DIGITAL MODULATION RECOGNITION USING CIRCULAR HARMONIC APPROXIMATION OF LIKELIHOOD FUNCTION

Alexander B. Sergienko, Senior Member, IEEE, and Andrey V. Osipov,

Saint Petersburg Electrotechnical University, 5 Prof. Popov Street, Saint Petersburg, 197376, Russia

ABSTRACT

New algorithm is proposed for recognition of linear digital modulation constellations. Its essence is to approximate the likelihood-based approach. Log-likelihood function is approximated by a truncated circular harmonic expansion. If only the first nonzero harmonic is retained in the expansion, simple solutions for dealing with phase and frequency offsets are obtained. Computer simulations confirmed applicability of the proposed method for M-PSK and 16-QAM signals.

Index Terms— Signal classification, modulation recognition, digital modulation, circular harmonic expansion

1. INTRODUCTION

Automatic modulation recognition (AMR) is an important task in such application areas as spectrum surveillance or cognitive radio. To accomplish this task in real-world conditions, many methods were proposed—see, for example, an extensive survey in [1]. According to this survey, two main classes of algorithms can be distinguished. Likelihood-based (LB) methods are based on the theory of statistical hypothesis testing, while feature-based (FB) methods rely on specific signal features, often chosen by heuristic reasoning. LB methods potentially offer better performance but they require a lot of computations, especially when the analyzed signal has many unknown parameters. FB methods are simpler, but the cost of this simplicity is higher error probability.

Approximation of the likelihood function (LF) in the form of truncated circular harmonic expansion (CHE) allows to capture its main features and greatly reduces the amount of required calculations. In [2] CHE was used for blind estimation of phase and frequency offsets, here we analyze its use for AMR as an approximation of several LB methods.

2. PROBLEM FORMULATION AND MAXIMUM LIKELIHOOD SOLUTION

The AMR problem is formulated as follows. We assume that the symbol rate is known or have been estimated (it can be done without knowledge of signal constellation, see [3] for an example). Observation $\{\dot{x}_k\}$ is a sequence of baud-rate samples after a matched filter:

$$\dot{x}_k = \dot{a}_k e^{j(\varphi_0 + k\varphi_f)} + \dot{n}_k, \quad k = 0, 1, \ldots, K-1, \tag{1}$$

where $\dot{a}_k$ are the data symbols independently and equiprobably drawn from one of P possible modulation constellations $\{\dot{C}_1^{(p)}, \dot{C}_2^{(p)}, \ldots, \dot{C}_{M_p}^{(p)}\}$, M_p is the p-th constellation size, $p = 1, 2, \ldots, P$, φ_0 is the phase offset, φ_f is the intersymbol phase shift due to the frequency offset Δf ($\varphi_f = 2\pi\Delta f T$, where T is the symbol period), $\dot{n}_k$ are the samples of complex white Gaussian noise with variance σ^2. The signal and noise levels are assumed to be known. Signal-to-noise ratio (SNR) is defined as the ratio between variances of signal and noise components:

$$\text{SNR} = \overline{|\dot{a}_k|^2} \big/ \sigma^2 . \tag{2}$$

The goal is to decide from which constellation data symbols $\dot{a}_k$ are taken.

According to the maximum likelihood (ML) approach we should choose the hypothesis $\hat{p}$ that maximizes the LF, or its logarithm (log-likelihood function, LLF) that is usually more convenient, conditioned on the observed sample sequence:

$$\text{LLF}\left(\hat{p} \mid \{\dot{x}_k\}\right) \geq \text{LLF}\left(p \mid \{\dot{x}_k\}\right) \quad \forall p . \tag{3}$$

LLF for hypothesis p constitutes a logarithm of joint probability density function (PDF) for K observed samples. For known phase and frequency offsets it can be written as [4], [5]

$$\text{LLF}\left(p \mid \{\dot{x}_k\}, \varphi_f, \varphi_0\right) = \sum_{k=0}^{K-1} \text{LLF}\left(p \mid \dot{x}_k e^{-j(\varphi_0 + k\varphi_f)}\right), \tag{4}$$

where $\text{LLF}(p \mid \dot{x}_k)$ is a log-PDF for a single sample with zero phase offset:

$$\text{LLF}\left(p \mid \dot{x}_k\right) = \log\left(\frac{1}{\pi\sigma^2 M_p} \sum_{m=1}^{M_p} \exp\left(-\frac{\left|\dot{x}_k - \dot{C}_m^{(p)}\right|^2}{\sigma^2}\right)\right). \tag{5}$$

As phase and frequency offsets are unknown in the majority of real-world scenarios, LF should be averaged or maximized over every parameter. These approaches are known, respectively, as average likelihood ratio test and generalized likelihood ratio test [1]. Any mix of these approaches (LF is averaged over a subset of parameters and

978-1-4799-2893-4/14/$31.00 ©2014 IEEE

maximized over remaining parameters) is called hybrid likelihood ratio test [1].

Two situations are considered in this paper (they seem to be the most reasonable from a practical standpoint):

a) Frequency offset is known, phase is treated as a uniformly distributed random variable (LF is phase-averaged) [6]:

$$\mathrm{LLF}_{\varphi_0}\left(p \mid \{\dot{x}_k\}, \varphi_f\right) = \log\left(\frac{1}{2\pi}\int_0^{2\pi} e^{\mathrm{LLF}\left(p \mid \{\dot{x}_k\}, \varphi_f, \varphi_0\right)} d\varphi_0\right). \quad (6)$$

b) Frequency offset is estimated and phase is treated as a uniformly distributed random variable (LF is maximized over frequency offset and phase-averaged):

$$\mathrm{LLF}_{\varphi_0, \varphi_f}\left(p \mid \{\dot{x}_k\}\right) = \max_{\varphi_f} \log\left(\frac{1}{2\pi}\int_0^{2\pi} e^{\mathrm{LLF}\left(p \mid \{\dot{x}_k\}, \varphi_f, \varphi_0\right)} d\varphi_0\right). \quad (7)$$

Note that to perform phase averaging in (6) and (7), we transform LLF into LF, average it over φ_0 and then transform the result back to log-domain.

Averaging in (6) and, especially, maximum search in (7) are extremely time-consuming. For that reason, unknown phase offset is often treated by differential processing [4]. As for frequency offset, as maximum search in (7) is computationally prohibitive, it is usually treated by using blind frequency estimation algorithms [7].

In the following sections, we propose approximations of (5)–(7) with reasonable computational complexity.

3. CIRCULAR HARMONIC EXPANSION OF LLF

Straightforward use of (5) for calculating LLF requires large amount of computations, especially when averaging or maximum search over parameters is needed (see (6) and (7)). In this section it will be shown how LLF can be expanded into a series that may be subsequently truncated to simplify its calculation.

First of all we explicitly separate magnitude and phase of the samples $\dot{x}$, so that LLF (5) for a single sample becomes

$$\mathrm{LLF}\left(p \mid \dot{x}\right) = \mathrm{LLF}\left(p \mid re^{j\phi}\right), \quad (8)$$

where $r = |\dot{x}|$ and $\phi = \arg \dot{x}$.

The dependence of LLF (8) on the phase ϕ is a periodic function with period $\Theta^{(p)}$ defined by the rotational symmetry of the *p*-th constellation. For M-PSK, $\Theta^{(p)} = 2\pi/M$, for QAM with square or cross constellations $\Theta^{(p)} = \pi/2$. As a result, LLF can be expanded in the Fourier series along phase ϕ, that gives its presentation in the form of circular harmonic expansion [8]:

$$\mathrm{LLF}\left(p \mid re^{j\phi}\right) = \frac{A_0^{(p)}(r)}{2} + \sum_{n=1}^{\infty} A_n^{(p)}(r)\cos\left(\frac{2\pi n\phi}{\Theta^{(p)}} + \theta_n^{(p)}(r)\right), \quad (9)$$

where $A_n^{(p)}(r)$ and $\theta_n^{(p)}(r)$ are (depending on the signal sample magnitude *r*) magnitude and phase of the *n*-th harmonic of the Fourier series:

$$A_n^{(p)}(r)e^{j\theta_n^{(p)}(r)} = \frac{2}{\Theta^{(p)}}\int_0^{\Theta^{(p)}} \mathrm{LLF}\left(p \mid re^{j\phi}\right)e^{-jn\phi}d\phi. \quad (10)$$

It should be noted that for a certain angular position of standard PSK and QAM constellations all complex coefficients (10) of the Fourier series appear real, so that $\theta_n^{(p)}(r) = 0$ or π. In the following formulas we assume for compactness that $\theta_n^{(p)}(r) = 0$, so that $A_n^{(p)}(r)$ can be negative. We will refer to these alternating versions of $A_n^{(p)}(r)$ as *weighting functions*.

In such a way LLF (5) for the whole signal sequence $\{\dot{x}_k\}$ can be written in the following form:

$$\begin{aligned}
&\mathrm{LLF}\left(p \mid \{\dot{x}_k\}, \varphi_f, \varphi_0\right) = \sum_{k=0}^{K-1} \mathrm{LLF}\left(p \mid \{r_k e^{j(\phi_k - \varphi_0 - k\varphi_f)}\}\right) \\
&= \sum_{k=0}^{K-1}\left(\frac{A_0^{(p)}(r_k)}{2} + \sum_{n=1}^{\infty} A_n^{(p)}(r_k)\cos\left(\frac{2\pi n}{\Theta^{(p)}}\left(\phi_k - \varphi_0 - k\varphi_f\right)\right)\right) \\
&= \sum_{k=0}^{K-1}\frac{A_0^{(p)}(r_k)}{2} + \sum_{n=1}^{\infty}\sum_{k=0}^{K-1}\mathrm{Re}\left(A_n^{(p)}(r_k)e^{j\left(\frac{2\pi n}{\Theta^{(p)}}\left(\phi(k) - \varphi_0 - k\varphi_f\right)\right)}\right) \\
&= \frac{f_0^{(p)}\left(\{\dot{x}_k\}\right)}{2} + \mathrm{Re}\sum_{n=1}^{\infty}\dot{f}_n^{(p)}\left(\left\{\dot{x}_k e^{-j\left(\varphi_0 + k\varphi_f\right)}\right\}\right),
\end{aligned} \quad (11)$$

where Re denotes the real part of a complex variable, and

$$\dot{f}_n^{(p)}\left(\{\dot{x}_k\}\right) = \sum_{k=0}^{K-1} A_n^{(p)}(r_k)\exp\left(j\frac{2\pi n\phi_k}{\Theta^{(p)}}\right). \quad (12)$$

It is seen from (12) that complex functions $\dot{f}_n$ possess the following important property (φ is arbitrary angle):

$$\dot{f}_n^{(p)}\left(\{\dot{x}_k e^{j\varphi}\}\right) = e^{j\frac{2\pi n\varphi}{\Theta^{(p)}}}\dot{f}_n^{(p)}\left(\{\dot{x}_k\}\right). \quad (13)$$

Due to this property, we can rewrite (11) in the form

$$\begin{aligned}
&\mathrm{LLF}\left(p \mid \{\dot{x}_k\}, \varphi_f, \varphi_0\right) \\
&= \frac{f_0^{(p)}\left(\{\dot{x}_k\}\right)}{2} + \mathrm{Re}\sum_{n=1}^{\infty} e^{-j\frac{2\pi n\varphi_0}{\Theta^{(p)}}}\dot{f}_n^{(p)}\left(\left\{\dot{x}_k e^{-jk\varphi_f}\right\}\right) \\
&= \frac{1}{2}F_0^{(p)}(0) + \mathrm{Re}\sum_{n=1}^{\infty} e^{-j\frac{2\pi n\varphi_0}{\Theta^{(p)}}}\dot{F}_n^{(p)}\left(\frac{2\pi n\varphi_f}{\Theta^{(p)}}\right),
\end{aligned} \quad (14)$$

where

$$\begin{aligned}
\dot{F}_n^{(p)}(\nu) &= \sum_{k=0}^{K-1} A_n^{(p)}(r_k)\exp\left(j\frac{2\pi n\phi_k}{\Theta^{(p)}}\right)\exp(-jk\nu) \\
&= \sum_{k=0}^{K-1}\dot{u}_n^{(p)}(\dot{x}_k)\exp(-jk\nu)
\end{aligned} \quad (15)$$

is the spectrum of nonlinearly transformed signal sequence

$$\dot{u}_n^{(p)}(\dot{x}) = \dot{u}_n^{(p)}(re^{j\phi}) = A_n^{(p)}(r)\exp\left(j\frac{2\pi n\phi}{\Theta^{(p)}}\right). \quad (16)$$

Weighting functions $A_n^{(p)}(r)$ can be implemented using lookup tables, so presenting the LLF in the form (14) leads to convenient computation procedures. It does not reduce the amount of computations but allows to truncate the series as will be shown in the next section.

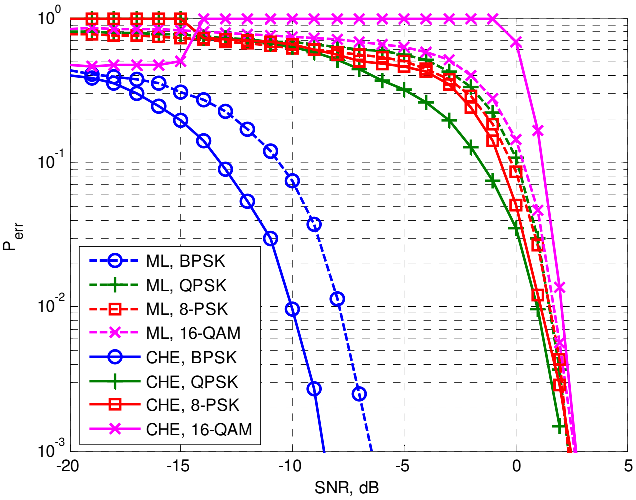

Figure 1. Error probability of recognition (coherent case)

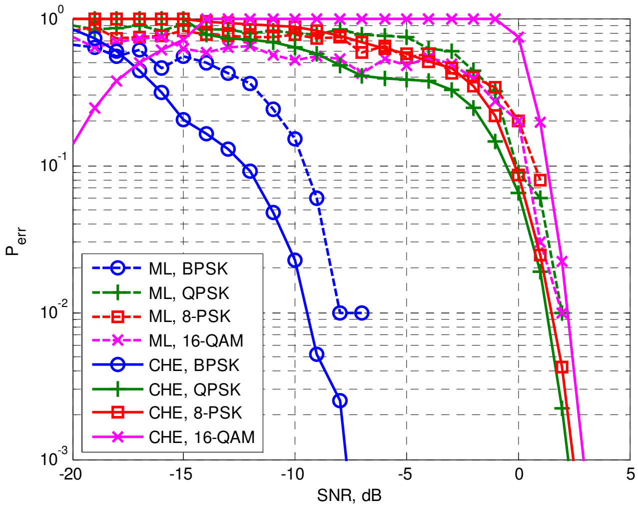

Figure 2. Error probability of recognition (phase averaging)

4. PROPOSED RECOGNITION METHODS

Circular harmonic expansion of LLF allows unified organization of its calculation for all considered constellations but averaging and/or maximization over phase and frequency offsets are still difficult. But as circular harmonic magnitudes decrease with their order it is possible to use LLF approximations by truncating the series in (14).

Especially simple approximation results from retaining in (14) only the angle-independent term ($n = 0$) and the first harmonic ($n = 1$):

$$\mathrm{LLF}\left(p \mid \{\dot{x}_k\}, \varphi_f, \varphi_0\right) \approx \frac{1}{2} F_0^{(p)}(0) + \mathrm{Re}\left(e^{-j\frac{2\pi\varphi_0}{\Theta^{(p)}}} \dot{F}_1^{(p)}\left(\frac{2\pi\varphi_f}{\Theta^{(p)}}\right)\right). \quad (17)$$

This approximation can be directly used instead of (5) when phase and frequency offsets are known. Note that to calculate (17) only two lookup tables for every hypothesis are needed that leads to a computationally efficient procedure.

Averaging or maximization over phase and frequency offsets leads to the following solutions:

a) Averaging over phase offset, frequency offset is known (approximation of (6)):

$$\mathrm{LLF}_{\varphi_0}\left(p \mid \{\dot{x}_k\}, \varphi_f\right) \approx \frac{1}{2} F_0^{(p)}(0) + \log I_0\left(\left|\dot{F}_1^{(p)}\left(\frac{2\pi\varphi_f}{\Theta^{(p)}}\right)\right|\right), \quad (18)$$

where I_0 is the modified Bessel function of the first kind of order zero.

b) Maximization over frequency offset, averaging over phase offset (approximation of (7)):

$$\mathrm{LLF}_{\varphi_0, \varphi_f}\left(p \mid \{\dot{x}_k\}, \varphi_f\right) \approx \frac{1}{2} F_0^{(p)}(0) + \log I_0\left(\max_{\nu}\left|\dot{F}_1^{(p)}(\nu)\right|\right). \quad (19)$$

Maximum search in (19) is performed for magnitude spectrum of nonlinearly transformed signal, so it can be efficiently implemented using fast Fourier transform algorithms.

5. SIMULATION RESULTS

To assess performance of proposed AMR methods, computer simulation was performed. In all cases sequences of $K = 1000$ symbols were used. The curves show dependences of recognition error probability on SNR for various signal constellations. The set of tested hypotheses included 2/4/8-PSK and 16-QAM.

The results for coherent case (phase and frequency offsets are known) are shown in Fig. 1. Dashed lines correspond to the ML method (5), solid—to the proposed approximation (17). The curves show that the proposed method allows to distinguish between all constellations, but the changes in performance (relative to the ML algorithm) depend on particular modulation modes: for all PSK signals SNR thresholds where error probability drops below 10^{-3} became lower than for the ML method, and only for 16-QAM SNR threshold moved slightly (about 1 dB) higher. There is no contradiction here, because ML approach minimizes *mean* recognition error probability averaged over the whole set of *a priori* equiprobable hypotheses, so any approximate implementation of ML method may lead to better results for a few (but not all) hypotheses. Also we can see that for very low SNR, unlike the ML case, decisions in favor of different hypotheses are not equiprobable: they are roughly evenly split between BPSK and 16-QAM, while QPSK and 8-PSK never win.

The results for the case of phase averaging (frequency offset is known) are shown in Fig. 2. Dashed lines correspond to the ML method (6), solid—to the proposed approximation (18). The curves show that, similar to the coherent case, the proposed algorithm allows recognition of

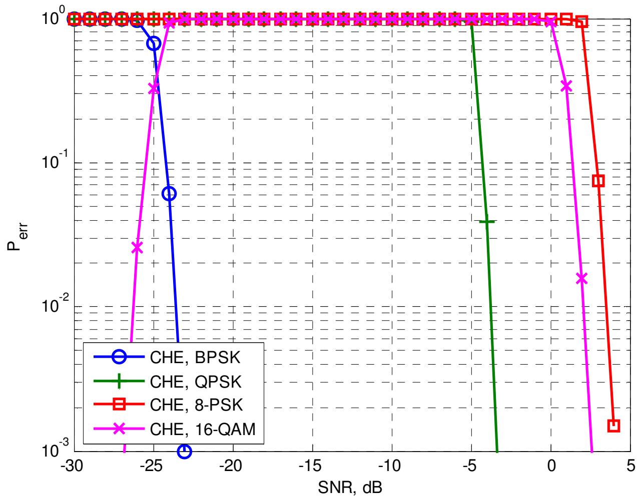

Figure 3. Error probability of recognition (phase averaging with frequency maximization)

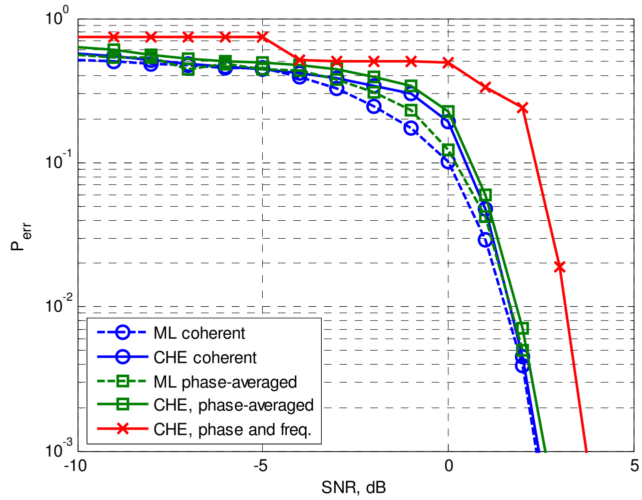

Figure 4. Mean probabilities of recognition error

the considered hypotheses. All performance curves for PSK signals show lower error probability than for the ML method, and only for 16-QAM there is performance loss about 1 dB. Also, we can see that at very low SNR almost all decisions of the proposed algorithm are in favor of 16-QAM hypothesis (recognition errors for this signal tends to zero as SNR drops below −15 dB).

The results for the case of phase averaging with frequency maximization are shown in Fig. 3. The curves correspond to the proposed approximation (19). Results for the ML method are not shown as the maximum search over frequency offset values in (7) is computationally prohibitive. Comparison between Fig. 3 and Fig. 2 shows that the performance loss about 1…2 dB is observed only in the case of 8-PSK modulation. Also, at very low SNR, bias of the decisions towards 16-QAM hypothesis became even stronger than in Fig. 2 (recognition errors for this signal quickly disappear as SNR drops below −24 dB).

In Fig. 4, error probabilities for all considered methods averaged over all modulation modes (as ML approach assumes equal *a priori* probabilities for all hypotheses) are shown. Comparing these curves we can see that in all cases except unknown frequency offset high-SNR performance is very close to each other, the losses do not exceed 0.5 dB. In the case of unknown frequency offset, the mean loss is about 1.5 dB.

Unfortunately, the formulas (17)–(19) can not be directly used to recognize higher-order QAM constellations—one circular harmonic is not enough for them. Adaptation of the presented idea to higher-order QAM constellations is the subject of our further studies.

6. CONCLUSION

The proposed approximation of LLF allows computationally efficient implementation of linear digital modulation recognition. It is especially attractive in the case of unknown phase and frequency offsets, as the proposed algorithms (18) and (19) effectively combine LLF calculation with phase averaging and maximization over frequency offset.

Possible direction of future work is optimization of weighting functions to maximize probability of correct recognition and to allow recognition of higher-order QAM constellations. Also, potential benefits from the use of more than one circular harmonic need to be investigated.

7. REFERENCES

[1] O. A. Dobre, A. Abdi, Y. Bar-Ness, and W. Su, “Survey of automatic modulation classification techniques: classical approaches and new trends,” IET Commun., Vol. 1, No. 2, April 2007, pp. 137–156.

[2] A. B. Sergienko and A. V. Petrov, “Joint blind estimation of carrier phase and frequency offset for QAM signals using circular harmonic decomposition,” Proc. 36th Int. Conf. on Acoustics, Speech and Signal Processing (ICASSP 2011), Prague, May 22–27, 2011, pp. 3460–3463.

[3] P. Ciblat, P. Loubaton, E. Serpedin, and G. B. Giannakis, “Asymptotic analysis of blind cyclic correlation-based symbol-rate estimators,” IEEE Trans. Inf. Theory, Vol. 48, No. 7, July 2002, pp. 1922–1934.

[4] J. A. Sills, “Maximum-likelihood modulation classification for PSK/QAM,” Proc. IEEE MILCOM, 1999, pp. 57–61.

[5] W. Wei and J. M. Mendel, “Maximum-likelihood classification for digital amplitude-phase modulations,” IEEE Trans. Commun., 2000, Vol. 48, pp. 189–193.

[6] A. Abdi, O. A. Dobre, R. Choudhry, Y. Bar-Ness, and W. Su, “Modulation classification in fading channels using antenna arrays,” Proc. IEEE MILCOM, 2004, Vol. 1, pp. 211–217.

[7] L. Hong, O. A. Dobre, Y. Bar-Ness, and W. Su, “Quasi-hybrid likelihood modulation classification with nonlinear carrier frequency offsets estimation using antenna arrays,” Proc. IEEE MILCOM, 2005, Vol. 1, pp.570–575.

[8] G. Jacovitti and A. Neri, “Multiresolution Circular Harmonic Decomposition,” IEEE Trans. Signal Processing, Vol. 48, No. 11, Nov. 2000, pp. 3242–3247.