

MULTIUSER FREQUENCY ALLOCATION WITH WIDEBAND POWER AMPLIFIER MODELS

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ABSTRACT

We consider the multiuser frequency allocation problem in single-input single-output (SISO) LTE-A type uplink with carrier aggregation (CA). The increased bandwidth in LTE-A system allows orthogonal allocation of subcarriers among users. We consider both consecutive and distributed frequency allocation strategies with per user transmit power constraints and more realistic power amplifier models accounting the dependence of the amplifier efficiency on the frequency allocation. A novel binary integer programming with water-filling power allocation is proposed for consecutive frequency allocation. The system level performance is evaluated via computer simulations. The results shed light to the problem of frequency allocation with real user devices both from theoretical and practical points of view.

Index Terms— Power minimization, subcarrier allocation, power amplifier model, carrier aggregation, LTE-A.

1. INTRODUCTION

The maximum allowed frequency band in the 3GPP Long Term Evolution – Advanced (LTE-A) standard has been increased 5-fold from 20 MHz to 100 MHz compared to LTE systems. A newly introduced technology, i.e., carrier aggregation (CA), enables a user to access LTE carriers at different frequency bands [1]. The wide bandwidth increases the frequency diversity and flexibility in frequency allocation.

The allocation of non-contiguous sub-bands in inter-band CA in the uplink transmission complicates the realization of the transceiver radio frequency (RF) chain in general and the transmitter power amplifier (PA) in particular. High power-efficiency is very critical especially in battery-powered devices. To achieve this, the PA is driven close to its saturation region, which, however, results in nonlinear intermodulation distortion (IMD) and leads to increased undesired emissions and can violate the spectrum emission limits. Therefore, it is of interest to make a controlled compromise between the freedom and flexibility of the frequency allocation on the system level and RF transmitter chain design on the device level. We aim at addressing this problem set-up in this paper by considering the resource allocation of non-contiguous inter-band CA and contiguous intra-band CA jointly with nonlinear power amplifier models.

The sum rate maximization (SRmax) and sum power minimization (SPmin) algorithms in multicarrier systems have been studied extensively. The sum power minimization for multiple-input multiple-output (MIMO) MAC with optimal non-linear receivers was investigated in [2–4]. The nonconvex binary frequency allocation problem was studied in [5–7]. To make the problem convex, the binary frequency allocation constraint was relaxed. Time sharing of users on each subcarrier was allowed [5, 8]. The optimization

problem can then be iteratively solved, for example, in its dual domain. For the weighted sum rate (WSR) maximization (WSRmax) with sum power constraint in the downlink, the optimal strategy is to allocate power to one MS per subcarrier [6]. The SPmin problem with per user rate constraints was further considered in [5, 9], but the optimal solution requires high computational efforts. Suboptimal low complexity solutions were proposed, e.g., in [8, 10], where the subcarrier and power allocations were solved separately. Another type of CA in LTE-A is continuous frequency allocation. If power is equally allocated over subcarriers, optimal solution can be found by a binary integer programming (BIP) approach [11].

In this paper, we investigate the sum power minimization (SPmin) subject to user-specific rate constraints in a single-antenna, single-cell multiuser system. The PA model and IMD are considered in the SPmin. A Lagrangian dual method is used for the inter-band CA and binary integer programming (BIP) [11] is utilized for the intra-band CA. We also extended the work in [11] by improving the power allocation from fixed to dynamic. The IMD is embedded in both resource allocation algorithms. Due to the presence of IMD, finding the optimal solution is challenging and not practically appealing. An intuitive heuristic approach is proposed to solve this problem. Simulations show that by embedding the nonlinear power amplifier model in the resource allocation, performance gains can be achieved. The performance of inter-band non-contiguous and intra-band contiguous CA with PA models are also compared.

2. SYSTEM MODEL AND PROBLEM FORMULATIONS

2.1. Power Amplifier Model

We assume a memoryless PA which has a baseband input/output expression as [12]

$$s(t) = \sum_{m=0}^M a_{2m+1} |x(t)|^{2m} x(t) \quad (1)$$

where $x(t)$ is the time domain PA input signal, $s(t)$ is the time domain PA output signal and a_{2m+1} is the $(2m+1)$ th order coefficient of PA. A closed form expression of power spectral density (PSD) of (1) is [12]

$$p_s(f) = \sum_{m=0}^M \alpha_{2m+1} \underbrace{p_x(f) \star \cdots \star p_x(f)}_{m+1} \star \underbrace{p_x(-f) \star \cdots \star p_x(-f)}_m \quad (2)$$

where $\star$ denotes convolution, $p_s(f)$ and $p_x(f)$ are the PSDs of $s(t)$ and $x(t)$, respectively, and

$$\alpha_{2m+1} = \frac{1}{m+1} \left| \sum_{i=m}^M a_{2m+1} \binom{i}{m} (i+1)! \left(\int_{-\infty}^{\infty} p_x(f) df \right)^{i-m} \right|. \quad (3)$$

It has been shown that a polynomial model with third order non-linearity is a good approximation and simplification of the full order of the nonlinearity [13]. Thus, the nonlinearity order is restricted to $K = 3$. The PSD of PA output (2) can be simplified to

$$p_s(f) = \underbrace{(a_1 + a_3)^2 p_x(f)}_{\text{desired signal}} + \underbrace{2a_3 p_x(f) \star p_x(f) \star p_x(-f)}_{\text{IMD}}. \quad (4)$$

The PSD of IMD generated by K users at the receiver is

$$I(f) = \sum_{k=1}^K 2a_3^2 |h_k(f)|^2 p_{x_k}(f) \star p_{x_k}(f) \star p_{x_k}(-f) \quad (5)$$

where $h_k(f)$ is the frequency domain channel transfer function and $P_{x_k}(f)$ denotes the PSD of input signal of user k .

2.2. System Level Model

We consider a perfectly synchronized single cell multicarrier system which consists of K mobile stations (MSs), and N subcarriers. The k th user signal from time domain are transformed into discrete frequency domain by an N -subcarrier discrete Fourier transform (DFT) resulting in $[s_{1,k}, \dots, s_{N,k}]$, which is a discrete version of $s(f)$. The received signal r_n on subcarrier n is

$$r_n = \sqrt{p_{n,k}} h_{n,k} s_{n,k} + I_n + \eta_n \quad (6)$$

where $p_{n,k}$, $h_{n,k}$ and I_n are subcarrier-indexed versions of $p_{x_k}(f)$, $h_k(f)$ and $I(f)$ in (4) and (5), respectively. $s_{n,k}$ is the DFT of $s(t)$ of MS k on subcarrier n , and $\eta_{n,k} \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise (AWGN) on subcarrier n . Denote the set of subcarriers allocated to user k by $\mathcal{N}_k$. It is assumed that subcarriers are orthogonally allocated among users, i.e., $\mathcal{N}_k \cap \mathcal{N}_{k'} = \emptyset, \forall k \neq k'$. The SINR of user k on subcarrier n is then $\gamma_{n,k} = \frac{p_{n,k} |h_{n,k}|^2}{I_n + \sigma^2}$ and the corresponding achievable rate is $R_{n,k} = \log_2(1 + \gamma_{n,k})$.

2.3. Problem Formulation

The SPmin with the inter-band CA can be expressed as [14]

$$\begin{aligned} & \underset{p_{n,k}, \mathcal{N}_k}{\text{minimize}} && \sum_{k=1}^K \sum_{n \in \mathcal{N}_k} p_{n,k} \\ & \text{subject to} && \sum_{n \in \mathcal{N}_k} \log_2 \left(1 + \frac{p_{n,k} |h_{n,k}|^2}{I_n + \sigma^2} \right) \geq \Gamma_k, \forall k \\ & && \mathcal{N}_k \cap \mathcal{N}_{k'} = \emptyset, \forall k \neq k' \\ & && \left| \bigcup_{k=1}^K \mathcal{N}_k \right| \leq N \end{aligned} \quad (P1)$$

where Γ_k is the rate target of user k . The first constraint is per user rate constraint. The second one is the orthogonal subcarrier allocation constraint and the third one is total bandwidth constraint.

The SPmin with the intra-band CA is

$$\begin{aligned} & \underset{p_{n,k}, \mathcal{N}_k}{\text{minimize}} && \sum_{k=1}^K \sum_{n \in \mathcal{N}_k} p_{n,k} \\ & \text{subject to} && \sum_{n \in \mathcal{N}_k} \log_2 \left(1 + \frac{p_{n,k} |h_{n,k}|^2}{I_n + \sigma^2} \right) \geq \Gamma_k, \forall k \\ & && \mathcal{N}_k \cap \mathcal{N}_{k'} = \emptyset, \forall k \neq k' \\ & && \left| \bigcup_{k=1}^K \mathcal{N}_k \right| \leq N \\ & && \mathcal{N}_k = \{ \min(\mathcal{N}_k), \min(\mathcal{N}_k) + 1, \dots, \max(\mathcal{N}_k) - 1, \max(\mathcal{N}_k) \}, \forall k \end{aligned} \quad (P2)$$

where the last constraint is the consecutive subcarrier constraint.

Note that without the IMD term I_n , both problems have been studied. The Lagrangian dual method can be applied [9, 14] to solve nonconvex (P1). (P2) has an extra consecutive subcarrier allocation constraint which prohibits application of the Lagrangian dual method. Instead, a BIP approach [11] is used, which will be explained in the next section.

3. ALGORITHMS

Problems (P1) and (P2) are non-convex and $p_{n,k}$ is a function of I_n which is in fact a function of transmit power from all users on all subcarriers. The presence of IMD makes finding the optimal solution difficult. Thus, we utilize [9] and [11] to tackle the problems assuming the IMD is fixed during the subcarrier allocation stage and update it after.

3.1. Dual Decomposition for (P1) of Inter-band CA

The Lagrangian of (P1) can be written as [9]

$$\mathcal{L}(p_{n,k}, R_{n,k}, \boldsymbol{\mu}) = \sum_{k=1}^K \sum_{n \in \mathcal{N}_k} p_{n,k} - \sum_{k=1}^K \mu_k \left(\sum_{n \in \mathcal{N}_k} R_{n,k} - \Gamma_k \right). \quad (7)$$

where $R_{n,k} = \log_2 \left(1 + \frac{p_{n,k} |h_{n,k}|^2}{I_n + \sigma^2} \right)$, $\boldsymbol{\mu}$ is the Lagrange multiplier. For a fixed $\boldsymbol{\mu}$, the minimization of (7) is equivalent to solving the dual $D(\boldsymbol{\mu})$ [9]

$$\begin{aligned} D(\boldsymbol{\mu}) = & \min \sum_{k=1}^K \sum_{n \in \mathcal{N}_k} p_{n,k} - \sum_{k=1}^K \mu_k R_{n,k} \\ & \text{subject to} \quad p_{n,k} \geq 0, \forall n, k \\ & \quad R_{n,k} \in C(h_{n,k}, \sum_{k=1}^K \sum_{n \in \mathcal{N}_k} p_{n,k}) \end{aligned} \quad (8)$$

$D(\boldsymbol{\mu})$ can be decoupled into N parallel subproblems. On the n th subcarrier, the subproblem is [9]

$$\begin{aligned} d_n(\boldsymbol{\mu}) = & \min \sum_{k=1}^K p_{n,k} - \sum_{k=1}^K \mu_k R_{n,k} \\ & \text{subject to} \quad p_{n,k} \geq 0, \forall n, k \end{aligned} \quad (9)$$

For a fixed $\boldsymbol{\mu}$, the power $p_{n,k}$ that minimizes (9) can be found at the point where the derivative of (9) is zero with respect to $p_{n,k}$, that is

$$p_{n,k} = \left[\frac{\mu_k}{\ln 2} - \frac{I_n + \sigma^2}{|h_{n,k}|^2} \right]^+ \quad (10)$$

where $[x]^+$ stands for $\max(x, 0)$. After the subcarrier and power allocation for all users, I_n is updated by (5).

For the fixed μ , a subcarrier is allocated to the user that minimizes (9). It needs K times search on each subcarrier. The dual variable μ that makes rate vector converge to the target is iteratively searched by the sub-gradient method or ellipsoid method [3]. The algorithm is summarized in Algorithm 1, where T_{MAX} is the maximum iteration limit, ϵ_r and ϵ_p are the convergence tolerance for rate and power, respectively. In practical systems, the small value of a_3 in (5) enforces the I_n to be small. The variation of the power is iteratively attenuated by a factor of $2a_3^2$. Thus, if the subcarrier allocation is fixed, the variation of updated power value becomes smaller and smaller until converge. It is possible that the subcarrier allocation oscillates among different solutions inside each iteration (while-loop in Algorithm 1). In such a case, the allocation is randomly chosen among possible solutions.

Algorithm 1 Lagrangian dual decomposition for inter-band CA with nonlinear PA model.

Input: $h_{n,k}, \forall n, k, \Gamma_k, \epsilon_r, \epsilon_p, T_{\text{MAX}}$.

Initialization: $\Delta r = +\infty, \Delta p = +\infty$

```

1: while  $\Delta r > \epsilon_r$  do
2:    $\mathcal{N}_k \leftarrow \emptyset, \forall k, t \leftarrow 0$ 
3:   while  $\Delta p > \epsilon_p$  and  $t < T_{\text{MAX}}$  do
4:      $t \leftarrow t + 1$ 
5:     for  $n \leftarrow 1, \dots, N$  do
6:       for  $k \leftarrow 1, \dots, K$  do
7:         Calculate  $p_{n,k}[t]$  by (10) and  $d_n(\mu)$  by (9) with
            $p_{n,k'}[t] = 0, \forall k' \neq k$ 
8:       end for
9:        $\hat{k} \leftarrow \arg \min_k d_n(\mu), \mathcal{N}_k \leftarrow \mathcal{N}_k \cup \{n\}$ 
10:    end for
11:    Calculate  $I_n, \forall n$  by (5)
12:     $\Delta p \leftarrow \sum_{n=1}^N \sum_{k=1}^K |p_{n,k}[t+1] - p_{n,k}[t]|$ 
13:  end while
14:  Calculate  $R_{n,k}, \forall n, k$ 
15:  Update  $\mu$  by ellipsoid method [3]
16:   $\Delta r \leftarrow \sum_{k=1}^K \left| \sum_{n \in \mathcal{N}_k} R_{n,k} - \Gamma_k \right|$ 
17: end while
Output:  $p_{n,k}, \mathcal{N}_k, \forall n, k$ 

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3.2. BIP for (P2) of Intra-band CA

Because of the additional subcarrier allocation constraint in the intra-band CA, the Lagrangian dual method cannot be applied to solve (P2). Instead, we will reformulate (P2) and solve it with BIP [11]. An allocation pattern (AP) is used to represent the orthogonal and consecutive subcarrier allocation and all users have the same AP. An example AP of $N = 4$ for each user is [11]

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \quad (11)$$

where the element $A_{n,m} = 1$ means the subcarrier n is allocated in allocation strategy m and '0' means not. Each column is a possible allocation for a user k and there are totally $M = 1 + \sum_{n=1}^N (N - (n - 1)) = \frac{1}{2}N^2 + \frac{1}{2}N + 1$ possible consecutive subcarrier allocations, i.e., $\mathbf{A} \in \mathbb{N}^{N \times M}$. A K -user AP is $[\mathbf{A}, \dots, \mathbf{A}] \in \mathbb{N}^{N \times KM}$. Let $\mathbf{b}_k \in \mathbb{N}^{M \times 1}$ denote a binary decision variable that decides which column of $\mathbf{A}$, i.e., the set of consecutive subcarriers, a user will

choose. $\mathbf{b}_k$ contains only one element of '1' and the rest are '0'. The K -user decision vector is then $\mathbf{b} = [\mathbf{b}_1^T, \dots, \mathbf{b}_K^T]^T \in \mathbb{N}^{KM \times 1}$ [11]. The orthogonal subcarrier allocation constraint can be expressed as [11]

$$[\mathbf{A}, \dots, \mathbf{A}]\mathbf{b} = \mathbf{1}^{N \times 1} \quad (12)$$

and the consecutive subcarrier constraint can be presented as

$$\begin{bmatrix} \mathbf{1}^{M \times 1^T} & \mathbf{0}^T & \dots & \mathbf{0}^T \\ \mathbf{0}^T & \mathbf{1}^{M \times 1^T} & \dots & \mathbf{0}^T \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}^T & \mathbf{0}^T & \dots & \mathbf{1}^{M \times 1^T} \end{bmatrix}^{K \times KM} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_K \end{bmatrix} = \mathbf{1}^{N \times 1} \quad (13)$$

Stacking (12) and (13), we have

$$\begin{bmatrix} \mathbf{A} & \dots & \mathbf{A} \\ \mathbf{1}^{M \times 1^T} & \dots & \mathbf{0}^T \\ \vdots & \ddots & \vdots \\ \mathbf{0}^T & \dots & \mathbf{1}^{M \times 1^T} \end{bmatrix}^{(N+K) \times KM} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_K \end{bmatrix} = \mathbf{1}^{2N \times 1}. \quad (14)$$

Let $\mathbf{p} = [\mathbf{p}_1^T, \dots, \mathbf{p}_K^T]^T \in \mathbb{R}^{KM \times 1}$ denote the sum power vector, where the m th element in $\mathbf{p}_k$ is the sum power that corresponds to the allocation of m th column in $\mathbf{A}$ of user k . For a given rate target and subcarrier allocation per user, i.e., a column in $\mathbf{A}$, it is equivalent to a single user multicarrier power control problem. We extended the fixed power allocation in [11] to a water-filling approach, from which $\mathbf{p}$ is pre-calculated.

The problem (P2) can now be transformed into an equivalent form

$$\begin{aligned} & \underset{\mathbf{b}}{\text{minimize}} && \mathbf{p}^T \mathbf{b} \\ & \text{subject to} && (14) \end{aligned} \quad (15)$$

which can be solved by a standard BIP approach, e.g., *bintprog* of Matlab. The algorithm is summarized in Algorithm 2. Similarly to Algorithm 1, the IMD is updated after the subcarrier allocation.

Algorithm 2 BIP for intra-band CA with nonlinear PA model.

Input: $h_{n,k}, \forall n, k, \Gamma_k, \epsilon_r, \epsilon_p, T_{\text{MAX}}, \mathbf{A}$.

Initialization: $\Delta p = +\infty$

```

1:  $\mathcal{N}_k \leftarrow \emptyset, \forall k, t \leftarrow 0$ 
2: while  $\Delta p > \epsilon_p$  and  $t < T_{\text{MAX}}$  do
3:    $t \leftarrow t + 1$ 
4:   for  $k \leftarrow 1, \dots, K$  do
5:     for  $m \leftarrow 1, \dots, M$  do
6:       Given  $m$ th allocation column in  $\mathbf{A}$ , corresponding sub-
         channels of user  $k$  and  $\Gamma_k$ , calculate  $p_{(k-1)*K+m}[t]$  by
         water filling solution
7:     end for
8:   end for
9:   Find  $\mathbf{b}$  in (15) by a standard BIP solver
10:  Calculate  $I_n, \forall n$  by (5)
11:   $\Delta p \leftarrow \sum_{n=1}^N \sum_{k=1}^K |p_{n,k}[t+1] - p_{n,k}[t]|$ 
12: end while
Output:  $\mathbf{p}, \mathbf{b}$ 

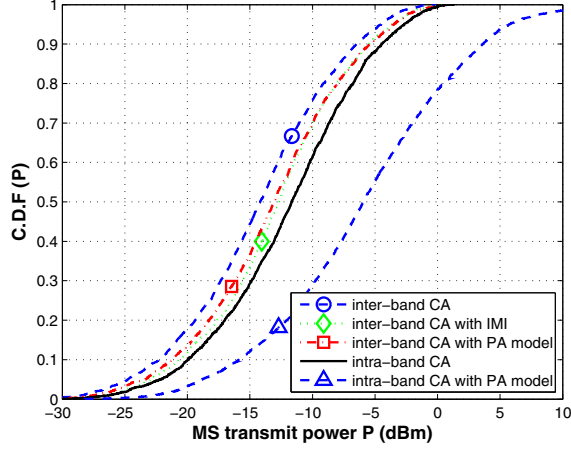
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4. SIMULATION RESULTS

We consider a single cell multicarrier system. The channel model is from [15]. The main simulation parameters are listed in Table 1. The

Table 1. Simulation parameters.

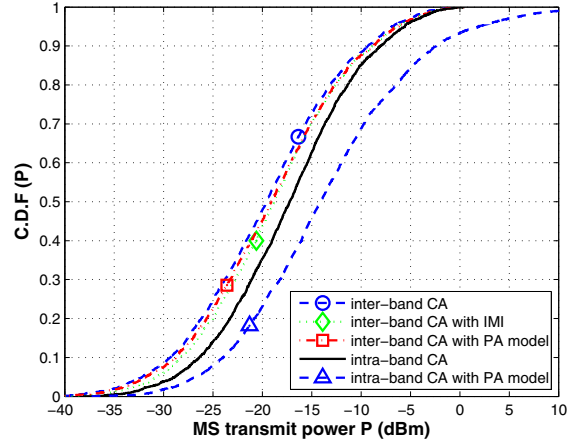
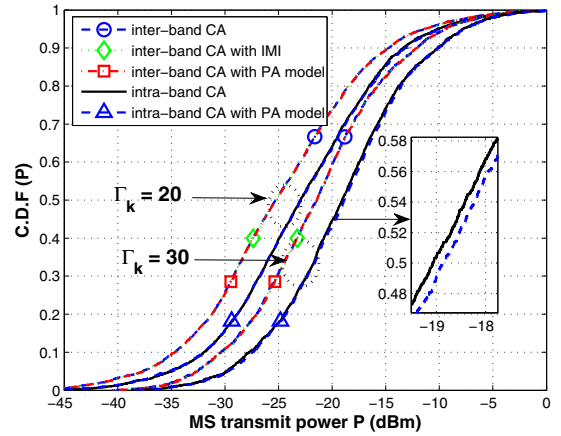
System level	
Cell radius	150 m
System bandwidth (CA)	2.5 MHz $\times 5$
Maximum MS transmit power	23 dBm
Maximum antenna gain	17 dBi
Thermal noise density	-174 dBm/Hz
Shadow fading	Log-Normal, 8 dB std
PA model	
PA gain	0 dB
3rd order input intercept point (IIP3)	30, 37 dBm
Output 1dB compression point	26 dB
Polynomial coefficients $[a_1, a_3]$	$[1, -1e^{-3}]$

**Fig. 1.** Comparison of inter/intra-band CA with and w/o PA model. IIP3 = 30dBm. Cell Radius is 150m. $\Gamma_k = 30$ bits.

PA gain is normalized to 0dB. We will show how well the algorithms perform in the wideband system with/without PA model.

The performances of the inter/intra-band resource allocation algorithms are compared in Fig. 1, where the CDF of the transmit power are plotted. The blue line with circle is inter-band CA without considering PA model. The green dotted line is that the IMD is only added at the end to the resource allocation that does not consider PA model. The red line with a square mark is the performance of PA is considered in the resource allocation stage. The black line is intra-band CA without PA and the red dashed line is resource allocation performed with PA model. It is clear that inter-band CA performs better than the intra-band CA. It can also be observed that with PA model, the performance of the system degrades. The intra-band CA degrades more than that of the inter-band CA. If the PA is considered in the resource allocation (green), then there is performance gain than that w/o considering PA (red). In Fig. 2, we increase the cell radius and lower the rate targets. It is equivalent to increasing the fading and reducing the average transmit power. The IMD level at the receiver side thus decreases. The performance order of the curves is similar as in Fig. 1. It is obvious that the performance gaps reduces. This is because the received IMD level has a smaller impact than that of Fig. 1.

In Fig. 3, the IIP3 of PA is increased to 37dBm, which results in a greater degradation in IMD level than that of IIP3 = 30dBm. With such a PA model, the IMDs can be even smaller than the noise

**Fig. 2.** Comparison of inter/intra-band CA with and w/o PA model. IIP3 = 30dBm. Cell Radius is 200m. $\Gamma_k = 20$ bits.**Fig. 3.** Comparison of different rate targets with IIP3 = 37dBm. Cell Radius is 200m.

power. Thus, the performance of resource allocation considering PA model approaches to that without PA model. The gaps are not as remarkably large as those of IIP3 = 30dBm in Figs 1 and 2.

5. CONCLUSIONS

We investigated the subcarrier allocation and power control problem in the wide band LTE-A type systems with PA model. Two resource allocation problems have been considered. The first is conventional multicarrier allocation problem, which is solved by a dual decomposition method. The second is a continuous-frequency constrained intra-band CA problem, which is solved by an BIP and water-filling approach. The PA model has been jointly considered in the frequency allocation procedure. The simulation results show that the inter-band CA performs significantly better than the intra-band CA due to the larger flexibility in frequency allocation. The IMD of PA degrades the performance of intra-band CA considerably. The PA model with large IIP3 results in a large performance gain. The current work considers a simple memoryless model. More sophisticated PA models and impact of I/Q imbalance will be studied in the future.

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