GREEN RESOURCE ALLOCATION IN RELAY-ASSISTED MIMO SYSTEMS WITH STATISTICAL CHANNEL STATE INFORMATION

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ABSTRACT

Green resource allocation in an amplify-and-forward (AF) relay-assisted MIMO system is considered, consisting of one source, one AF relay, and one destination, in which the relay-to-destination channel is only statistically known to the source and relay. The source covariance matrix and the relay AF matrix are optimized so as to maximize the system energy efficiency (EE), defined as the ratio of the system ergodic achievable rate over the total consumed power. The resulting optimization problem is a challenging non-convex problem, which is tackled employing fractional programming in conjunction with the alternating maximization algorithm. In addition, the regime of single-stream transmission is investigated and a sufficient condition for its optimality is derived.

***Index Terms*—** Energy Efficiency, Relay-Assisted communications, Multiple-antenna systems, Fractional programming, Statistical CSI.

1. INTRODUCTION AND RELATION TO PRIOR WORK

Relaying is a well-established technique to increase coverage in wireless networks and serve cell-edge users with high throughput and agile frequency reuse[1, 2]. Moreover, deploying relays in areas subject to a strong shadowing helps to improve the network reliability. In this context, amplify-and-forward (AF) is one of the most widely used choices because it does not require the relays to decode and know the users' codebooks, thus allowing a faster and simpler design and placement of the relays. AF relaying strategy is also one candidate approach in the standard LTE-Advanced and is usually referred to as layer-1 relaying [3].

In this paper, we consider a half-duplex AF MIMO relay channel in which the source, the relay, and the destination are all equipped with multiple antennas. Most previous literature on AF relaying focuses on achievable rate maximization with power constraints [4, 5] or sum-power minimization with QoS constraints [6, 7]. However, green considerations as well as the need to extend the lifetime of battery-powered terminals in mobile networks, have resulted in a great research effort towards a more efficient use of the energy available in a communication system. From a mathematical point of view, the energy efficiency (EE) optimization problem is well-modeled as the maximization of fractional performance measures which are measured in bit/Joule, thus representing the efficiency with which each Joule of energy is being used to transmit information. Among the most widely accepted performance measures, the ratio of the throughput over the consumed power has been considered in [8, 9] for multiple access channels, in [10] for ultra-wideband systems, and in [11], for OFDMA interference networks, whereas the ratio of the achievable rate over the consumed power has been considered in [12, 13] for multi-carrier interference networks and in [14, 15] for multiple-antenna systems.

All of the mentioned references do not deal with relay-assisted systems. The problem of bit/Joule EE optimization in relay-assisted systems has been tackled only more recently in [16, 17], where competitive power control algorithms for EE maximization in multiuser wireless networks are devised. However, most previous works assume perfect channel state information (CSI) is available at all network nodes, which may be unrealistic in real-world system, since it implies a significant amount of feedback and overhead, especially for time-varying channels.

Motivated by this consideration, in this paper we relax the perfect CSI assumption, and consider the problem of EE maximization in a single-user relay-assisted MIMO system with statistical CSI at both source and relay. The system EE is defined as the ratio between the ergodic achievable rate and the average consumed power. EE maximization is carried out subject to both power and rate requirements. The considered problem belongs to the class of fractional programs and is neither concave nor pseudo-concave. In such a challenging scenario, the following contributions are made: 1) A closed-form expression of the optimal energy-efficient source and relay transmit directions is provided; 2) Fractional programming coupled with the alternating maximization is used to solve the resulting power control problem; 3) A sufficient condition for the optimality of source single-stream transmission is provided. Moreover, numerical results are provided that contrast the performance of the proposed scheme with that obtained with other CSI assumptions.

Notation: In the sequel, vectors and matrices are denoted by capital and lower-case bold letters, respectively. $\mathbf{E}[\cdot]$ is the statistical expectation operator, $\boldsymbol{I}_n$ denotes a $n \times n$ identity matrix, $(\cdot)^H$, $\mathrm{tr}(\cdot)$, $|\cdot|$, and $(\cdot)^+$ denote Hermitian, trace, determinant, and pseudo-inversion of a matrix, respectively. Matrix inequalities will be intended in the Löwner sense[1].

2. SYSTEM MODEL AND PROBLEM STATEMENT

Consider an AF relay-assisted MIMO system, wherein source, relay, and destination have N_S, N_R, and N_D antennas respectively. Denote by $\boldsymbol{s}$ the source's unit-norm symbol vector, and let $\boldsymbol{x} = \boldsymbol{Q}^{1/2}\boldsymbol{s}$, with $\boldsymbol{Q} = E[\boldsymbol{x}\boldsymbol{x}^H]$ the source transmit covariance matrix. Let us

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[1] $\boldsymbol{X} \succeq \boldsymbol{Y}$ means $\boldsymbol{X} - \boldsymbol{Y}$ is positive semidefinite.

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also denote by $\boldsymbol{H}$ and $\boldsymbol{G}$ the source-relay and relay-destination channels, and by $\boldsymbol{A}$ the AF relay matrix, Then, the signals $\boldsymbol{y}_R$ and $\boldsymbol{y}_D$ received at the relay and destination respectively, can be written as $\boldsymbol{y}_R = \boldsymbol{H}\boldsymbol{Q}^{1/2}\boldsymbol{s} + \boldsymbol{n}_R$ and $\boldsymbol{y}_D = \boldsymbol{G}\boldsymbol{A}\boldsymbol{H}\boldsymbol{Q}^{1/2}\boldsymbol{s} + \boldsymbol{G}\boldsymbol{A}\boldsymbol{n}_R + \boldsymbol{n}_D$, wherein $\boldsymbol{n}_R$ and $\boldsymbol{n}_D$ denote the thermal noise at relay and destination, modeled as zero-mean complex circular Gaussian vectors with covariance matrices $\sigma_R^2\boldsymbol{I}_{N_R}$ and $\sigma_D^2\boldsymbol{I}_{N_D}$, respectively. In the sequel, it will be assumed that source and relay have perfect CSI as far as the source-relay channel $\boldsymbol{H}$ is concerned, but only statistical CSI as for the relay-destination channel. It should be remarked that such an assumption is typical in the downlink of a cellular network, in which the base station and the relay are typically both fixed terminals, thus implying that the source-relay channel is strictly static or slowly time-varying, whereas the destination is a mobile terminal which results in the relay-destination channel to be rapidly varying and therefore more difficult to estimate.

To elaborate, the channel matrix $\boldsymbol{G}$ is modeled according to the Kronecker model [18, 19, 20, 21] as

$$\boldsymbol{G} = \boldsymbol{R}_{r,G}^{1/2}\boldsymbol{Z}_G\boldsymbol{R}_{t,G}^{1/2}\ , \tag{1}$$

where $\boldsymbol{Z}_G$ is a random matrix with independent, zero-mean, unit-variance, proper complex Gaussian entries, which neither the source nor the relay know, whereas $\boldsymbol{R}_{r,G}$ and $\boldsymbol{R}_{t,G}$ are the receive and transmit correlation matrices associated to $\boldsymbol{G}$, which are instead available. This model admits as special cases the two relevant scenarios of completely uncorrelated transmit and receive antennas, which is obtained by setting $\boldsymbol{R}_{r,G}$ and $\boldsymbol{R}_{t,G}$ to identity matrices, and the case of completely correlated transmit and receive antennas, obtained for unit-rank $\boldsymbol{R}_{r,G}$ and $\boldsymbol{R}_{t,G}$. The former case corresponds to the situation in which $\boldsymbol{G}$ is completely unknown, while the latter to the situation in which $\boldsymbol{G}$ is known up to a complex scalar coefficient.

In this scenario, the system EE is defined as the ratio between the ergodic achievable rate[2], measured in bit/s/Hz, and the average consumed power, namely

$$\frac{\mathbf{E}_{Z_G}\left[\log\frac{\left|\sigma_D^2\boldsymbol{I}_{N_D} + \boldsymbol{G}\boldsymbol{A}\left(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H + \sigma_R^2\boldsymbol{I}_{N_R}\right)\boldsymbol{A}^H\boldsymbol{G}^H\right|}{\left|\sigma_D^2\boldsymbol{I}_{N_D} + \sigma_R^2\boldsymbol{G}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{G}^H\right|}\right]}{\operatorname{tr}\left(\boldsymbol{A}\left(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H + \sigma_R^2\boldsymbol{I}_{N_R}\right)\boldsymbol{A}^H\right) + \operatorname{tr}(\boldsymbol{Q}) + P_c} \tag{2}$$

wherein, $\operatorname{tr}(\boldsymbol{Q})$ is the source transmit power, $\operatorname{tr}(\boldsymbol{A}\left(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H + \sigma_R^2\boldsymbol{I}_{N_R}\right)\boldsymbol{A}^H)$ is the relay transmit power, while P_c is the circuit power dissipated in order to operate the devices. For more details on how to model the dissipated circuit power in the nodes of wireless networks, we refer to [13]. It is important to stress that (2) is measured in bit/Joule, thus representing a natural measure of the efficiency with which each Joule of energy is used. In the following, the problem of EE maximization will be tackled, subject to power and QoS constraints.

3. PROPOSED ALGORITHM FOR EE MAXIMIZATION

For future reference, let us define the EVD of $\boldsymbol{Q}$, $\boldsymbol{R}_{r,G}^{1/2}$ and $\boldsymbol{R}_{t,G}^{1/2}$ as $\boldsymbol{Q} = \boldsymbol{U}_Q\boldsymbol{\Lambda}_Q\boldsymbol{U}_Q^H$, $\boldsymbol{R}_{r,G}^{1/2} = \boldsymbol{U}_{r,G}\boldsymbol{\Lambda}_{r,G}^{1/2}\boldsymbol{U}_{r,G}^H$, and $\boldsymbol{R}_{t,G}^{1/2} = \boldsymbol{U}_{t,G}\boldsymbol{\Lambda}_{t,G}^{1/2}\boldsymbol{U}_{t,G}^H$. Moreover, let us define the SVD of $\boldsymbol{A}$ and $\boldsymbol{H}$ as $\boldsymbol{A} = \boldsymbol{U}_A\boldsymbol{\Lambda}_A^{1/2}\boldsymbol{V}_A^H$ and $\boldsymbol{H} = \boldsymbol{U}_H\boldsymbol{\Lambda}_H^{1/2}\boldsymbol{V}_H^H$. The EE optimization problem can be formulated as the maximization of (2) subject to the constraints

$$\begin{aligned}
&\operatorname{tr}(\boldsymbol{A}(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H + \sigma_R^2\boldsymbol{I}_{N_R})\boldsymbol{A}^H) \leq P_R^{max}\\
&\operatorname{tr}(\boldsymbol{Q}) \leq P_S^{max}\ ,\ \boldsymbol{Q} \succeq 0\\
&\mathbf{E}_{Z_G}\left[\log\left(\frac{\left|\sigma_D^2\boldsymbol{I}_{N_D} + \boldsymbol{G}\boldsymbol{A}\left(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H + \sigma_R^2\boldsymbol{I}_{N_R}\right)\boldsymbol{A}^H\boldsymbol{G}^H\right|}{\left|\sigma_D^2\boldsymbol{I}_{N_D} + \sigma_R^2\boldsymbol{G}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{G}^H\right|}\right)\right]\\
&\geq R_S^{min}
\end{aligned} \tag{3}$$

with P_S^{max}, P_R^{max}, and R_S^{min}, the maximum source feasible power, the maximum relay feasible power, and the minimum source acceptable rate, respectively. The following result provides the optimal $\boldsymbol{U}_Q$, $\boldsymbol{U}_A$, and $\boldsymbol{V}_A$ for the considered problem.

Proposition 1 *Assume $\boldsymbol{H}$ is a tall full-rank matrix[3]. The optimal $\boldsymbol{Q}$ and $\boldsymbol{A}$ are such that $\boldsymbol{U}_Q = \boldsymbol{V}_H$, $\boldsymbol{U}_A = \boldsymbol{U}_{t,G}$ and $\boldsymbol{V}_A = \boldsymbol{U}_H$, while the optimal $\boldsymbol{\Lambda}_Q$ and $\boldsymbol{\Lambda}_A$ are obtained as the solution of Problem (4), shown at the top of the next page.*

Proof: The proof is omitted due to space constraints. It is available in [22].

Even if the optimal source and relay transmit directions have been determined in closed form, the resulting power allocation problem (4) is neither concave nor pseudo-concave in $(\boldsymbol{\Lambda}_Q, \boldsymbol{\Lambda}_A)$. However, it can be tackled employing fractional programming tools and the alternating maximization approach. First of all, let us recall that the following known result.

Proposition 2 *Consider the fractional function $f(x) = \dfrac{N(x)}{D(x)}$. If $N(x) \geq 0$ is a concave function and $D(x) > 0$ is a linear function, then $f(x)$ is a pseudo-concave function. Moreover, maximizing $f(x)$ is equivalent to finding the positive zero of the function $F(\mu) = \max_x \{N(x) - \mu D(x)\}$.*

Proof: See [23, 24]

Thus, a pseudo-concave function can be maximized by finding the zero of the auxiliary function $F(\mu)$. If the constraint set enforced in the maximization of $f(x)$ defines a convex set, then the zero of $F(\mu)$ can be found with a superlinear convergence by means of Dinkelbach's algorithm [24].

Unfortunately, the objective of Problem (4) is not even pseudo-concave in $(\boldsymbol{\Lambda}_Q, \boldsymbol{\Lambda}_A)$. Therefore, Dinkelbach's algorithm can not be implemented directly in (4). However, it can be seen that, for fixed $\boldsymbol{\Lambda}_A$, the objective of (4) is the ratio of a concave function of $\boldsymbol{\Lambda}_Q$ over a linear function of $\boldsymbol{\Lambda}_Q$, and hence is a pseudo-concave function. Moreover, with respect to $\boldsymbol{\Lambda}_Q$, the constraints are all linear or concave. The same properties can be shown to hold with respect to $\boldsymbol{\Lambda}_A$, if $\boldsymbol{\Lambda}_Q$ is fixed. To see this, first observe that the denominator of the objective is clearly linear in $\boldsymbol{\Lambda}_A$. As for the numerator, it can be rewritten as $\mathbb{E}_{\widetilde{F}}\left[\log\left|\boldsymbol{I}_{N_D} + (\sigma_D^2\boldsymbol{I}_{N_D} + \sigma_R^2\widetilde{\boldsymbol{F}}\boldsymbol{\Lambda}_A\widetilde{\boldsymbol{F}}^H)^{-1}\widetilde{\boldsymbol{F}}\boldsymbol{\Lambda}_A\boldsymbol{\Lambda}_B\widetilde{\boldsymbol{F}}^H\right|\right]$, with $\widetilde{\boldsymbol{F}} = \boldsymbol{\Lambda}_{r,G}^{1/2}\boldsymbol{Z}_G\boldsymbol{\Lambda}_{t,G}^{1/2}$ and $\boldsymbol{\Lambda}_B = \boldsymbol{\Lambda}_H^{1/2}\boldsymbol{\Lambda}_Q\boldsymbol{\Lambda}_H^{H/2}$, which is concave in $\boldsymbol{\Lambda}_A$ as shown in [25]. Then, also the constraints are all linear

[2] MMSE reception coupled with successive interference cancellation is considered, and the receiver is assumed to have perfect CSI of both $\boldsymbol{G}$ and the product $\boldsymbol{G}\boldsymbol{A}\boldsymbol{H}$, which can be achieved for example by means of data-aided channel estimation.

[3] This assumption is not critical and could be relaxed.

$$\begin{cases} \max\limits_{\mathbf{\Lambda}_Q,\mathbf{\Lambda}_A} \dfrac{\mathbb{E}_{Z_G}\left[\log\left(\dfrac{\left|\sigma_D^2 \boldsymbol{I}_{N_D} + \mathbf{\Lambda}_{r,G}\boldsymbol{Z}_G\mathbf{\Lambda}_{t,G}\mathbf{\Lambda}_A\left(\mathbf{\Lambda}_H^{1/2}\mathbf{\Lambda}_Q\mathbf{\Lambda}_H^{H/2} + \sigma_R^2\boldsymbol{I}_{N_R}\right)\boldsymbol{Z}_G^H\right|}{\left|\sigma_D^2\boldsymbol{I}_{N_D} + \sigma_R^2\mathbf{\Lambda}_{r,G}\boldsymbol{Z}_G\mathbf{\Lambda}_{t,G}\mathbf{\Lambda}_A\boldsymbol{Z}_G^H\right|}\right)\right]}{\mathrm{tr}\left(\mathbf{\Lambda}_A\left(\mathbf{\Lambda}_H^{1/2}\mathbf{\Lambda}_Q\mathbf{\Lambda}_H^{H/2} + \sigma_R^2\boldsymbol{I}_{N_R}\right)\right) + \mathrm{tr}(\mathbf{\Lambda}_Q) + P_c} \\ \text{s.t.} \quad \mathrm{tr}\left(\mathbf{\Lambda}_A\left(\mathbf{\Lambda}_H^{1/2}\mathbf{\Lambda}_Q\mathbf{\Lambda}_H^{H/2} + \sigma_R^2\boldsymbol{I}_{N_R}\right)\right) \leq P_R^{max}\ , \ \ \mathrm{tr}(\mathbf{\Lambda}_Q) \leq P_S^{max}\ , \ \mathbf{\Lambda}_Q \succeq 0\ , \ \mathbf{\Lambda}_A \succeq 0 \\ \mathbb{E}_{Z_G}\left[\log\left(\dfrac{\left|\sigma_D^2 \boldsymbol{I}_{N_D} + \mathbf{\Lambda}_{r,G}\boldsymbol{Z}_G\mathbf{\Lambda}_{t,G}\mathbf{\Lambda}_A\left(\mathbf{\Lambda}_H^{1/2}\mathbf{\Lambda}_Q\mathbf{\Lambda}_H^{H/2} + \sigma_R^2\boldsymbol{I}_{N_R}\right)\boldsymbol{Z}_G^H\right|}{\left|\sigma_D^2\boldsymbol{I}_{N_D} + \sigma_R^2\mathbf{\Lambda}_{r,G}\boldsymbol{Z}_G\mathbf{\Lambda}_{t,G}\mathbf{\Lambda}_A\boldsymbol{Z}_G^H\right|}\right)\right] \geq R_S^{min} \end{cases} \quad (4)$$

or concave in $\mathbf{\Lambda}_A$ for fixed $\mathbf{\Lambda}_Q$. Accordingly, Dinkelbach's algorithm can be used to solve (4) with respect to $\mathbf{\Lambda}_Q$, if $\mathbf{\Lambda}_A$ is fixed, and viceversa. This motivates us to tackle (4) employing the alternating maximization algorithm [26], according to which an objective function can be cyclically maximized with respect to one variable, while keeping the other variables fixed. The formal procedure is stated next in Algorithm 1.

Algorithm 1 Alternating maximization for problem (4)

Initialize $\mathbf{\Lambda}_Q$ to a feasible value $\mathbf{\Lambda}_Q^{(0)}$. Set a tolerance ϵ. Set $n = 0$;
repeat
  Given $\mathbf{\Lambda}_Q^{(n)}$, compute $\mathbf{\Lambda}_A^{(n+1)}$ as the solution to Problem (4) with respect to $\mathbf{\Lambda}_A$;
  Given $\mathbf{\Lambda}_A^{(n+1)}$, compute $\mathbf{\Lambda}_Q^{(n+1)}$ as the solution to Problem (4) with respect to $\mathbf{\Lambda}_Q$;
  Compute $\mathrm{EE}^{(n)}$;
  $n = n + 1$;
until $|\mathrm{EE}^{(n+1)} - \mathrm{EE}^{(n)}| \leq \epsilon$.

Convergence is guaranteed since each iteration does not decrease the EE.

4. OPTIMALITY OF SOURCE SINGLE-STREAM TRANSMISSION.

The aim of this section is to derive a sufficient condition such that the optimal power allocation policy at the source is to support only one data-stream. Otherwise stated, we look for a condition under which the solution to Problem (4) with respect to $\mathbf{\Lambda}_Q$ is a rank-one matrix. This is helpful because it greatly reduces the computational complexity of the resource allocation process, since if single-stream transmission is optimal, then the whole system reduces to an equivalent SISO system. Moreover, receiver design and channel coding are also simpler, because just one data stream needs to be decoded, and it is possible to make use of the well-developed theory of channel coding for SISO systems. However, single-stream transmission is not the optimal power allocation policy in general, and therefore it is of interest to derive condition that allow to determine when we can support only one data-stream without suffering any performance loss. The problem of single-stream transmission optimality has been addressed in several papers before with respect to achievable rate optimality in single-user one-hop systems [20, 21], multiple access systems [19, 27] and relay-assisted systems[28]. However, to the best of our knowledge, single-stream optimality has never been dealt with as far as EE is concerned. To begin with, in order to obtain a mathematically tractable problem, single-stream optimality will be addressed in the simplified scenario in which the QoS constraint is relaxed in Problem (4) and $\mathbf{\Lambda}_{t,G}\mathbf{\Lambda}_A = \boldsymbol{I}_{N_R}$. Considering the relaxed problem, the condition is provided in the following proposition.

Proposition 3 *For all $i = 1, \ldots, N_S$, denote by $\lambda_{i,G}^t$, $\lambda_{i,A}$ and $\lambda_{i,H}$ the i-th diagonal element of $\mathbf{\Lambda}_{t,G}$, $\mathbf{\Lambda}_A$ and $\mathbf{\Lambda}_H^{1/2}\mathbf{\Lambda}_H^{H/2}$, respectively and by $\boldsymbol{f}_i$ the i-th column of the matrix $\boldsymbol{F} = \left(\sigma_D^2\boldsymbol{I}_{N_D} + \sigma_R^2\mathbf{\Lambda}_{r,G}^{1/2}\boldsymbol{Z}_G\boldsymbol{Z}_G^H\mathbf{\Lambda}_{r,G}^{1/2}\right)^{-1/2}\mathbf{\Lambda}_{r,G}^{1/2}\boldsymbol{Z}_G$. Next, define $P = \min\left(\lambda_{1,A}\lambda_{1,H}P_S^{max}, P_R^{max} + P_c - b\right)$, $b = \sigma_R^2\sum_{i=1}^{N_R}\lambda_{i,A} + P_c$, $d_i = 1 + \frac{1}{\lambda_{i,A}\lambda_{i,H}}$ and*

$$C_{i,G} = \lambda_{i,G}^t\left(\mathbb{E}_{\boldsymbol{f}_2}\left[\|\boldsymbol{f}_2\|^2\right] - P\lambda_{1,G}^t\mathbb{E}_{\boldsymbol{f}_1,\boldsymbol{f}_2}\left[\frac{|\boldsymbol{f}_2^H\boldsymbol{f}_1|^2}{1 + P\lambda_{1,G}^t\|\boldsymbol{f}_1\|^2}\right]\right) \times(b + Pd_1) - d_i\mathbb{E}_{\boldsymbol{f}_1}\left[\log\left(1 + P\lambda_{1,G}^t\|\boldsymbol{f}_1\|^2\right)\right] . \quad (5)$$

Then, if $C_{2,G} \geq 0$, then a sufficient condition for beamforming optimality is

$$P\lambda_{2,G}^t\left(\mathbb{E}_{\boldsymbol{f}_2}\left[\|\boldsymbol{f}_2\|^2\right] - P\lambda_{1,G}^t\mathbb{E}_{\boldsymbol{f}_1,\boldsymbol{f}_2}\left[\frac{|\boldsymbol{f}_2^H\boldsymbol{f}_1|^2}{1 + P\lambda_{1,G}^t\|\boldsymbol{f}_1\|^2}\right]\right) + \mathbb{E}_{\boldsymbol{f}_1}\left[\frac{1}{1 + P\lambda_{1,G}^t\|\boldsymbol{f}_1\|^2}\right] + \frac{P(d_1 - d_2)}{b + Pd_1}\mathbb{E}_{\boldsymbol{f}_1}\left[\log\left(1 + P\lambda_{1,G}^t\|\boldsymbol{f}_1\|^2\right)\right] \leq 1 \quad (6)$$

Instead, if $C_{2,G} \leq 0$, then a sufficient condition for beamforming optimality is

$$\mathbb{E}_{\boldsymbol{f}_1}\left[\frac{1}{1 + P\lambda_{1,G}^t\|\boldsymbol{f}_1\|^2}\right] \quad (7)$$

$$+\frac{Pd_1}{b + Pd_1}\mathbb{E}_{\boldsymbol{f}_1}\left[\log\left(1 + P\lambda_{1,G}^t\|\boldsymbol{f}_1\|^2\right)\right] \leq 1 . \quad (8)$$

Proof: The proof is omitted due to space constraints. It is available in [22].

The provided condition can be checked before solving for $\mathbf{\Lambda}_Q$. If it holds, then the optimal $\mathbf{\Lambda}_Q$ is immediately known in closed-form.

5. NUMERICAL RESULTS

In our simulations, a MIMO AF relay-assisted system with $N_S = N_R = N_D = 3$ has been considered. The matrices $\boldsymbol{R}_{r,G}$ and $\boldsymbol{R}_{t,G}$ have been generated according to the exponential correlation model, with equal transmit and receive correlation index ρ. Instead, the relay-destination channel matrix $\boldsymbol{H}$ has been generated as a realization of a random matrix with zero-mean, unit variance circular Gaussian entries. The QoS constraint has been set to $R_S^{min} = 1$ bit/s/Hz. The performance has been evaluated in terms of the instantaneous EE, i.e. the deterministic counterpart of (2),

$$\mathrm{EE}_{in} = \frac{\log \frac{\left|\sigma_D^2 \boldsymbol{I}_{N_D} + \boldsymbol{G}\boldsymbol{A}\left(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H + \sigma_R^2 \boldsymbol{I}_{N_R}\right)\boldsymbol{A}^H\boldsymbol{G}^H\right|}{\left|\sigma_D^2 \boldsymbol{I}_{N_D} + \sigma_R^2 \boldsymbol{G}\boldsymbol{A}\boldsymbol{A}^H\boldsymbol{G}^H\right|}}{\mathrm{tr}\left(\boldsymbol{A}\left(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H + \sigma_R^2 \boldsymbol{I}_{N_R}\right)\boldsymbol{A}^H\right) + \mathrm{tr}(\boldsymbol{Q}) + P_c}, \quad (9)$$

versus the SNR, defined as $\mathrm{SNR} = P_{max}/\sigma^2$, wherein $P_{max} = P_R^{max} = P_S^{max}$ and $\sigma^2 = \sigma_R^2 = \sigma_D^2$. The presented results have been averaged over 1000 independent channel realizations.

In Fig. 1, the correlation index has been set to $\rho = 0.5$ and the following scenarios have been contrasted.

1. Instantaneous EE (9) achieved when the resources are allocated with statistical CSI on $\boldsymbol{G}$ and perfect CSI on $\boldsymbol{H}$, i.e. the proposed Algorithm 1.
2. Instantaneous EE (9) achieved when the resources are allocated with statistical CSI on $\boldsymbol{H}$ and perfect CSI on $\boldsymbol{G}$. This scenario has been tackled in [22].
3. As a benchmark, we also report the instantaneous EE (9) achieved when perfect CSI on both $\boldsymbol{H}$ and $\boldsymbol{G}$ is available and therefore $\boldsymbol{Q}$ and $\boldsymbol{A}$ can be allocated so as to optimize (9). This scenario has been tackled in [22].

The results indicate that having perfect CSI on $\boldsymbol{H}$ and statistical CSI on $\boldsymbol{G}$ grants better performance than the opposite case in which $\boldsymbol{G}$ is perfectly known, while $\boldsymbol{H}$ is only statistically available. This is explained observing that $\boldsymbol{H}$ appears in both the numerator and denominator of the EE (2), while $\boldsymbol{G}$ only affects the numerator. Consequently, it can be expected that it is more convenient to know the source-to-relay channel than the relay-to-receiver.

In Fig. 2, a similar scenario is considered with the difference that two extreme values of ρ are considered, namely $\rho = 0.1$ and $\rho = 0.9$. Similar remarks as for Fig. 1 hold. Moreover, it is interesting to observe how the gap with respect to the perfect CSI case is smaller for $\rho = 0.9$. Indeed, ρ can be regarded as the degree of information about the channel. The higher ρ, the more information about the channel is contained in the correlation matrices $\boldsymbol{R}_{r,G}$ and $\boldsymbol{R}_{t,G}$.

Finally, in Fig. 3, the number of iterations required for Algorithm 1 to converge are reported, considering 10 different initialization points. It is seen that Algorithm 1 converges after a handful of iterations, thus implying that it lends itself to be implemented in real world scenarios.

6. CONCLUSIONS

Energy-efficient resource allocation in an AF relay-assisted MIMO system subject to power and QoS constraints has been studied. Perfect CSI has been assumed for the source-relay channel, while only statistical CSI for the relay-destination channel is available. The optimal transmit directions at the source and relay have been provided in closed-form and the resulting power allocation problem has been tackled by means of fractional programming and alternating maximization. The overall algorithm has limited computational complexity and is guaranteed to converge. Next, a sufficient condition for the optimality of source single-stream transmission has been provided. Finally, numerical results have been provided to assess the performance of the proposed resource allocation algorithm.

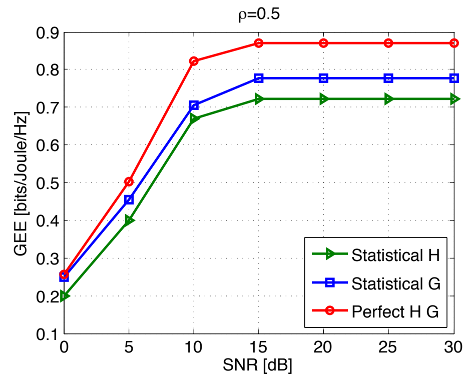

Fig. 1. EE versus SNR for: EE optimization with statistical CSI on $\boldsymbol{G}$, i.e. Algorithm 1; EE optimization with statistical CSI on $\boldsymbol{H}$; EE optimization with perfect CSI.

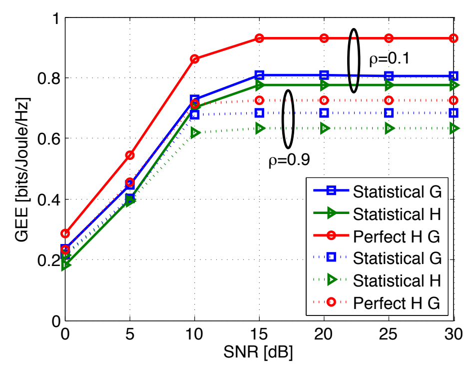

Fig. 2. EE versus SNR for: EE optimization with statistical CSI on $\boldsymbol{G}$, i.e. Algorithm 1; EE optimization with statistical CSI on $\boldsymbol{H}$; EE optimization with perfect CSI.

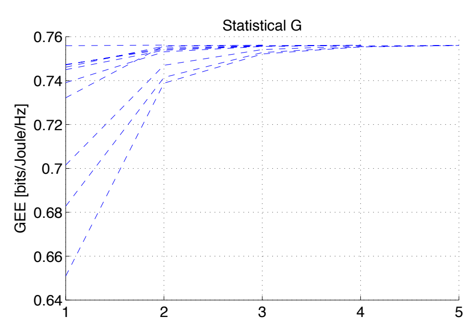

Fig. 3. Number of iterations required for Algorithm 1 to converge.

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