COMBINING SPREAD SPECTRUM COMPRESSIVE SENSING WITH RAKENESS FOR LOW FREQUENCY MODULATION IN RMPI ARCHITECTURE

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ABSTRACT

In this work we combing two novelty in the area of Analog Information Converter based on Compressed Sensing. A new architecture, the Spread Spectrum Random Modulation Pre-Integration and a new design flow, the rakeness based design of a Compressed Sensing system. We demonstrate that combining these approaches produces a strong reduction of the internal chipping frequency in the sensing coupled with a high compression ratio with respect to standard Analog to Digital Converter.

Index Terms— Compressive Sensing, Spread Spectrum, Rakeness, ECG, Analog to Information Converter, RMPI

1. INTRODUCTION

In the last half decade, Compressive Sensing (CS) [1, 2] has triggered a great deal of interest thanks to its capability to merge signal acquisition and compression tasks, thus paving the way for the design and implementation of Analog to Information Converters (AICs) [3][4], which are able to acquire all the signal information using less samples with respect to a Nyquist-rate ADC.

This is possible thanks to the fact that the N-dimensional input signals x to be processed are K-sparse, i.e., such that a suitable basis $\Psi = \{\psi_1, \ldots, \psi_N\}$ exits so that $x = \Psi\alpha$, where Ψ is a $N \times N$ matrix with column vectors ψ_j, $j = 1, \ldots, N$ and the coefficient vector α has at most K non-vanishing elements.

Several possible approached have been proposed to implement an AIC (see [4] and references therein). We here focus on the classic Random Modulation Pre Integration (RMPI) architecture, first presented in [3]. In a RMPI architecture, CS acquisition is achieved by integrating the product of a input signal time window with a suitable set of N-dimensional sensing functions ϕ_j, $j = 1, \ldots, M$. With this, the signal information content is represented by a measurement vector $y = \Phi x$ where Φ is a $M \times N$ sensing matrix whose row are the sensing functions ϕ_j. The reconstruction of x can be achieved by solving the following optimization problem, which relies on the sparsity assumption [2]

$$\begin{aligned} \hat{\alpha} &= \min \|\alpha\|_{l1} \\ \text{s.t. } & y = \Phi\Psi\alpha \end{aligned} \tag{1}$$

where $\|\cdot\|_{l_1} = \sum|\cdot|$ is the standard l_1 norm and $\hat{x} = \Psi\hat{\alpha}$ is the reconstructed input signal. The convergence of $\hat{x}$ to x is guaranteed by the so-called *restricted isometry* property of the matrix $\Phi\Psi$ which roughly ensures that its application must be able to conserve the input signal l_2 norm [1, 2, 5]. When restricted isometry holds, the input signal reconstruction is guaranteed also with $M \ll N$, i.e., with a related compression ratio equal to N/M. Interesting enough, restricted isometry is always is achieved by considering Φ composed by instances of a collection of independent and identically distributed random variables: in this case classical CS theory [2] guarantees reconstruction for $M \geq M_{\min} = 4K\log(N/K)$.

To improve the performance of an RMPI, two different approaches were recently proposed. The first [8, 6] relaxes the restricted isometry property when the class of signals to acquire is also *localized*,i.e., the information content is not only sparse, but also non-uniformly distributed in the whole signal domain. In this setting, random sensing functions are also designed to maximize their *rakeness*, that is the average energy which one is able to collect (i.e. rake) when the input signal is projected into them. Exploiting such an approach one is able to reduce $M_{\min}$ guaranteeing a correct reconstruction [8]. The second, called Spread Spectrum RMPI (SRMPI) [10], exploits an innovative encoder architecture that increase the RMPI performance by introducing a pre-spreading of the input signal. Its main advantage is to reduce the system power consumption by lowering the frequency switching in the sensing stage [1].

In this paper, we combine both the previous approaches. With respect to a standard SRMPI, we will be able to both further decrease the internal switching frequency and to re-

[1] In [10] it shown that it is possible to halve the internal frequency switching.

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duce the amount of needed projections to reach a properly reconstructed signal. Despite our approach is fully general, to make a fair comparison, we demonstrate the achievable advantages referring to electrocardiograms (ECGs), i.e., the same signal employed when SRMPI and rakeness were originally presented [9, 8].

The paper is organized as follow: Sec. 2 reports the mathematical models grounding SRMPI and rakeness-based design of a CS system and presents the methodology we propose to combine SRMPI and rakeness. Sec 3 describes the system setting and obtained results are reported.

2. SRMPI ARCHITECTURE AND RAKENESS-BASED DESIGN OF A CS SYSTEM

2.1. SRMPI

As in the RMPI architecture, also for the SRMPI case we assume that the the input signal is acquired at Nyquist rate in a time window of length T. With this, $x \in \mathbb{R}^N$ in the block diagram of fig. 1(a) is a vector collecting the corresponding N samples (i.e. the sampling rate of the input signal is N/T, which is then multiplied by a spreading sequence (SS) composed by $n \geq N$ random antipodal symbols (i.e. its chipping frequency is n/T) and finally put at the the input of M parallel RMPI circuits.

To satisfy the restricted isometry property, all sensing functions ϕ_j, $j = 1, \ldots, M$ are assumed composed by antipodal independent and identically distributed random symbols with length $m \leq N \leq n$ (i.e. with chipping frequency m/T) . The output of the SRMPI is a measurements vector $y \in \mathbb{R}^M$ sampled at frequency $1/T$.

To get an intuitive flavor of the system behavior one may refer to the simple case $n = m = N$, so that $y = \Phi\, S\, x = F\, x$, where $S \in \mathbb{R}^{N,N}$ is a diagonal matrix whose non-null elements are the SS coefficients. Hence, the matrix $F = \Phi S$ represents the linear operator that links the input signal to the measurements vector.

The main goal of this architecture is to enable a reduction of the operating frequency of all switches used in a possible implementation of this architecture, which means that one needs to consider an operating condition where both m and n are as low as possible. Noting that $n \geq N$ by system construction, the most obvious choice is $n = N$. The additional degree of freedom is the value of m which we set $< N$, to achieve an undersampling factor $r = \frac{N}{m}$ with respect to the a system where all switches operate at the Nyquist frequency. With these assumptions, the generic measurement can be expressed as

$$y_i = \sum_{j=1}^{N} \Phi_{i,\lceil \frac{j}{r} \rceil} S_{j,j}\, x_j \qquad j = 1, \ldots, M$$

where $\lceil \cdot \rceil$ is the ceiling operator. In this case we can write

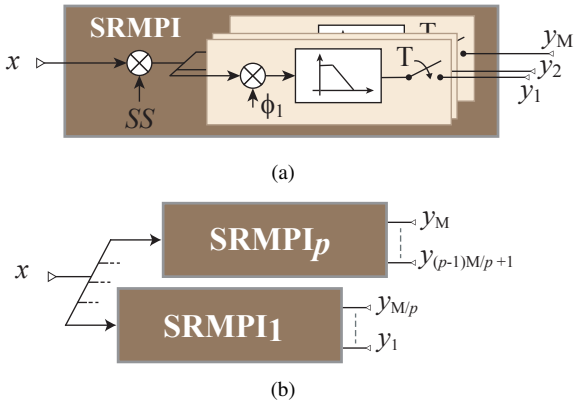

Fig. 1: block diagrams of spread spectrum random modulation pre-integration (a) and multi spread spectrum random modulation pre-integration (b).

again $y = F\, x$ where $F \in \mathbb{R}^{M,N}$ with

$$F_{i,j} = \Phi_{i,\lceil \frac{j}{r} \rceil} S_{j,j} \tag{2}$$

Hence an SRMPI system is equivalent to a classical RMPI architecture where a sensing matrix F is assumed.

2.2. CS based on Rakeness

As mentioned in the Introduction, the aim of this technique is to increase the energy collected by the sensing stage preserving at the same time the restricted isometry property of the acquisition operator. To do so, we need to define the *rakeness* between the two stochastic processes $\underline{\phi}$ and $\underline{x}$ generating the sensing functions and the signals instances as

$$\rho(\underline{\phi}, \underline{x}) = \mathbf{E}_{\underline{\phi},\underline{x}} \left[|\langle \phi, x \rangle|^2 \right]$$

where ϕ is a generic sensing function and x a single input instance. With this, the acquisition of the maximum energy during the sampling phase can be obtained using sensing functions ϕ solving

$$\begin{array}{ll} \max_{\phi} & \rho(\underline{\phi}, \underline{x}) \\ \text{s.t.} & \langle \phi, \phi \rangle = e \\ & \rho(\underline{\phi}, \underline{\phi}) \leq \tau e^2 \end{array} \tag{3}$$

where e is the energy of each sampling vector and where the second constraint is need to ensure that the solutions cope with the restricted isometry property. The parameter τ can be tuned on a proper range to set the randomness of $\underline{\phi}$, but whose specific value is not critical since it does not appreciably alter the overall system performance [6, 7].

As amply discussed in [8], the optimization problem (3) can be solved for a fixed class of input signal, and its output is the second-order statistical characterization of $\underline{\phi}$ represented by its correlation matrix C_ϕ. Roughly speaking, adopting rakeness means shaping the statistical characterization of $\underline{\phi}$ on

the characteristic of $\underline{x}$ with a consequently improvement in terms of signal reconstruction with respect to standard CS exploiting sampling functions based on independent and identically distributed random symbols.

2.3. Combining SRMPI with Rakeness

As shown in Sec. 2.1, for a SRMPI architecture the sensing stage is represented by the matrix F, so that adopting a rakeness-based acquisition in this case reduces to obtain the correlation matrix C_F which solves the optimization problem (3) and which characterizes the process generating the rows of F.

Taking (2) into account, C_F needs to be changed by tuning the correlations of the processes generating the rows of Φ and different spreading sequences SS. Consequently, in principle we should consider the correlation of the product of the instances of two stochastic processes, which is very hard task. To overcome this impasse and find a suitable solution to our problem, we will fix the statistics of Φ, whose element will be assumed i.i.d. antipodal random variables, so that the statistical features of SS will be our degrees of freedom.

Let us to indicate with $C_{F_{\{j,k\}}}$ the correlation between two elements of F in the same row and at columns j and k

$$C_{F_{\{j,k\}}} = \mathbf{E}\left[F_{\cdot,j}F_{\cdot,k}\right] = \mathbf{E}\left[\Phi_{\cdot,\lceil \frac{j}{r} \rceil}\Phi_{\cdot,\lceil \frac{k}{r} \rceil}S_{j,j}S_{k,k}\right] =$$

$$= \begin{cases} \mathbf{E}\left[S_{j,j}S_{k,k}\right] & \lceil \frac{j}{r} \rceil = \lceil \frac{k}{r} \rceil \\ \mathbf{E}\left[\Phi_{\cdot,\lceil \frac{j}{r} \rceil}\Phi_{\cdot,\lceil \frac{k}{r} \rceil}S_{j,j}S_{k,k}\right] & \lceil \frac{j}{r} \rceil \neq \lceil \frac{k}{r} \rceil \end{cases}$$

where the second equality holds since $\Phi_{\cdot,j}\Phi_{\cdot,j} = 1$. Furthermore since $\Phi_{\cdot,j}$ are composed by i.i.d random variable we can further obtain

$$C_{F_{\{j,k\}}} = \begin{cases} \mathbf{E}\left[S_{j,j}S_{k,k}\right] & \lceil \frac{j}{r} \rceil = \lceil \frac{k}{r} \rceil \\ 0 & \lceil \frac{j}{r} \rceil \neq \lceil \frac{k}{r} \rceil \end{cases} \quad (4)$$

As a result, in this setting, fixing the correlation of the spreading sequences SS to the one resulting by solving (3) is equivalent to impose the desired correlation to the elements of the rows of F *only* in the cases where $\lceil \frac{j}{r} \rceil = \lceil \frac{k}{r} \rceil$. As a consequence, to achieve a better rareness-based design one would need to increase r as much as possible. Yet, adopting high values of r can degrade the *sensing diversity*, i.e., it may increase the probability that two or more rows of F are quite similar. Hence, a too large value of r may result in destroying the total amount of collected information in the sensing stage. On the contrary, with lower values of r we preserve the amount of collected signal information but we may lose part of the benefit introduced by the rakeness approach. In addition to the above difficulty, the value of r achievable by directly applying a rakeness based approach to an SRMPI architecture is hardly limited by the fact that in the original SRMPI proposal $r > 2$ is cannot be obtained [9].

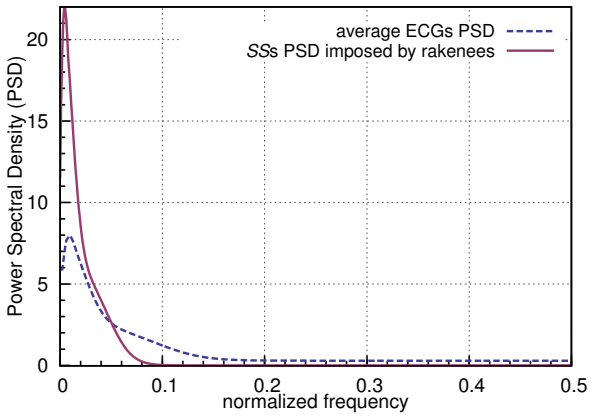

Fig. 2: average of normalized power spectral density of ECG signals (dashed curve) and power spectral densities obtained by solution of rakeness optimization problem (solid curve).

To cope with this, we propose a Multi Spread Spectrum RMPI (MSRMPI) approach, whose block scheme is reported in Fig. 1(b). It is composed by p different SRMPI where: *i*) the SSs are generated following the correlation imposed by the solution of (3), *ii*) each block produce M/p measurements which can be collected to have the measurement vector y. This architecture possesses the additional advantage of improving the behavior with respect to the *diversity reduction* imposed by the direct combination of the rakeness based approach with the SRMPI architecture. In fact the MSRMPI uses p different spreading sequences with the aim to preserving the collected input signal information by a reduction of the probability that similar F rows appear.

3. SIMULATION SETTING AND RESULTS

To test the performance of the MSRPI architecture we rely on ECGs which can be considered sparse signals with respect to a collection of Gabor Functions (GF) [8, 11].

The signal instances are synthetic ECG generated by [12] with the same setting described in [8], sampled at 256 Hz with a 1 second time window. The optimization problem (3) is solved with a solution expressed in terms of power spectral density (PSD) instead of correlation matrix, which is of course fully equivalent since the ECG process is cycle stationary (see [8] for more details). In other words, assuming the estimation of the average ECG PSD as the input, the solution of the rakeness optimization problem is the average PSD of the sequences that characterize the sensing stage. For our setting, both profiles are shown in Fig. 2.

It is important to note that the average PSD of ECGs is close to zero in the high frequency range, but this does not mean that no instance possesses components at high fre-

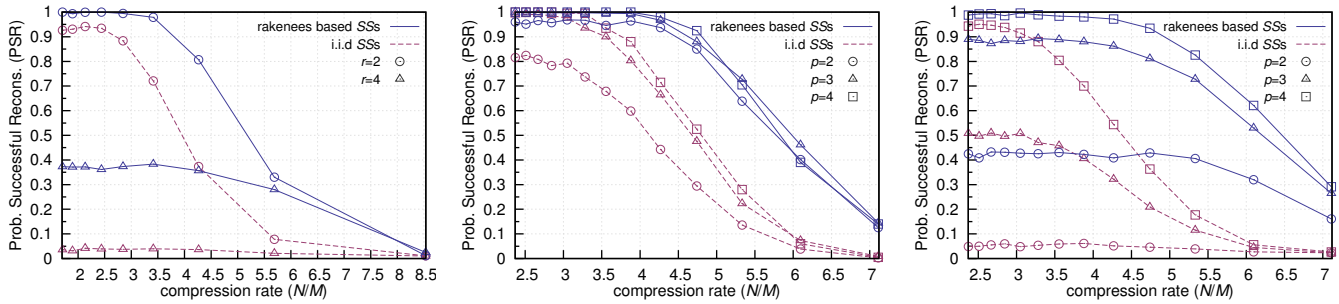

Fig. 3: plot a: PSR as function of compression rate (N/M) where the encoding is done by SRMPI with i.i.d. SSs (dashed lines) and SSs with PSD imposed by rakenees (solid lines) for $r = \{2, 4\}$; plot b: PSR as function of compression rate (N/M) where the encoding is done by MSRMPI with i.i.d. SSs (dashed lines) and SSs with PSD imposed by rakenees (solid lines) for $r = 4$ and $p = \{2, 3, 4\}$; plot c: PSR as function of compression rate (N/M) where the encoding is done by MSRMPI with i.i.d. SSs (dashed lines) and SSs with PSD imposed by rakeness (solid lines) for $r = 8$ and $p = \{2, 3, 4\}$.

quency, but only that the probability of such an event is low. As it can be seen, a rakeness-based design copes with this features by imposing a non-zero PSD of the spreading sequences also at high frequencies.

To assess performance improvement of the MSRMPI architecture we consider two cases: i.i.d SSs [9], which will be our reference case, and SSs with PSD imposed by a rakeness-based design and generated by a Linear probability Feedback Process [13, 14, 15].

All results are obtained by performing montecarlo simulation over 2000 trials, where the input signal reconstruction is done by (1) adopting Cplex optimization toolbox[2]. As a figure of merit, we use the probability of successful reconstruction (PSR), defined as PSR $= Pr(\|\alpha\| \geq 20\|\alpha - \hat{\alpha}\|)$.

We simulated first the SRMPI architecture with $r = \{2, 4\}$. The related results are shown on Fig. 3-(a). When $r = 2$, a slight improvement can be obtained by rakeness based design with respect to the reference i.i.d. case and that PSR=1 can be obtained only exploiting rakeness. As expected, such an improvement with respect to the reference case increases when $r = 4$, but PSR=1 is never achieved due to the intrinsic limitation of the SRMPI architecture.

The MSRMPI architecture was tested first for $r = 4$. The PSR trends are shown in Fig. 3-(b) which clarifies the advantages introduces by this approach. Here we are able to reach PSR=1 using $p = 3$, i.e., three different spreading sequence, in both considered environment either with i.i.d or rakenees based spreading sequences but rakenees approach reach PSR=1 with a $N/M \leq 4$ while i.i.d. sequences reach PSR=1 only for $N/M \leq 2.5$.

The case $r = 8$ was also analyzed. This correspond to $m = 32$ and implies a strong reduction of the chipping frequency in the sensing. The results in terms of PSR are shown on Fig. 3-(c) which show similar improvement as Fig. 3-(b) for $r = 4$.

[2]web site:
http://www-01.ibm.com/spftwere/integration/optimization/cplex-optimizer

4. CONCLUSIONS

We demonstrated that combining rakeness with SRMPI produces a strong reduction of the internal chipping frequency in the sensing if a multi spread architecture is used. We have shown that MSRMPI is able to reach perfect signal reconstruction by means of 4 SSs operating at Nyquist rate and 85 projections done with an internal frequency reduction by a factor 8 with respect to the Nyquist rate. In the same setting, classical RMPI arrives to PSR=1 only with more then 100 projections all of them computed at Nyquist rate[3].

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[3]As a further term of comparison note that classical RMPI needs a minimum amount of projections related to the sparse level of the input signal K by the low $M > C\,K\log_{10}(N/K)$, where C is a constant usually equal to $4; 5$. For ECG sparse on GF and in the considered simulation setting we have K in average greater then 20, so M is around 100.

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