MULTIPLE TESTING FOR SEQUENTIAL PROBABILITY RATIO TESTS WITH APPLICATION TO MULTIBAND SPECTRUM SENSING

Fiky Y. Suratman and Abdelhak M. Zoubir

Signal Processing Group,Technische Universität Darmstadt
Merckstrasse 25, 64283, Darmstadt, Germany

ABSTRACT

Literature on multiple testing is mostly concerned with fixed sample number testing. In this paper, we propose a sequential multiple testing procedure. This work is motivated by an application in multiband spectrum sensing for cognitive radio, in which a primary user or a cognitive radio user can use several bands at a time. The proposed procedure simultaneously controls the false alarm and miss detection rate not only for a single band, but also for the system (familywise). The common method to individually testing the hypotheses fails to achieve this. Furthermore, simulation results show that the proposed method has a smaller average sensing time (sample number) than Bonferroni's procedure, which makes it suitable for the scenario at hand.

Index Terms— Multiple testing procedure, sequential probability ratio tests, multiband spectrum sensing, cognitive radio.

1. INTRODUCTION

A multiple testing procedure (MTP) refers to the testing of more than one hypothesis at a time. It is intended to solve the multiplicity effect [1], by making the individual tests more conservative to arrive at rejecting a hypothesis. Some classical MTPs such as Bonferroni's method and its extensions can be found in [1] and references therein. The main issue in classical MTPs is the tight control of type-I errors. Thus, they tend to have substantially less power than individual testing procedures of the same levels, particularly when the number of hypotheses increases. To solve this issue, Benjamini and Hochberg [2] propose to control the false discovery rate instead of the familywise error rate (see Section 3 for definitions), which is more attractive to use in fixed sample number cases. The implementation of an MTP to source enumeration can be found, for example, in [3] and to the identification of optimal sensor positions can be found in [4]. It also has been implemented in distributed detections and wireless sensor networks, such as in [5–8]. In this paper, we propose the implementation of an MTP in multiband (wideband) spectrum sensing to jointly detect availabilities of multiple bands, which are then to be used by cognitive radio (CR) users.

Multiband spectrum sensing is relatively less studied than single band (narrow-band) spectrum sensing [9]. To mention a few, multiband spectrum sensing can be found in [10–12]. The motivation behind the use of MTPs in multiband spectrum sensing is the capability of MTPs to provide control over decision errors not only per single band but also for the overall system. In this respect, [13] and [14] propose to use Benjamini-Hochberg procedure [2] in multiband spectrum sensing. The results show a better tradeoff between type I errors and type II errors at the system level, compared to individual testing procedures. In all the above literature, MTPs are used in the context of a fixed sample number. In this paper, we propose an MTP for sequential detections, more precisely for sequential probability ratio tests (SPRT), where the sample number is random. To the best of our knowledge, the study of MTP for the SPRT is scarce. Note that with an MTP for the SPRT we do not mean sequential multihypothesis testing, such as in [15–17] and the like. At least, three points mark the differences between the implementation of an MTP in fixed sample number cases (FSN) and in the SPRT we propose here. *First*, unlike FSN, the p-value [18] as the level of evidence is difficult to get in the SPRT since the number of samples N changes. However, we can use random stopping times (random sample numbers) instead. The smallest stopping time can be considered to be equivalent to the smallest p-value. Simply said, an MTP in FSN works based on the ordered p-values, while in the SPRT it works based on the ordered stopping times. *Second*, ordering p-values in FSN is done after all tests finished, while ordering stopping times in the SPRT is done while the SPRTs are in progress, successively one test after another, depending on the one that finished earlier. *Third*, the main objective in MTP for FSN is to maximize the power, while in the SPRT context, the aim is to minimize the average sample number.

2. SYSTEM MODEL

We assume that the primary network operates over a wide frequency bandwidth which is divided into K nonoverlapping subbands, such as in multicarrier-based system. Whenever possible, a primary user can be assigned to use a number of subbands K_p simultaneously, where $1 \leq K_p \leq K$. The binary hypothesis testing problem for spectrum sensing of the subband k is

$$\begin{aligned} &\mathcal{H}_{k,0} : f_{k,0}(\mathbf{x}_k[n]; \boldsymbol{\theta}_{k,0}) \\ &\mathcal{H}_{k,1} : f_{k,1}(\mathbf{x}_k[n]; \boldsymbol{\theta}_{k,1}), \quad k = 1, \ldots, K. \end{aligned} \tag{1}$$

where $\mathbf{x}_k[n]$ denotes a scalar or a vector observation, and $f_{k,i}(\cdot)$ is the density function of the subband k under hypothesis $\mathcal{H}_i,\ i = 0, 1$. $\boldsymbol{\theta}_{k,0}$ and $\boldsymbol{\theta}_{k,1}$ are the parameters for the subband k which could be scalars or vectors, under the respective hypotheses. Here, we assume that the observations are identically independent distributed (i.i.d.) within subbands and also independent accross subbands. Suppose that within a particular time interval, K_0 out of K subbands might not be used by the primary users and are available for cognitive access. Let us assume that the CR network supports some CR users to use several unoccupied subbands simultaneously. The number of subbands $K_c,\ 1 \leq K_c \leq K_0$, assigned to a specific CR user is, say, based on priority and currently active CR users. To accommodate the use of multiple subbands by the primary and the CR users, we need an overall view for the performance of spectrum sensing, e.g., false alarm and miss detection not only per subband, but for $1 \leq K_c \leq K_0$ and $1 \leq K_p \leq (K - K_0)$, respectively. Note that

978-1-4799-2893-4/14/$31.00 ©2014 IEEE

K_0 could span from 0 (all subbands occupied) to K (all subbands unoccupied).

For the sake of clarity, we give an illustration. Suppose that each subband is individually tested with nominal probabilities of false alarm $\alpha_k = \alpha$ and miss detection $\beta_k = \beta,\ k = 1, \ldots, K$. Let us assume that there are $K_0 = 5$ unoccupied subbands and two active CR users, that is one with high priority (can use $1 \leq K_c \leq 4$ subbands) and the other with low priority ($K_c = 1$). The low priority user could have a subband to use, while the high priority user might defer to use all four unoccupied subbands simultaneously, and thus uses lower K_c, due to the higher false alarm rate caused by the multiplicity effect (the actual probability of false alarm for the four subbands = $1 - (1 - \alpha)^4$). With the same argument, any primary user that uses a higher number of subbands K_p simultaneously will experience a higher aggregate interference level due to a higher probability of miss detection. In this case, an MTP should be implemented in multiband spectrum sensing to jointly detect the subbands and hence to provide control on the decision errors at the system level.

3. PERFORMANCE MEASURES

Referring to Table 1, the so-called *familywise error rate* is defined as the probability of committing any type I error or false alarm in families of comparisons, formally

$$\mathrm{FWE} = P_0(V \geq 1), \tag{2}$$

where $P_0(\cdot)$ represents the probability of an event under $\mathcal{H}_0$, which could be complete null hypotheses (all $H_{0,k}$ are true) or partial null hypotheses (some subsets of nulls, say $\mathcal{H}_{0,j1}, \ldots, \mathcal{H}_{0,jk}$, are true). An MTP is said to control the FWE in the weak sense if FWE $\leq \alpha$ only under complete null hypotheses and in the strong sense if FWE $\leq \alpha$ under partial null hypotheses, regardless of which subsets of null hypotheses is true [19], including complete null hypotheses. Here, we define *familywise miss detection* (FWM). It refers to the probability of committing any type II error or miss detection in families of comparisons, formally

$$\mathrm{FWM} = P_1(T \geq 1), \tag{3}$$

which also could be under complete or partial alternative hypotheses. In addition, since we do sequential detection, other important measures are the average sample number over all subbands that are under $\mathcal{H}_0$, denoted as ASN_0, and over all subbands that are under $\mathcal{H}_1$, denoted as ASN_1. They are defined formally as

$$\mathrm{ASN}_0 = E_0\left[\frac{\sum_{i=1}^{K_0} N_{s,i}}{K_0}\right],\ \mathrm{ASN}_1 = E_1\left[\frac{\sum_{l=1}^{K-K_0} N_{s,l}}{K-K_0}\right], \tag{4}$$

where $N_{s,i}$ denotes the stopping time of the subband S_i. Note that a false discovery rate (FDR $= E\left[V/R\right]$) [2] controlling procedure is now commonly used in the fixed sample number case. This is because FDR controlling tests are more powerful than those controlling the FWE. However, the use of the FDR in MTP for the SPRT is unclear due to the capability of the SPRT to simultaneously control the probabilities of false alarm and miss detection.

4. MTP FOR THE SPRT

Let $\mathbf{x}_{k,N} = (\mathbf{x}_k[1]\ \mathbf{x}_k[2]\ \cdots\ \mathbf{x}_k[N])$ be a sequence of i.i.d. observations of a signal recorded up to the sample N at subband S_k. Here,

Table 1. Number of correct and false decisions for testing K subbands

	Declared $\mathcal{H}_0$	Declared $\mathcal{H}_1$	Total
True $\mathcal{H}_0$	U	V	K_0
True $\mathcal{H}_1$	T	S	$K - K_0$
	$K - R$	R	K

$\mathbf{x}_k[n]$ is assumed to admit the distribution described by the density function in (1) under each hypothesis. The sequential probability ratio test (SPRT) can then be defined as

$$Z_{k,N} = \begin{cases} \geq A, & \text{accept } \mathcal{H}_{k,1} \\ \leq B, & \text{accept } \mathcal{H}_{k,0}, \\ A < Z_{k,N} < B, & N \leftarrow N+1. \end{cases} \tag{5}$$

where $Z_{k,N} = \sum_{n=1}^{N} \log \dfrac{f_{k,1}(\mathbf{x}_k[n]; \boldsymbol{\theta}_{k,1})}{f_{k,0}(\mathbf{x}_k[n]; \boldsymbol{\theta}_{k,0})},\ k = 1, 2, \ldots, K$. For the implementation, we use the thresholds [20]

$$A \approx \log \frac{1}{\alpha'},\ \ B \approx \log \beta', \tag{6}$$

to have the actual probabilities of false alarm $P_{f,k} \leq \alpha'$ and miss detection $P_{m,k} \leq \beta'$ of the subband S_k, where α' and β' denote the respective nominal values of the probabilities of false alarm and miss detection per subband [21]. Note that we use the same thresholds for all subbands. We further assume that the SPRTs start simultaneously at all K subbands whenever the sensing period starts, and the SPRT running in each subband fulfills the condition of having a finite random stopping time $N_{s,k},\ k = 1, \ldots, K$ [22]. The objective is to have

$$\mathrm{FWE} \leq \alpha, \quad \mathrm{FWM} \leq \beta, \tag{7}$$

with ASN_0 and ASN_1 as small as possible, by jointly testing all K subbands.

Simple Bonferroni procedure (SBF) for the SPRT. For K hypotheses (subbands), the simplest way of conducting an MTP is to follow the simple Bonferroni procedure [19], like in the fixed sample number case. More precisely, we test each subband individually at the level $\alpha_k = \alpha/K$ and $\beta_k = \beta/K,\ \forall k \in \{1, \ldots, K\}$ and set the thresholds accordingly using (6). Thus, it will guarantee to fulfill the sensing objective (7). However, this approach is too conservative in protecting decision errors and hence it results in large ASN_0 and ASN_1 for the SPRT. To solve this issue, we propose a stepwise procedure that will be elaborated in the sequel.

Stepwise procedure (SWP) for the SPRT. Inspired by the work of Holm [23] that uses the ordered p-values for the fixed sample number case, the SWP is based on the ordered stopping times $N_{s,(1)} \leq N_{s,(2)} \leq \cdots \leq N_{s,(K)}$ corresponding to subbands $S_{(1)}, S_{(2)}, \ldots, S_{(K)}$. The procedure in the SPRT is more involved since we have two thresholds to consider and we perform ordering while the sample number N is increasing. In principle, we start the SPRT in each subband with the largest value of the threshold A and the lowest value of the threshold B. The largest A and the lowest B depend on the nominal values of FWE and FWM and the number of subbands K. Whenever one or more SPRTs stop and favor $\mathcal{H}_1$ (the subbands are declared occupied), update A with a smaller value to conduct the SPRT at the other subbands, and simultaneously, whenever one or more SPRTs stop and favor $\mathcal{H}_0$ (the subbands are declared unoccupied), update B with a larger value to conduct the

SPRT at the other subbands. In addition, after any SPRT stops we proceed with a procedure to find the other SPRTs that probably have crossed the respective smaller value of A or larger value of B in the past. If any, declare the respective subbands as occupied or unoccupied and repeat the procedure with the next smaller A or larger B. Otherwise, the SPRTs proceed for the rest of subbands. The whole process continues until all subbands have been declared as occupied or unoccupied.

Suppose that the nominal values of the FWE and FWM are α and β, respectively. The detail of the SWP is as follows:

Step 1). Initialize the sample number $N = 0$, variables $I_1 = 0$ and $I_0 = 0$, and a set of terminated subbands $\mathbf{S}_T = \emptyset$. Two sets of thresholds

$$\begin{aligned}\mathbf{\Lambda}_A &= \{\log(1/\alpha), \log(2/\alpha), \ldots, \log((K-I_1)/\alpha)\} \\ \mathbf{\Lambda}_B &= \{\log(\beta), \log(\beta/2), \ldots, \log(\beta/(K-I_0))\},\end{aligned} \tag{8}$$

correspond to the FWE and FWM of Benferroni's method for the number of subbands $1, 2, \ldots, K$. The size of the sets $\mathbf{\Lambda}_A$ and $\mathbf{\Lambda}_B$ will be shrinking while in progress, which depends on the variables I_1 and I_0. I_1 indicates the number of subbands which have been declared occupied and I_0 indicates the number of subbands which have been declared unoccupied.

Step 2). For all subbands $S_k \notin \mathbf{S}_T$, take a sample $N \leftarrow N+1$, calculate and then compare $Z_{k,N}$ according to (5) where $A = \max\{\lambda_A : \lambda_A \in \mathbf{\Lambda}_A\}$ and $B = \min\{\lambda_B : \lambda_B \in \mathbf{\Lambda}_B\}$. If one of the two thresholds is crossed at any subband, continue to **Step 3**, otherwise repeat **Step 2**. Note that, at each stage N, $Z_{k,N}$ is also inspected by the processor in the subband k for whether it has crossed the upper thresholds $\{\log(1/\alpha), \ldots, \log((K-I_1-1)/\alpha)\}$ or the lower thresholds $\{\log(\beta), \ldots, \log(\beta/(K-I_0-1))\}$. The results are stored, say, in a memory $\mathbf{u}_k = \left[u_{k,1}, u_{k,2}, \ldots, u_{k,(K-1)}\right]$, where $u_{k,l}$ is set to 1 if $Z_{k,N} \geq \log(l/\alpha)$ or set to 0 if $Z_{k,N} \leq \log(\beta/l)$, otherwise keep $u_{k,l}$ empty.

Step 3). If the SPRTs in a subset of subbands $\mathbf{S}_1 = \{S_{(1)}, \ldots, S_{(L_1)}\}$ stop, due to the respective $Z_{(k),N} \geq A$, the subbands $S_{(1)}, \ldots, S_{(L_1)}$ are declared occupied, and the variable $I_1 \leftarrow I_1 + L_1$ is updated. Simultaneously, if the SPRTs in a subset of subbands $\mathbf{S}_0 = \{S_{1^*}, \ldots, S_{L_0^*}\}$, where $\mathbf{S}_0 \cap \mathbf{S}_1 = \emptyset$, stop, due to the respective $Z_{k^*,N} \leq B$, the subbands $S_{1^*}, \ldots, S_{L_0^*}$ are declared unoccupied, and the variable $I_0 \leftarrow I_0 + L_0$ is updated. The set of terminated subbands should also be updated, i.e., $\mathbf{S}_T = \{S_T, S_1, S_0\}$.[1] If not all subbands have been declared occupied or unoccupied, the decisions for the rest of the subbands depend on the results of the following procedure, which inspects the memory $\mathbf{u}_k$, $\forall S_k \notin \mathbf{S}_T$. Initially, set index variables $i_0 = 1$ and $i_1 = 1$,

(a) If none of $S_k \notin \mathbf{S}_T$ having $u_{k,(K-I_1-i_1)} = 1$ or $u_{k,(K-I_1-i_1)} = 0$, update the sets $\mathbf{\Lambda}_A$ and $\mathbf{\Lambda}_B$ in (8) and repeat **Step 2**.

(b) Otherwise, if $u_{k,(K-I_1-i_1)} = 1$ in a subset of subbands $\mathbf{S}'_1 = \{S_{1'}, \ldots, S_{l'_1}\}$, the subbands $S_{1'}, \ldots, S_{l'_1}$ are declared occupied, update the variable $i_1 \leftarrow i_1 + l_1$. Simultaneously, if $u_{k,(K-I_1-i_0)} = 0$ in a subset of subbands $\mathbf{S}'_0 = \{S'_{1^*}, \ldots, S'_{l_0^*}\}$ where $\mathbf{S}'_0 \cap \mathbf{S}'_1 = \emptyset$, the subbands $S'_{1^*}, \ldots, S'_{l_0^*}$ are declared unoccupied, and then update the variable $i_0 \leftarrow i_0 + l_0$. The set of terminated subbands should also be updated, i.e., $\mathbf{S}_T = \{\mathbf{S}_T, \mathbf{S}'_1, \mathbf{S}'_0\}$. As long as $\mathbf{S}'_0 \cup \mathbf{S}'_1 \neq \emptyset$, continue from the beginning of (b). Otherwise, if not all subbands have been declared occupied or unoccupied, update the variables $I_1 \leftarrow I_1 + i_1 - 1$ and $I_0 \leftarrow I_0 + i_0 - 1$ and accordingly the sets $\mathbf{\Lambda}_A$ and $\mathbf{\Lambda}_B$ in (8), then repeat **Step 2**.

[1] Note that the possibility to have the total number of terminations $L_1 + L_0 > 1$ is small when the SNRs (more precisely, the increments [24]) are small, since under this condition, the variance of the stopping time at each subband is large and thus the SPRTs most likely will not stop at the same time (either $\mathbf{S}_0$ or $\mathbf{S}_1$ is mostly an empty set). However, when the SNRs are high, the possibility is larger, since the variance of stopping time in each subband is small.

Note that we only update the upper threshold A (not A and B) whenever one or more SPRTs stop and favor $\mathcal{H}_1$. The same holds for the opposite, when favoring $\mathcal{H}_0$. This can be explained as follows. Suppose that an SPRT at the subband S_k stops and favors $\mathcal{H}_{k,1}$ with the thresholds $A = \log(K/\alpha)$ and $B = \log(\beta/K)$. In this case, when $\mathcal{H}_{k,0}$ has been rejected, using the nominal value of the probability of false alarm $\alpha' = \alpha/K$ ($A = \log(K/\alpha)$), we should believe that $\mathcal{H}_{k,0}$ is false ($\mathcal{H}_{k,1}$ is true). Therefore, there are only $K-1$ null hypotheses which might be still true, implying the critical value now to be $\alpha' = \alpha/(K-1)$ (update the upper threshold to $A = \log((K-1)/\alpha)$). However, for the lower threshold B, we should believe that there are still K alternative hypotheses which might be true (including the one that has been declared), since we have no evidence that any of $\mathcal{H}_{k,1}$ has been rejected. This implies the nominal value for the probability of miss detection is still $\beta' = \beta/K$ (the lower threshold is maintained at $B = \log(\beta/K)$). The same argument applies when an SPRT at the subband S_k stops and favors $\mathcal{H}_{k,0}$. If we were to update both thresholds each time an SPRT stops, regardless of which hypothesis is rejected, then the objective (7) will not be achieved.

5. EXAMPLE

As an example, we assume that a CR user receives complex Gaussian signals in each subband, i.e

$$\begin{aligned}&\mathcal{H}_{k,0} : \mathbf{x}_k[n] \sim \mathcal{CN}\left(0, \sigma_{k,0}^2\right), \\ &\mathcal{H}_{k,1} : \mathbf{x}_k[n] \sim \mathcal{CN}\left(0, \sigma_{k,1}^2\right), \quad k = 1, 2, \ldots, K.\end{aligned} \tag{9}$$

The log-likelihood ratio $Z_{k,N}$ can then be calculated from (5) to perform spectrum sensing in K subbands. For all simulations we assume that the noise power $\sigma_{k,0}^2 = 1,\ k = 1, \ldots, K$, where $\sigma_{k,1}^2$ depends on the SNR at each subband, which is defined as $\text{SNR}_k = 10\log_{10}((\sigma_{k,1}^2 - \sigma_{k,0}^2)/\sigma_{k,0}^2)$. The nominal values of the FWE and FWM are set to $\alpha = \beta = 0.1$. All the results are generated using 10^4 Monte Carlo runs.

Fig. 1 shows the resulting FWE and FWM for the SBF and the SWP. The number of subbands that are jointly tested is $K = 8$ and the number of unoccupied subbands K_0 varies from 0 to 8. Note that K_0 unoccupied subbands were randomly selected from the K subbands in each Monte Carlo run. Here, the occupied subbands have the same SNR$= -10$ dB. The results when all subbands are individually tested without MTP at the nominal values $\alpha' = \beta' = 0.1$, are also shown. In general, Fig. 1 indicates that the SBF and SWP control the FWE and the FWM in the strong sense. More precisely, regardless of how many available subbands might be opportunistically used by a CR user, the probability of losing opportunity to use the respective subbands is no larger than $\alpha = 0.1$, and regardless of how many subbands are used by a primary user, the probability of the respective primary user to receive interferences is no larger than $\beta = 0.1$. However, full protection resulting from the SBF is too restrictive, particularly for the FWE when K_0 is large and for the FWM when K_0 is small. Meanwhile, lack of multiplicity control by testing individually is too permissive, and hence reducing the

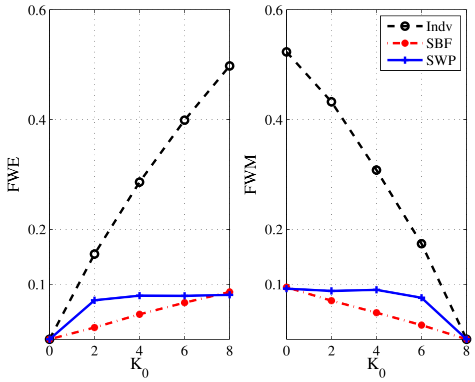

Fig. 1. FWE and FWM vs. K_0 for individually tested without MTP (Indv), the SBF, and the proposed SWP.

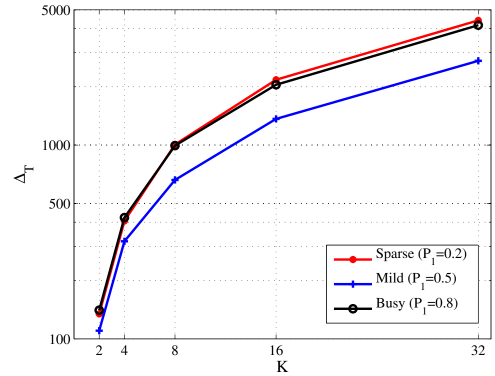

Fig. 2. The gap Δ_T between the total ASN of the SBF and the SWP vs. K, when the subbands occupancies are busy, mild and sparse.

Table 2. ASN_0 and ASN_1 for the SBF and the SWP as a function of K_0.

		K_0				
		0	2	4	6	8
ASN_0	SBF	-	980	977	977	978
	SWP	-	928	910	861	779
ASN_1	SBF	934	935	934	932	-
	SWP	743	820	864	888	-

overall throughput of the CR network (due to higher FWE for higher numbers of unoccupied subbands) and increasing the interferences to the primary users (due to higher FWM for higher number of occupied subbands), especially for the primary users that use several subbands at a time. The proposed procedure SWP handles the problem appropriately. It pulls the FWE and the FWM closer to the nominal values while still preserving the objective (7). As a result, the SWP has smaller average sample numbers than the SBF, as shown in Table 2. By using the SWP, the gain that we obtain on the ASN_1 is larger when the subbands are busier (mostly occupied), and the gain is larger for the ASN_0 when the subbands are sparser (mostly unoccupied).

According to (4), the ASN_0 and ASN_1 represent the average sample numbers per subband. The total average sample number can be defined as

$$\begin{aligned}\text{ASN}_T =& E\left[\sum_{i=1}^{K} N_{s,i}\right] = E\left[\sum_{i=1}^{K_0} N_{s,i} + \sum_{l=1^*}^{(K-K_0)^*} N_{s,l}\right] \\ \approx& K\left\{(1-P_1)\text{ASN}_0 + P_1\text{ASN}_1\right\}, \qquad (10)\end{aligned}$$

where we have assumed that the probability of each subband being occupied by a primary user is equal, i.e., $P(\mathcal{H}_{k,1}) = P_1$, $k = 1, \ldots, K$. In Fig. 2, we plot the gap $\Delta_T = \text{ASN}_T(\text{SBF}) - \text{ASN}_T(\text{SWP})$, between the total ASN of the SBF and the SWP against the number of subbands K, in which the occupied subbands have the same SNR$= -10$ dB. We evaluate the performance when the channel occupancy is considered to be busy ($P_1 = 0.8$), mildly busy ($P_1 = 0.5$) and sparse ($P_1 = 0.2$). It can be remarked that the gap is larger when either K subbands are busy or sparse. The reasoning is as follows. When the subbands are busy, the alternative hypotheses dominate the MTP. In this case, the probability of the SWP updating the upper threshold A down to the lowest value in each realization is high, and hence the probability of SPRTs stop with smaller sample numbers is also high. The same case applies when the subbands are sparse, namely the null hypotheses dominate the MTP. In this case, the probability of the SWP updating the lower threshold B up to the largest value in each realization is high, and hence the probability of SPRTs to stop with smaller sample numbers is also high. However, this does not apply when the occupancies of the subbands are mild since, then, the null hypotheses and the alternative hypotheses are competing. In this case, all SPRTs will mostly have stopped immediately after reaching the $[K/2]$ smallest of the thresholds A and the $[K/2]$ largest of the thresholds B. Therefore, the average sample number of the SWP of mildly occupied subbands is higher than that of busy and sparse subbands. This explains the smaller gap when the occupancies of subbands are mild.

It is noteworthy that in this paper we present our preliminary results on MTP for sequential testing that are based on a heuristic approach. An analytical approach such as using optimization theory is possible. From our perspective, some questions are still open. It includes, for example, what is the optimum way to conduct MTP in the SPRT? What is the role of FDR and some other measures in MTP for sequential testing? Is it possible to use these measures to further reduce the sample number and if so, how? The analytical approach and answering these questions will be the challenges in our future works on MTP for sequential testing.

6. CONCLUSIONS

The paper presents the implementation of multiple testing procedure in sequential probability ratio tests. The results show that the proposed method fulfills the objective to have control over the decision errors at the system level which is required in a scenario where the primary and the cognitive radio networks provide their users accesses to several bands at a time. The total ASN of the proposed method is significantly smaller than that of the simple Bonferroni procedure. Therefore, the proposed method is a promising technique to increase the overall throughput of cognitive radio networks without making harmful interferences to the primary networks.

7. REFERENCES

[1] Y. Hochberg and A.C. Tamhane, *Multiple comparison procedures*, John Wiley & Sons, Inc., 1987.

[2] Y. Benjamini and Y. Hochberg, "Controlling the false discovery rate: a practical and powerful approach to multiple testing," *Journal of the Royal Statistical Society. Series B (Methodological)*, pp. 289–300, 1995.

[3] P.J. Chung, J.F. Böhme, C.F. Mecklenbrauker, and A.O. Hero, "Detection of the number of signals using the benjamini-hochberg procedure," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2497–2508, 2007.

[4] A.M. Zoubir and J.F. Böhme, "Bootstrap multiple tests applied to sensor location," *IEEE Transactions on Signal Processing*, vol. 43, no. 6, pp. 1386–1396, 1995.

[5] P. Ray and P.K. Varshney, "False discovery rate based sensor decision rules for the network-wide distributed detection problem," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, no. 3, pp. 1785–1799, 2011.

[6] P. Ray and P.K. Varshney, "Distributed detection in wireless sensor networks using dynamic sensor thresholds," *International Journal of Distributed Sensor Networks*, vol. 4, no. 1, pp. 5–12, 2008.

[7] E. B. Ermis and V. Saligrama, "Distributed detection in sensor networks with limited range multimodal sensors," *IEEE Transactions on Signal Processing*, vol. 58, no. 2, pp. 843–858, 2010.

[8] E.B. Ermis and V. Saligrama, "Adaptive statistical sampling methods for decentralized estimation and detection of localized phenomena," in *Information Processing in Sensor Networks, 2005. IPSN 2005. Fourth International Symposium on*. IEEE, 2005, pp. 143–150.

[9] E. Axell, G. Leus, E.G. Larsson, and H.V. Poor, "Spectrum sensing for cognitive radio: State-of-the-art and recent advances," *IEEE Signal Processing Magazine*, vol. 29, no. 3, pp. 101–116, 2012.

[10] W.Y. Lee and I.F. Akyildiz, "Optimal spectrum sensing framework for cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 7, no. 10, pp. 3845–3857, 2008.

[11] Z. Quan, S. Cui, A.H. Sayed, and H.V. Poor, "Optimal multiband joint detection for spectrum sensing in cognitive radio networks," *IEEE Transactions on Signal Processing*, vol. 57, no. 3, pp. 1128–1140, 2009.

[12] Z. Quan, S. Cui, H.V. Poor, and A.H. Sayed, "Collaborative wideband sensing for cognitive radios," *IEEE Signal Processing Magazine*, vol. 25, no. 6, pp. 60–73, 2008.

[13] S.H. Wu, C.Y. Yang, and D.H.T. Huang, "Cooperative sensing of wideband cognitive radio: a multiple-hypothesis-testing approach," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 4, pp. 1835–1846, 2010.

[14] G. Atia, E.B. Ermis, and V. Saligrama, "Robust energy efficient cooperative spectrum sensing in cognitive radios," in *Statistical Signal Processing, 2007. SSP'07. IEEE/SP 14th Workshop on*. IEEE, 2007, pp. 502–506.

[15] V.P. Dragalin, A.G. Tartakovsky, and V.V. Veeravalli, "Multihypothesis sequential probability ratio tests. i. asymptotic optimality," *IEEE Transactions on Information Theory*, vol. 45, no. 7, pp. 2448–2461, 1999.

[16] V.P. Dragalin, A.G. Tartakovsky, and V.V. Veeravalli, "Multihypothesis sequential probability ratio tests. ii. accurate asymptotic expansions for the expected sample size," *IEEE Transactions on Information Theory*, vol. 46, no. 4, pp. 1366–1383, 2000.

[17] A.G. Tartakovsky and X.R. Li, "Sequential testing of multiple hypotheses in distributed systems," in *Information Fusion, 2000. FUSION 2000. Proceedings of the Third International Conference on*. IEEE, 2000, vol. 2, pp. THC4–17.

[18] A.C. Davison and D.V. Hinkley, *Bootstrap methods and their application*, vol. 1, Cambridge university press, 1997.

[19] P.H. Westfall and S.S. Young, *Resampling-based multiple testing: Examples and methods for p-value adjustment*, vol. 279, Wiley. com, 1993.

[20] A. Wald, "Sequential tests of statistical hypotheses," *The Annals of Mathematical Statistics*, pp. 117–186, 1945.

[21] Q. Zou, S. Zheng, and A.H. Sayed, "Cooperative sensing via sequential detection," *IEEE Transactions on Signal Processing*, vol. 58, no. 12, pp. 6266–6283, 2010.

[22] A. Wald, "On cumulative sums of random variables," *The Annals of Mathematical Statistics*, pp. 283–296, 1944.

[23] S. Holm, "A simple sequentially rejective multiple test procedure," *Scandinavian journal of statistics*, pp. 65–70, 1979.

[24] F.Y. Suratman and A.M. Zoubir, "Effects of model errors on multiple antenna based spectrum sensing using sequential detection," in *Communications, Signal Processing, and their Applications (ICCSPA), 2013 1st International Conference on*. IEEE, 2013, pp. 1–6.