

Iterative Blind Estimation of Nonlinear Channels

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Abstract—Nonlinear distortions in analog frontends are becoming a growing problem which is not limited to power amplifiers. Modern modulation methods such as OFDM and next generation standards have high linearity requirements on all components in the signal path. A radio system that can tolerate a certain degree of nonlinear distortion without substantial loss of performance could enable high cost savings in development and production. In this paper we present a novel iterative blind estimator for nonlinear distortions. It complements existing mitigation algorithms by providing them with accurate estimates of the nonlinearity characteristic. It is shown that there is a negligible performance gap between perfect and estimated knowledge. The method is designed to be computationally inexpensive and can be readily implemented on today's digital signal processing systems.

I. INTRODUCTION

Nonlinear distortion caused by analog frontend components can severely reduce the performance of communications systems. OFDM signals are particularly affected because they exhibit a very high peak to average power ratio (PAPR) and hence require amplifiers with linear characteristic over a large input range. This is not only expensive to develop and produce, it also often renders the amplifier very energy inefficient [1]. If OFDM signals are nonlinearly distorted, inter carrier interference is induced [2].

Predistortion is an approach that has drawn a lot of research attention for linearization of amplifiers [3]. Here, knowledge of the nonlinear amplifier characteristic is used to design a predistortion nonlinearity such that the overall characteristic is linearized. However, since temperature, aging and tolerances change the amplifier characteristics over time, calibration circuitry or specially designed pilots are required. Otherwise, residual nonlinearity will remain. Similarly, many models and algorithms exist for receiver based mitigation of nonlinear distortions [4]–[6]. Those methods usually require knowledge of the nonlinearity characteristic which is often assumed to be perfectly known in publications.

To avoid additional pilots that break compatibility with current methods, blind estimation based on observations of the received signal is desired. In [7] we presented a low-complexity method for parameter estimation of a memoryless AM/AM nonlinearity. The method was implemented on a software defined radio (SDR) platform and verified to give performance gains with real amplifiers driven in saturation [8]. In [9] this method was extended to AM/PM nonlinearities and it was shown that in this case computational efforts are very high. All of these techniques employed maximum likelihood

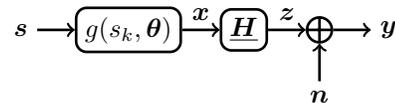


Fig. 1. Transmission Model

estimation based only on the received samples. In this publication we present a novel iterative method that improves the estimate by incorporating feedback from the OFDM detector. It also has a much lower complexity compared to the earlier approaches. To the best knowledge of the authors, not much research has been done in this field. A recent approach for blind estimation of AM/AM characteristics using a Kalman filter is presented in [10].

The rest of the paper is structured as follows. In Section II we present the system model. In Section III the iterative blind estimator is derived. Section IV shows the simulation results and the paper is concluded in Section V

II. SYSTEM MODEL

A. Transmission Model

The transmission model is shown in Fig.1. The input vector s corresponds to the time-domain baseband samples belonging to one OFDM symbol. It has been shown in [11] that s can be statistically described as a zero-mean complex normally distributed random vector, i.e. $s \sim CN(\mathbf{0}, \underline{C}_s)$. The covariance matrix \underline{C}_s depends on the power allocation per subcarrier and if there is no guardband and all subcarriers have equal power then $\underline{C}_s = \sigma_s^2 \mathcal{I}$. The nonlinearity $g(s_k, \theta)$ is modelled as a memoryless nonlinearity (i.e., the output of the nonlinearity at a certain time depends only on the input at the same time) that can be decomposed into an amplitude (AM/AM) and phase (AM/PM) distortion as follows:

$$g(s_k, \theta) = g_A(|s_k|, \theta) \cdot e^{j(\angle(s_k) + g_\phi(|s_k|, \theta))}. \quad (1)$$

$\angle(s_k)$ denotes the phase of the complex-valued sample s_k . The nonlinearity is assumed to be known at the receiver except for a parameter vector θ . The nonlinearity is modelled on the transmitter side and hence before the frequency selective channel which is represented by the matrix \underline{H} . In most scenarios, \underline{H} will be a circular convolution matrix but the estimator is not limited to this case. In the following, \underline{H} is generally assumed to be known perfectly at the receiver. Finally, \mathbf{n} is a vector representing an additive noise contribution. For the sake of

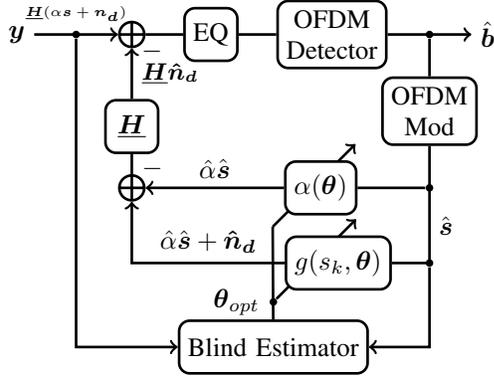


Fig. 2. Decision Feedback Receiver with estimator

the estimator it is only important that \mathbf{n} is independent of \mathbf{x} . A common model is additive white Gaussian noise which models the elements n_k as i.i.d. zero-mean complex normally distributed, i.e. $n_k = CN(0, \sigma_n^2)$ where σ_n^2 is the average noise power.

B. TWTA Nonlinearity

A concrete example of a nonlinearity is the travelling wave tube amplifier (TWTA) [14], which is a very popular model exhibiting AM/AM and AM/PM distortions. According to this model, the amplitude and phase nonlinearities acting on the input signal s_k are described by the following equations:

$$g_A(|s_k|, \theta) = \frac{\alpha_A |s_k|}{1 + \beta_A |s_k|^2} \quad (2)$$

$$g_\phi(|s_k|, \theta) = \frac{\alpha_\phi |s_k|^2}{1 + \beta_\phi |s_k|^2} \quad (3)$$

The parameter α_A is entirely linear and indistinguishable from other attenuation such as pathloss. Hence, in many real systems it will be handled as part of gain control. Therefore, to achieve unity gain in the linear range of the amplifier, $\alpha_A = 1$. Hence, the parameter vector which is to be estimated becomes $\theta = [\beta_A, \alpha_\phi, \beta_\phi]$. This model is used in the simulations below.

C. Decision Feedback Receiver

Fig.2 shows the decision feedback receiver as presented in [4]. It is based on the Bussgang theorem [12] which states that the output of a memoryless nonlinearity can be decomposed into a linear attenuation and phase rotation α and an additive term as follows:

$$g(s) = \alpha s + \mathbf{n}_d \quad (4)$$

if the input signal s is normally distributed. The receiver assumes \mathbf{n}_d as an additive noise signal which is to be removed. To that end, in each iteration, the signal is equalized and demodulated using common OFDM techniques. The estimated information bits $\hat{\mathbf{b}}$ are then remodulated to form an estimate of the transmit signal $\hat{\mathbf{s}}$. This signal is run through a simulated nonlinearity and separately multiplied with $\hat{\alpha}$ and the resulting signals are subtracted to generate an estimate $\hat{\mathbf{n}}_d$. The channel is then applied to this signal and it is subtracted from the input. It can be seen that in the best case where both, the

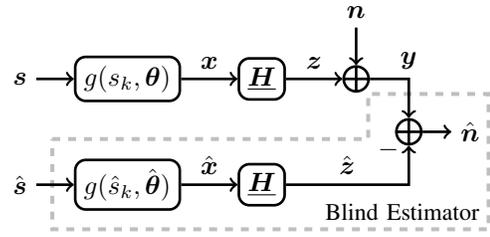


Fig. 3. Blind Decision Feedback Estimator

feedback and nonlinearity knowledge are perfect, $\hat{\mathbf{n}}_d = \mathbf{n}_d$ so the resulting signal at the input of the detector will be $\underline{\mathbf{H}}\alpha\mathbf{x}$. Hence, in the best case this receiver will exhibit a SNR-loss ρ_{df} of

$$\rho_{df} = 20 \log_{10} |\alpha| \quad (5)$$

compared to linear transmission. The reason is that the receiver assumes \mathbf{n}_d as noise and removes it completely. A different architecture could use the information in \mathbf{n}_d for better reception performance.

It can be seen that the decision feedback receiver requires knowledge about the nonlinearity $g(s_k)$. It was described earlier that the nonlinearity is assumed as known despite a parameter vector θ which needs to be found. It is not desired to introduce additional pilots into the system so the information should be acquired using blind estimation methods. A method employing the decision feedback is described in the following.

III. BLIND DECISION FEEDBACK ESTIMATOR

The blind decision feedback estimator is shown in Fig.3. The estimator itself is visualized by a dashed gray frame. It can be seen that its input are the received signal \mathbf{y} and the estimate of the transmit signal $\hat{\mathbf{s}}$ generated from the detected bits $\hat{\mathbf{b}}$ as shown in Fig.2. For every candidate solution θ the estimator calculates

$$\hat{\mathbf{z}} = g(\hat{\mathbf{s}}, \hat{\theta})\underline{\mathbf{H}} = \hat{\alpha}\hat{\mathbf{s}}_k\underline{\mathbf{H}} + \hat{\mathbf{n}}_d\underline{\mathbf{H}}, \quad (6)$$

where the Bussgang theorem is used to decompose the effect of the nonlinearity into a linear complex multiplication and an addition. The resulting signal $\hat{\mathbf{z}}$ is then subtracted from the received signal \mathbf{y} to obtain estimates of the AWGN signal:

$$\hat{\mathbf{n}} = \mathbf{y} - \hat{\mathbf{z}} = \mathbf{n} + \underbrace{(\alpha\mathbf{s} - \hat{\alpha}\hat{\mathbf{s}} + \mathbf{n}_d - \hat{\mathbf{n}}_d)\underline{\mathbf{H}}}_{\mathbf{n}_e}. \quad (7)$$

\mathbf{n}_e represents the combined error signal that arises due to estimation errors in $\hat{\mathbf{s}}$ and $\hat{\theta}$. Following the same line of thought as with the Bussgang theorem, \mathbf{n}_e can be modelled as a zero-mean complex random variable which, except in special cases, will usually not be Gaussian distributed.

A. Likelihood estimation

The PDF $p_n(\mathbf{n})$ of the additive noise \mathbf{n} is usually known and easy to evaluate. Following the same line of thought as the very common maximum likelihood estimator, the following optimization

$$\theta_{opt} = \arg \max_{\hat{\theta}} p_n(\hat{\mathbf{n}}) \quad (8)$$

leads to the parameter vector θ_{opt} that yields the highest likelihood of all possible candidates $\hat{\theta}$. Strictly spoken this is different from maximum likelihood estimation where the PDF changes depending on the parameters while here the argument of the PDF changes but it is the same idea of finding the candidate yielding the highest likelihood.

The method has the advantage of being very general since it does not require \mathbf{n} to be Gaussian distributed. However, for the very common case of \mathbf{n} being additive white Gaussian noise, an especially simple formulation emerges.

B. Minimum variance estimation

Under the assumption of \mathbf{n} being white and Gaussian, the distribution $p_{\mathbf{n}}$ is given as

$$p_{\mathbf{n}}(\hat{\mathbf{n}}) = \prod_{k=0}^{N_c-1} p_n(\hat{n}_k) = \prod_{k=0}^{N_c-1} \frac{1}{\pi\sigma_n^2} e^{-\frac{|\hat{n}_k|^2}{\sigma_n^2}}, \quad (9)$$

where N_c denotes the number of subcarriers and hence, the amount of time domain samples belonging to one OFDM symbol. Since it is strictly monotonous, optimization is commonly done using the logarithm of (9) as follows:

$$\log p_{\mathbf{n}}(\hat{\mathbf{n}}) = -N_c \log(\pi\sigma_n^2) - \frac{1}{\sigma_n^2} \sum_{k=0}^{N_c-1} |\hat{n}_k|^2 \quad (10)$$

Since both, N_c and σ_n^2 are independent of $\hat{\theta}$ they are insignificant for the optimization and only the sum remains. Therefore, (8) can be rewritten as:

$$\theta_{opt} = \arg \min_{\hat{\theta}} \sum_{k=0}^{N_c-1} |\hat{n}_k|^2. \quad (11)$$

This is equivalent to minimizing the sample variance of the noise estimate $\hat{\mathbf{n}}$. This simplification is especially useful since it does not require knowledge about the noise variance σ_n^2 which is not always available.

C. Imperfect feedback

It can be seen from (7) that in case of perfect feedback, i.e., $\hat{\mathbf{s}} = \mathbf{s}$, the estimate θ_{opt} will on average be correct, hence $E\{\theta_{opt}\} = \theta$. However, this is generally not the case for imperfect feedback, i.e., $\hat{\mathbf{s}} \neq \mathbf{s}$. Instead, the best estimates will usually exhibit a bias that tends to grow the larger the bigger the deviation between the feedback and the original transmit signal is. It has proven to be rather difficult to quantize this bias in an analytical way.

D. Low complexity

The main advantage of the blind decision feedback estimator is the simplicity especially in the common case of AWGN. For each candidate $\hat{\theta}$, only the nonlinearity needs to be applied which is possible with a complexity of order $O(N_c)$. If present, the frequency selective channel has to be incorporated as well which is a matrix multiplication with \underline{H} and can usually be done in the frequency domain with a complexity of order $O(N_c \log N_c)$. This yields a total complexity of $O(N_c \log N_c)$ which is on par with standard OFDM methods. Hence, the method is eligible for implementation on current real-time systems.

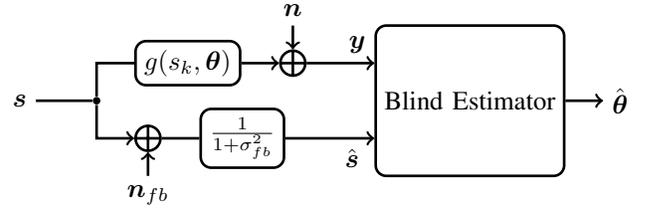


Fig. 4. Block diagram for principal estimation performance

E. Generality

The method is not limited to input signals exhibiting a normal distribution as long as the Bussgang theorem applies. It was shown in [13] that the theorem applies for all signals that can be described as a zero-mean circularly complex random process.

Compared to the feedforward based results from [7], [9], there are much less restrictions on the shape of the nonlinearity. The method can cope with nonlinearities exhibiting non-monotonous or discontinuous characteristics.

Finally, the method can cope with non-AWGN noise. However, to remain computationally inexpensive, the noise density should be easy to evaluate. The generality paired with a very low complexity renders this a very powerful framework for nonlinearity estimation.

IV. NUMERICAL RESULTS

All of the following simulations employ the TWTA nonlinearity model which is described above.

A. Estimation performance

Since the estimator uses feedback from the OFDM receiver, its performance strongly depends on the remaining receiver structure. In order to obtain more independent results, the estimator is investigated as shown in Fig.4. The signal \mathbf{s} is generated as white Gaussian noise with unit variance in accordance to the Gaussian assumption for OFDM signals. Afterwards, the signal is nonlinearly distorted with a TWTA nonlinearity and superimposed by AWGN to produce the received signal \mathbf{y} . The feedback signal is generated by superimposing the original transmit signal \mathbf{s} with AWGN \mathbf{n}_{fb} of variance σ_{fb}^2 and afterwards renormalized to unit variance to generate a feedback signal $\hat{\mathbf{s}}$ that resembles the result of a remodulation with bit errors. The nonlinearity is then jointly estimated by the blind estimator.

Firstly, the estimation performance is tested for the case of perfect feedback, i.e., $\sigma_{fb}^2 = 0$. Fig.5 shows the respective estimation mean square error for different observation sizes. Firstly it can be seen that β_A is estimated more accurately due to the larger influence of this parameter. Increasing the amount of observations consistently decreases the estimation error which indicates that the estimator is free of bias and hence, estimation accuracy can be controlled almost arbitrarily with the amount of observations.

When the assumption of perfect feedback is not met, the estimation is subject to bias errors and hence increasing the

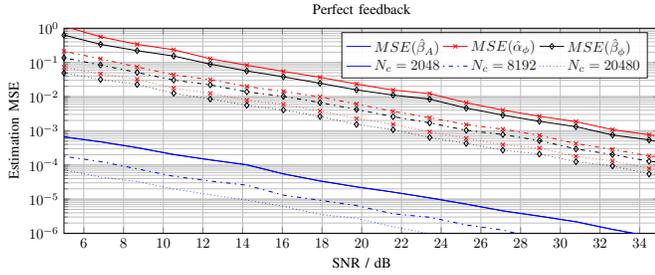


Fig. 5. Performance with perfect feedback for $\beta_A = 0.3, \alpha_\phi = \beta_\phi = 2$

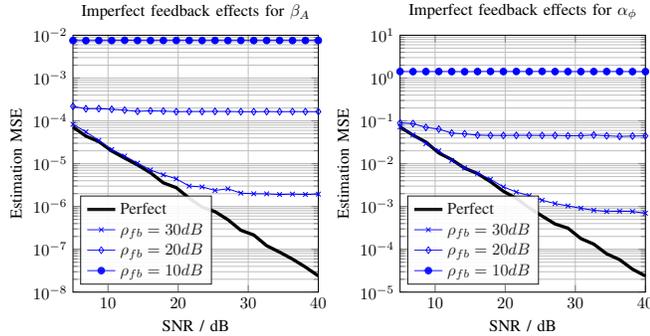


Fig. 6. Effect of imperfect feedback for different feedback SNRs ρ_{fb}

amount of observations will not improve the estimation error any further. This can be seen in Fig.6 for the same nonlinearity that was used in Fig.5. The resulting estimation performance is shown for $N_c = 20480$ and different values of the feedback SNR ρ_{fb} which is defined as follows:

$$\rho_{fb} = 10 \log_{10} \frac{1}{\sigma_{fb}^2}. \quad (12)$$

It can be seen that the performance flattens out similar to an error floor. It also shows that the estimation is much more sensitive to a noisy feedback signal than to AWGN in the received signal and hence, the feedback noise quickly dominates the overall estimation performance. These curves cannot be improved any further by increasing the amount of observations.

B. System simulation results

Finally, the method was tested in a system simulation with the following parameters:

- 512 subcarrier OFDM, 256 guardband carriers
- QPSK, 16-QAM and 64-QAM modulation
- TWTA nonlinearity with $\beta_A = 0.3, \alpha_\phi = \beta_\phi = 2$
- 40 symbols observation size (20480 samples)
- AWGN channel

The results are shown in Fig.7 for multiple iterations of the decision feedback receiver. The ideal curve depicts the case of perfect feedback and perfect nonlinearity knowledge and hence only includes the SNR-loss ρ_{df} originating from the factor α that the decision feedback receiver cannot mitigate. The linear curve corresponds to the performance of a OFDM receiver

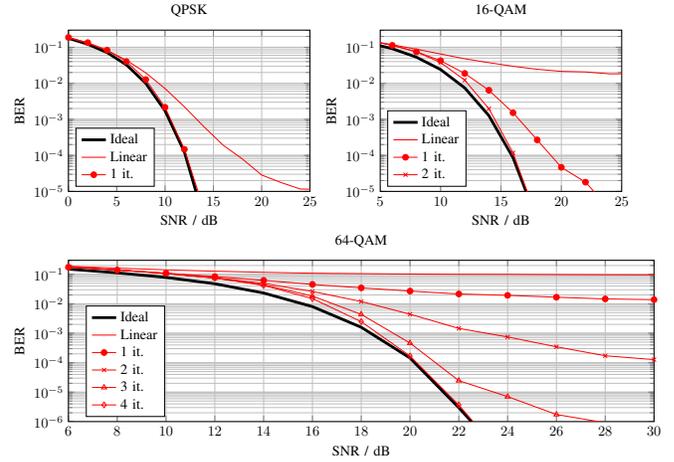


Fig. 7. Bit error rates for TWTA nonlinearity and AWGN channel

that is unaware of the nonlinearity. This receiver exhibits an error floor in every case caused by the additive n_d .

It can be seen that in case of QPSK and 16-QAM the best possible performance is reached after just one or two iterations. In the case of 64-QAM, four iterations are required to reach the ideal performance in the high SNR region. The often important BER of 10^{-3} is reached in all cases with an SNR loss of less than 0.3 dB. It has also shown that the curves for estimated nonlinearity knowledge are basically indistinguishable from the curves where the nonlinearity is assumed as perfectly known. The SNR gap in those cases is well below 0.1 dB consistently over all simulations.

V. CONCLUSIONS AND OUTLOOK

A blind decision feedback estimator for memoryless distortions in OFDM systems has been proposed. It has been shown that it exhibits a very low computational complexity and hence is fit for implementation on today's hardware. Bit error rate simulations show that a system using this estimator is capable of reaching nearly optimal performance with just a very small SNR gap. Furthermore it has shown that the results with an estimated nonlinearity are virtually indistinguishable from the case of perfectly known nonlinearity.

Some open questions still remain. For once it has shown that the estimator is very general and could theoretically be applied to a very broad range of nonlinearity estimation scenarios. These presumptions need to be investigated. Furthermore, the system simulations only covered the case of an AWGN channel. The case of an actual frequency selective channel needs to be investigated. Of special interest is the case of imperfect channel knowledge since this knowledge is used in the estimation process. The interdependence of channel and nonlinearity estimation needs to be investigated.

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