O(KN_s)$ (N_s is number of samples) due to the avoidance of FFT operation and peak search process. Two practical problems are solved. Firstly, the requirement of high sampling rates is relaxed by using large window sizes. Secondly, the RTVF can efficiently separate multicomponent signals into individual components at low SNR levels because of its superior performance in suppressing interferences.

The RTVF can be viewed as a novel time-varying filter compared to conventional time-varying filters [14–17], which obtain the underlying signal by an inverse TF operation of a masked TFD. However, the masked TFD is generally not valid for inversion process. The RTVF is not based on TF inversion, thus the validity problem is avoided. Besides, conventional time-varying filters require accurate IF estimate [18–20]. Although an initial IF estimation is also required, the iterative RTVF process can tolerate certain amount of IF errors and gradually obtain more accurate IF information.

6. REFERENCES

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