ON NOISE PROPAGATION IN CLOSED-LOOP LINEAR PREDICTIVE CODING

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ABSTRACT

A new noise production and propagation model for open- and closed loop linear predictive coding (LPC) is proposed in this paper. The model allows to accurately predict the overall SNR even at lower bit rates where the conventional high rate theory fails. Moreover, a source of LPC *encoder* instabilities is pointed out which is due to the interaction between the quantizer and the (filtered) feedback of the quantization error. The new model is verified by measurements.

Index Terms— linear predictive coding, quantization

1. INTRODUCTION

Theoretical analyses of linear predictive coding (LPC) are usually based on the assumption of high bit rates, e.g., [1]. Yet, in practical applications such as speech coding, high rate theory is not able to explain specific phenomena observed for low bit rates, for example *encoder* instabilities. The observation of instabilities in LPC indeed seems to be very astonishing since for variants such as Delta Modulation [2], Differential Pulse Code Modulation (DPCM), and Adaptive Differential Pulse Code Modulation (ADPCM) [3–5] deterministic and stochastic stability has been proven. Despite these proofs, for the more complex approaches which use block adaptive LPC with higher order linear prediction and feedback of the quantization noise, unstable behavior at very low bit rates was already described in [6]. As a solution, the author proposes to manipulate the LP spectrum at high frequencies. In [7], it is stated that in noise-feedback coding (NFC), a limiter is required to guarantee stability. The instability is attributed to overload effects in the quantizer.

To further investigate these phenomena, a new theoretical analysis of LPC is devised in this paper. Key element of our investigation is a scalar *quantization noise production and propagation model* which is valid for open- *and* closed-loop LPC with both scalar and vector quantization of the prediction residual. In LPC, quantization noise is effectively processed by the cascade of an "error weighting filter" and autoregressive synthesis filter. We use a *noise propagation network* to model the effect of this processing on the quantization noise. In the model, the noise is generated by a *power controlled additive noise source*, motivated by the fact that practical quantizers for LPC produce a nearly constant quantization SNR [7, 8].

It will be shown that the new model not only confirms the high bit rate results known from literature but that it is also valid for *lower bit rates* and explicitly accounts for the interaction between the quantizer and the noise feedback. This interaction is mostly neglected by previous analyses of LPC. The model explains why closed-loop quantization may become unstable and allows to compute a theoretical overall coding SNR which deviates from the SNR predicted by the conventional high rate theory.

2. NOISE PROPAGATION MODEL FOR LPC

The new quantization noise production and propagation model assumes that all signals are stationary with zero mean. The goal is to compute relations between signal variances and to determine the overall coding SNR as a function of the core quantization SNR related to the involved quantizer (here: SNR_0).

The impact of all filters will be considered in the form of *filter gains*. These gains are derived from the Wiener-Lee relation and Parseval's theorem [9] which state that, if an uncorrelated, stationary, and spectrally white signal $x(k)$ is filtered by a filter with system function $H(z)$, the power of the filter output signal $y(k)$ is

$$E\{y^2(k)\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \phi_y(\Omega)\,\mathrm{d}\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} |H(\Omega)|^2 \sigma_x^2\,\mathrm{d}\Omega. \quad (1)$$

In this context, the filter gain $G_{x,y}$ is defined as the relation between the variances of the filter output $y(k)$ and the filter input $x(k)$,

$$G_{x,y} \doteq \frac{E\{y^2(k)\}}{E\{x^2(k)\}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |H(\Omega)|^2\,\mathrm{d}\Omega. \quad (2)$$

2.1. Model Definition

The signal flow chart in Figure 1-a) defines the encoding and decoding process of LPC, e.g., [10]. To highlight the impact of this processing on the overall coding SNR, Figure 1-b) is derived from Figure 1-a) and introduces the novel *noise production and propagation model*. Its components are summarized in the following.

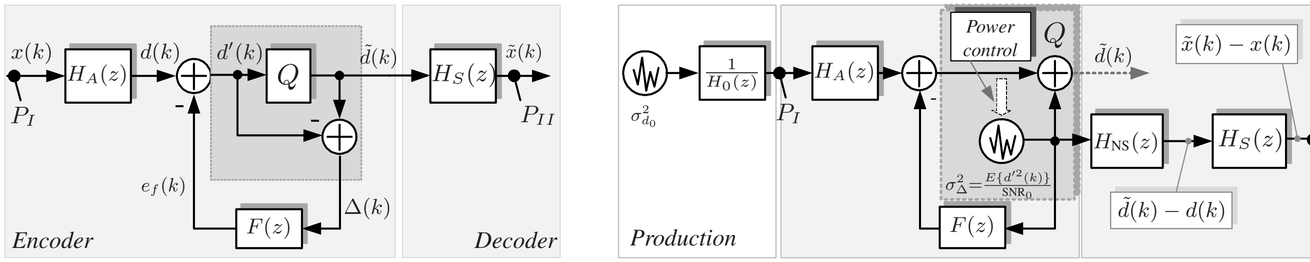
1. Signal generation: The stationary signal $x(k)$ to be coded is assumed to be the output of an auto-regressive (AR) process which is realized with an all-pole filter of order N_{ar} with the AR coefficients $\mathbf{a}_{\mathrm{ar}} = (a_{\mathrm{ar},0}, a_{\mathrm{ar},1} \ldots a_{\mathrm{ar},N_{\mathrm{ar}}})^{\mathrm{T}}$ whereby $a_{\mathrm{ar},0} = 1$. The filter is fed by an uncorrelated, zero mean, and spectrally white excitation signal $d_0(k)$. Note that the corresponding magnitude spectrum has zero-mean property since all poles of

$$H_{\mathrm{ar}}(z) = \frac{1}{H_0(z)} = \frac{1}{\sum_{i=0}^{N_{\mathrm{ar}}} a_{\mathrm{ar},i} \cdot z^{-i}} \quad (3)$$

are located inside the unit circle.

2. LP analysis: The LP filter coefficients are computed from the signal $x(k)$ and used in the LP analysis filter $H_A(z)$ of order N_{lpc}. The output is the LP residual signal $d(k)$. It is assumed that $N_{\mathrm{lpc}} \approx N_{\mathrm{ar}}$ so that $H_A(z)$ is a good approximation of $H_0(z)$. Hence, $d(k)$ is similar to $d_0(k)$.

3. Quantization with noise feedback: This block comprises the quantizer Q and the error weighting filter $F(z)$. The quantizer input



a) Encoding/decoding signal model b) Noise production and propagation model

Fig. 1. Signal production and quantization noise propagation in open- and closed-loop LPC with noise feedback.

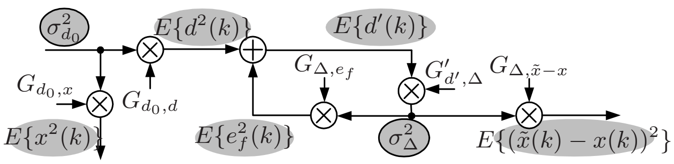


Fig. 2. Noise propagation network with variances and filter gains.

$d'(k)$ is the difference between $d(k)$ and the weighted quantization noise $e_f(k)$. The quantization error $\Delta(k)$ is assumed to be spectrally white, which clearly holds for high bit rates, e.g. [7]. Here, this is generally assumed to be true, see the detailed discussion in Section 4.1. As, furthermore, the quantizer is assumed to yield a constant SNR_0, the variance of $\Delta(k)$ depends on the variance of the quantizer input $d'(k)$. Hence, the quantizer is modeled as a *power controlled noise source* (white arrow in Figure 1-b) with variance

$$E\{\Delta^2(k)\} = \sigma_\Delta^2 = \frac{E\{d'^2(k)\}}{\text{SNR}_0}. \tag{4}$$

4. Noise shaping: The effective quantization noise $\tilde{d}(k) - d(k)$ within the quantized LP residual $\tilde{d}(k)$ is a filtered (shaped) version of $\Delta(k)$. The transfer function $H_{\text{NS}}(z) = 1 - F(z)$ is responsible for this noise shaping.

5. LP synthesis: The LP synthesis filter is simply the inverse of the LP analysis filter, i.e., $H_S(z) = (H_A(z))^{-1}$.

The signal flow chart of Figure 1-b) is now transformed into a *noise propagation network* according to Figure 2. In that diagram, instead of concrete signals and filters, only the corresponding variances and filter gains are shown. The signals $d_0(k)$ and $\Delta(k)$ are assumed to be spectrally white and statistically independent. $e_f(k)$ and $d(k)$ are assumed to be mutually uncorrelated; the corresponding variances can therefore be added (Bienaymé formula). The set of the five involved filter gains is therefore given as follows:

$$G_{d_0,x} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{1}{H_0(\Omega)}\right|^2 \mathrm{d}\Omega \tag{5}$$

$$G_{d_0,d} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{H_A(\Omega)}{H_0(\Omega)}\right|^2 \mathrm{d}\Omega \tag{6}$$

$$G_{\Delta,e_f} = \frac{1}{2\pi}\int_{-\pi}^{\pi}|F(\Omega)|^2 \,\mathrm{d}\Omega \tag{7}$$

$$G_{\Delta,\tilde{x}-x} = \frac{1}{2\pi}\int_{-\pi}^{\pi}|H_{\text{NS}}(\Omega)\cdot H_S(\Omega)|^2 \,\mathrm{d}\Omega = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{1-F(\Omega)}{H_A(\Omega)}\right|^2 \mathrm{d}\Omega \tag{8}$$

$$G'_{d',\Delta} = \frac{1}{\text{SNR}_0}, \tag{9}$$

whereby the filter gain G_{Δ,e_f} of the error weighting (noise feedback) filter $F(z)$ will be denoted as the *feedback gain* in the following. The gain $G'_{d',\Delta}$ accounts for the *power controlled noise source* which models the quantizer with SNR_0, cf. (4). The LP coefficients are assumed to be constant since $x(k)$ is stationary ($H_0(z)$ is constant). Therefore, also all filter gains are constant.

2.2. Derivation of the overall coding SNR (SNR_{lpc})

The overall coding SNR is defined as the division of the variance of the signal to be encoded $x(k)$ (position P_I in the figure) by that of the quantization noise in the decoder output, i.e., $\tilde{x}(k) - x(k)$ (position P_{II} in the figure):

$$\text{SNR}_{\text{lpc}} = \frac{E\{x^2(k)\}}{E\{(\tilde{x}(k)-x(k))^2\}}. \tag{10}$$

In the following, $E\{(\tilde{x}(k)-x(k))^2\}$ shall be computed as a function of the input signal variance $E\{x^2(k)\}$. With respect to the signal generation model, $E\{x^2(k)\}$ is given as

$$E\{x^2(k)\} = E\{d_0^2(k)\}\cdot G_{d_0,x} = \sigma_{d_0}^2 \cdot G_{d_0,x}. \tag{11}$$

Now the variance of the LP residual signal $d(k)$ can be written as

$$E\{d^2(k)\} = E\{d_0^2(k)\}\cdot G_{d_0,d} = \frac{E\{x^2(k)\}}{G_{d_0,x}}\cdot G_{d_0,d}. \tag{12}$$

The signals $e_f(k)$ and $d(k)$ are assumed to be statistically independent. Therefore, the corresponding variances can be added to produce the variance of $d'(k)$:

$$E\{d'^2(k)\} = E\{d^2(k)\} + E\{e_f^2(k)\}. \tag{13}$$

The relation between the variances of the quantization error $\Delta(k)$ and of its filtered version $e_f(k)$ is given as

$$E\{e_f^2(k)\} = E\{\Delta^2(k)\}\cdot G_{\Delta,e_f}, \tag{14}$$

and the variance of the effective quantization error that is inherent to the decoder output signal is

$$E\{(\tilde{x}(k)-x(k))^2\} = \sigma_\Delta^2 \cdot G_{\Delta,\tilde{x}-x}. \tag{15}$$

Inserting (4) into (14) and the result into (13) yields

$$E\{d'^2(k)\} = E\{d^2(k)\}\cdot\left(1 - \frac{G_{\Delta,e_f}}{\text{SNR}_0}\right)^{-1}, \tag{16}$$

and, with (12) and (15), the effective noise can be expressed as

$$E\{(\tilde{x}(k)-x(k))^2\} = E\{x^2(k)\} \cdot \frac{G_{\Delta,\tilde{x}-x}}{\text{SNR}_0} \cdot \frac{G_{d_0,d}}{G_{d_0,x}} \cdot \frac{1}{1-\frac{G_{\Delta,e_f}}{\text{SNR}_0}}. \quad (17)$$

The overall coding SNR (10) therefore amounts to

$$\text{SNR}_{\text{lpc}} = \frac{G_{d_0,x}}{G_{\Delta,\tilde{x}-x} \cdot G_{d_0,d}} \cdot \left(1 - \frac{G_{\Delta,e_f}}{\text{SNR}_0}\right) \cdot \text{SNR}_0. \quad (18)$$

3. EVALUATION OF THE NOISE PROPAGATION MODEL

By defining different constraints for $H_A(z)$, $H_S(z)$ and $F(z)$, the noise propagation model can be configured for open- and closed-loop quantization. The open-loop case for $F(z) = 0$ and $G_{\Delta,e_f=0}$ (i.e., the maximum amount of noise shaping is applied) has been investigated in [11] where it is shown that open-loop LPC can benefit from correlation in the input signal by *partially* decorrelating $x(k)$ in the LP analysis filter. The maximum open-loop SNR is achieved by a "half-whitening" LP analysis filter $H_A(z)$, cf. [11]. The performance is of course still inferior to that of closed-loop quantization where $F(z) \neq 0$. In closed-loop LPC, which is considered in the following, the LP analysis filter is commonly configured to decorrelate the input signal as much as possible, i.e., $H_A(z) \approx H_0(z)$.

3.1. Source of LPC Encoder Instabilities

The noise variance of (17) may only assume positive values, hence

$$G_{\Delta,e_f} \overset{!}{<} \text{SNR}_0. \quad (19)$$

Now if $G_{\Delta,e_f} \rightarrow \text{SNR}_0$ an increasingly *unstable feedback loop* evolves, cf. Figure 2:

- The variance of $e_f(k)$ increases if the variance of the quantization error $\Delta(k)$ increases.
- The variance of $\Delta(k)$ increases if the variance of $d'(k)$ increases since the quantizer produces a constant SNR_0.
- The variance of $d'(k)$ increases if the variance of $e_f(k)$ increases because $e_f(k)$ is independent from $d(k)$.

The effective quantization error (17) may therefore increase without bounds which is obvious for $G_{\Delta,e_f} = \text{SNR}_0$.

3.2. High Bit Rate Approximation

The overall quantization SNR (18) shall now be approximated for closed-loop quantization at high bit rates *with and without noise shaping*. At high bit rates the noise variance can be assumed to be very low, hence $\text{SNR}_0 \gg G_{\Delta,e_f}$. When noise shaping is deactivated, i.e., $F(z) = A(z/\gamma) \doteq 1 - H_A(z/\gamma)$ with $\gamma = 1$, the overall coding SNR according to (18) reduces to $\text{SNR}_{\text{lpc,hr}} = G_{d_0,x} \cdot \text{SNR}_0$ since $1 - G_{\Delta,e_f}/\text{SNR}_0 \approx 1$, $G_{d_0,d} = 1$, and $G_{\Delta,\tilde{x}-x} = 1$. With the zero-mean property of the AR filter $H_0(z)$, it can be shown that $G_{d_0,x}$ is equal to the conventional *maximum prediction gain* G_p which is also given as the inverse of the *spectral flatness measure* [12] of $x(k)$.The resulting logarithmic SNR in dB is therefore

$$10\log(\text{SNR}_{\text{lpc,hr}})_{\gamma=1} = 10\log(G_p) + 10\log(\text{SNR}_0) \quad (20)$$

which is the well-known result from the literature under high bit rate assumptions, e.g., [1, 10, 12]. The correlation of $x(k)$ can hence be transformed into a benefit with respect to the overall coding SNR.

Set	$G_{d_0,x}$	$G_{d_0,d}$	γ	G_{Δ,e_f}	$G_{\Delta,\tilde{x}-x}$
$\mathbf{a}_{\text{ar},1}$	19.62 dB	0 dB	1	15.46 dB	0 dB
			0.9	12.14 dB	2.45 dB

$\mathbf{a}_{\text{ar},1} = (1, -2.58425, 2.95464, -2.08111, 1.23315, -0.920031, 0.969564, -1.22493, 1.51283, -1.76435, 1.72109, -1.14735, 0.471528, -0.199035, -0.18829, 0.667626, -0.514674, 0.0287184, 0.0887971)^{\mathrm{T}}$

Table 1. Model parameters for example AR coefficients $\mathbf{a}_{\text{ar},1}$.

In closed-loop quantization *with* noise shaping the error weighting filter is chosen as $F(z) = A(z/\gamma)$ with, e.g., $\gamma = 0.9$. Here, compared to the case where $\gamma = 1$, the filter gain $G_{\Delta,\tilde{x}-x}$ is increased, and (18) yields a reduced SNR:

$$10\log(\text{SNR}_{\text{lpc,hr}})_{\gamma=0.9} = 10\log(\text{SNR}_{\text{lpc,hr}})_{\gamma=1} - 10\log(G_{\Delta,\tilde{x}-x}). \quad (21)$$

3.3. A Case Study for the Exact Solution

In order to evaluate the new noise propagation model for lower bit rates, some assumptions about the correlation properties of the input signal $x(k)$ shall be made. As a realistic example, a typical set of AR coefficients $\mathbf{a}_{\text{ar},1}$ with $N_{\text{ar}} = 18$ has been computed from a representative short-term stationary audio segment with 22 kHz sampling rate. Then, by approximating the required magnitude spectra with a (long) DFT, the filter gains from Eqs. (5)–(9) can be easily computed. All relevant model parameters for the present example are listed in Table 1 for $F(z) = A(z/\gamma) \doteq 1 - H_A(z/\gamma)$ with $\gamma = 1$ (closed-loop quantization without noise shaping) and $\gamma = 0.9$ (closed-loop quantization with noise shaping). In addition, the quality of the quantizer is assumed to be $10\log(\text{SNR}_0) = 16\,\text{dB}$. According to the *conventional high rate theory*, the overall SNR (20) without noise shaping ($\gamma = 1$) would amount to

$$10\log(\text{SNR}_{\text{lpc,hr}})_{\gamma=1} = 35.62\,\text{dB}. \quad (22)$$

With noise shaping ($\gamma = 0.9$), i.e., following (21), we get

$$10\log(\text{SNR}_{\text{lpc,hr}})_{\gamma=0.9} = 33.17\,\text{dB}. \quad (23)$$

Now, following the *new noise propagation model* (18) which is also valid for low bit rates, it can firstly be concluded that the system is always stable since, in the example of Table 1, condition (19) is fulfilled for both values of γ. Without noise shaping ($\gamma = 1$), the overall logarithmic SNR according to the new model is

$$10\log(\text{SNR}_{\text{lpc}})_{\gamma=1} = 26.28\,\text{dB}. \quad (24)$$

This is substantially lower than the value as predicted by the conventional theory (22).

If noise shaping is used ($\gamma = 0.9$), the filter gain $G_{\Delta,\tilde{x}-x}$ and the feedback gain G_{Δ,e_f} are changed according to Table 1 and the overall logarithmic SNR according to the new model is

$$10\log(\text{SNR}_{\text{lpc}})_{\gamma=0.9} = 30.86\,\text{dB} \quad (25)$$

which is still lower than predicted by the conventional high rate theory (21) but it is—and this is remarkable—*higher* than the SNR for $\gamma = 1$. As a conclusion, according to our new model, the choice of $\gamma < 1$ is not only beneficial due to psychoacoustic reasons but also improves the overall coding SNR of LPC in the low bit rate regime. Moreover, we conjecture that, given a signal with a specific correlation, there is an optimum γ to maximize the SNR.

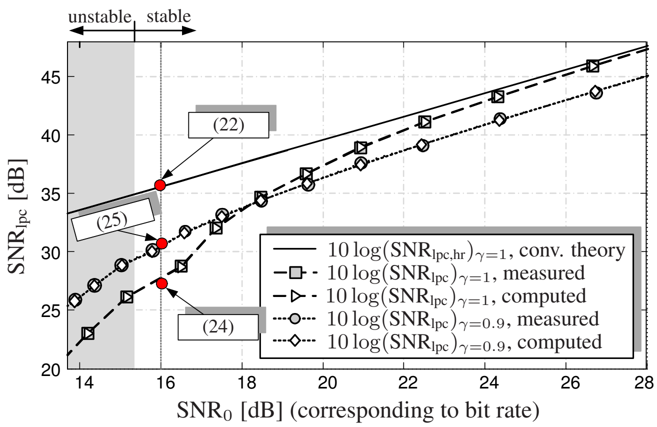


Fig. 3. Comparison of conventional and new theory with the verification measurements. The example setup of Table 1 was used here.

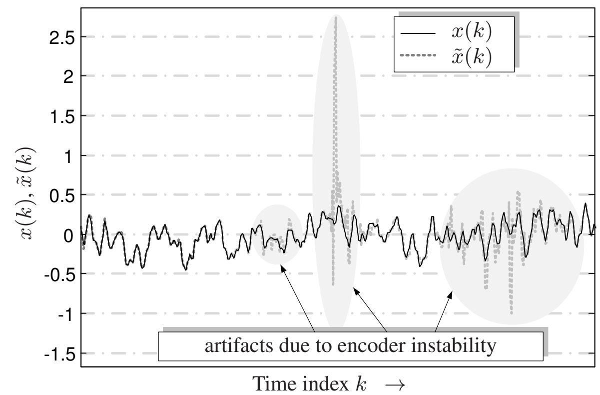


Fig. 4. Coding artifacts resulting from unstable operation conditions.

3.4. Example for Encoder Stabilization by Noise Shaping

With a slightly worse quantizer in the case study from above, e.g., $10\log(\text{SNR}_0) = 13\,\text{dB}$, an unstable system would result for $\gamma = 1$ since (19) would no longer be fulfilled. In this case, the choice of $\gamma = 0.9$ would be the solution to stabilize the complete system since the feedback gain G_{Δ,e_f} is reduced and (19) is fulfilled again.

3.5. Verification by Measurements

In order to verify the new noise propagation model, its theoretical predictions were confirmed by measurements of a real LP based coding scheme whereby the example coefficient set $\mathbf{a}_{\text{ar},1}$ and a Gaussian noise excitation $d_0(k)$ were used to generate an artificial stationary input signal $x(k)$. Apart from the overall SNR, also the required filter gains G_{Δ,e_f} and $G_{d_0,x}$ were measured by evaluating the signal powers of $x(k)$, $d(k)$, $\Delta(k)$ and $e_f(k)$. To ensure a constant SNR_0, a logarithmic (scalar) quantizer without overload was employed, cf. [13].

In Figure 3, the measurements are compared with the predictions from conventional high rate theory and with that of our new noise propagation model, demonstrating that the latter is much more consistent with the measured results. The area for which the new model predicts that the encoder becomes unstable is marked with a gray background ($\text{SNR}_0 < 15.5\,\text{dB}$ for $\gamma = 1$).

4. DISCUSSION

So far, issues related to the feedback of the quantization error in LPC have not attracted much attention from the speech coding community. Nevertheless, there are circumstances under which also real codecs produce severe artifacts, but, to be noticeable as such, a sequence of larger quantization errors must occur. Artifacts develop more quickly if the feedback gain is high and are more likely at low bit rates, i.e., when SNR_0 is low. An example of typical symptoms is shown in Figure 4 for a sequence from the artificial stationary signal of Section 3.5. Such artifacts are encountered more frequently with audio (or music) input signals where segments with very high prediction and feedback gains are common. Speech signals usually do not exhibit such extreme characteristics.

4.1. Validity of the Model for Low Bit Rates

In the above derivations, a spectrally white and uncorrelated quantization error $\Delta(k)$ has been assumed. Normally, this can only be guaranteed for sufficiently high bit rates which may be perceived as a contradiction. However, the key to resolve this contradiction are different notions of "low": In the present context of coding of correlated signals, the bit rate (and thus the SNR_0) are assumed to be "low" if the conventional high rate theory fails to accurately predict the overall coding SNR. In contrast, bit rates at which the assumptions regarding $\Delta(k)$ do not hold anymore are typically much lower (and usually lie within the unstable region of the encoder system, cf. (19)).

4.2. Applicability to CELP Coding

So far, the quantizer has been modeled by a *scalar* additive noise source. Yet, the results can, to a large extent, be generalized to the case of closed-loop LPC with gain-shape vector quantization (i.e., CELP [14]) because also here, due to the gain-shape decomposition, a constant SNR_0 ensues, see [13]. As a consequence, the instability effects predicted by the new model can actually be observed in CELP codecs under specific conditions.

However, the joint optimization of sequential samples (codevector search) implies a certain interdependence of the quantized samples which can not be achieved by scalar quantization. This interdependence, effectively, acts as an *implicit error weighting* which must be considered in addition to the *explicit weighting* by the noise shaping filter. As a conclusion, the measured gains G_{Δ,e_f} in a CELP encoder differ from the theoretical values of (7) which can be explained by a non-white quantization error $\Delta(k)$.

5. CONCLUSIONS & OUTLOOK

In this paper a new quantization noise production and propagation model has been devised which generalizes the conventional high rate theory of LPC towards lower bit rates. In particular, it was shown that for lower bit rates, the overall SNR is significantly lower than the value predicted by the conventional theory. Moreover, the interaction between the quantizer and the feedback of the quantization error in LPC has been explicitly considered. This feedback loop can lead to encoder instabilities and overall performance losses.

The new aspects pointed out in this paper contribute to a deeper fundamental understanding of LPC. Moreover, they are practically relevant for low delay audio coding which will be the subject of a follow-up paper: It will be shown how to combat the encountered instabilities and how several techniques encountered in modern speech codecs already help to mitigate such effects (although they have initially been introduced for other reasons).

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