# A NEW NONPARAMETRIC METHOD FOR TESTING STATIONARITY BASED ON TREND ANALYSIS IN THE TIME MARGINAL DISTRIBUTION

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# ABSTRACT

In this manuscript, we propose a novel nonparametric test for nonstationarities that are seen as a trend or an evolution in the local energy of the signal. The idea of the proposed technique consists in applying empirical mode decomposition for estimating and further quantifying the trend in the time marginal of the estimated time-frequency representation. Such methodology allows for the detection of slowly-varying nonstationarities of first and second-order.

*Index Terms*— Stationarity test, time-frequency analysis, time marginals, empirical mode decomposition, trends

# 1. INTRODUCTION

Over the last few decades, much has been done in developing methods to test signals with respect to stationarity. Stationarity tests are important not only in signal processing and in many areas of technical science [1][2][3], but also in the domain of environmental science [4][5][6]. Thus, having stationarity tests that are sensitive to different forms of nonstationarities is required in order to approach the analysis of nonstationary phenomena. In this paper, we present a test that can detect both trends and slowly-varying nonstationarities.

Different procedures for testing stationarity can be found in the literature, some based on time series modeling [7], spectral analysis [8], or detection of changes [9]. As an efficient representation for the signal is often desirable, a number of techniques consists in analyzing the signal in timefrequency (TF) domain, which can offer much valuable information about the structure of the underlying signal in the absence of an external specification. Among the emerging alternatives in the literature, one could categorize parametric [10][3] and nonparametric [11][12] approaches. Recently, a novel TF-based approach was proposed in [13] for testing stationarity. Originally designed for 2nd-order stationarity relatively to a global horizon of observation, it was modified in [14] in order to characterize also 1st-order nonstationarity.

A problem involving these nonparametric TF-based techniques is their computational load, as we often need to compute various TF representations, one for each stationary resample [13]. Thus, in this paper, we propose a simpler nonparametric approach, which is efficient for detecting a trend and/or an evolution of the local energy of the signal. For that, we use the method of [15] for estimating the trend in the time marginal of the TF representation. We further develop a measure to quantify the significance of the trend and a test is designed to detect these 2nd-order trend nonstationarities.

In Section 2, we recall some elements of TF analysis for testing stationarity, and the chosen method to extract trends. In Section 3, we develop a procedure to quantify the trend contamination in the time marginal distribution, and also the condition to reject stationarity. The experimental study and the conclusions are shown in Section 4 and 5, respectively.

#### 2. BACKGROUND ELEMENTS

## 2.1. Time-Frequency analysis and stationarity test

A possible estimation of a TF spectrum of a signal x(t) is offered by multitaper spectrogram [16]:

$$\widehat{S}_{K}(t,f) = \frac{1}{K} \sum_{k=0}^{K-1} S^{(h_{k})}(t,f)$$
(1)

which is computed by averaging K spectrograms  $S^{(h_k)}(t, f)$  with windows  $h_k(t)$ . These are the squared magnitude of the Short-Time Fourier Transform (STFT) of the signal:

$$S^{(h_k)}(t,f) = \left| \int x(s)h_k(s-t)e^{j2\pi fs} ds \right|^2$$
(2)

where  $h_k(t)$  are the first K Hermite functions, which are orthonormal and maximally concentrated in TF domain [13]. The time instants t = 1, ..., n where the spectrograms are evaluated are computed considering the length of the signal and an adjustable fraction of the length of the window  $h_k(t)$ .

Testing for stationarity in TF domain amounts to quantify whether  $\hat{S}_K(t, f)$  is statistically consistent with the null hypothesis that it reduces to a stationary power spectral density:  $\hat{S}_K(t, f) \simeq \text{PSD}(f)$ . We show in Fig. 1 the estimated TF spectra of a stationary Gaussian process  $(x_1(t))$ , and a nonstationary one  $(x_2(t))$  with a slowly-varying variance that



**Fig. 1.** (a) A stationary Gaussian process with  $\mu = 0$ ,  $\sigma^2 = 1$  and length N = 450. (b) The same Gaussian process with a varying variance starting at t = N/2. The TF representation of  $x_1(t)$  and  $x_2(t)$  are shown in (c) and (d). Their time marginal distributions are shown in (e) and (f).

starts at t = N/2. The time marginals are also shown and a temporal structure in the second time marginal is noted. For this type of signals, the TF spectrum should generally read as:

$$S(t,f) = m(t)^2 \delta(f) + a(t) (\widetilde{\text{PSD}}(f) + \epsilon(t,f))$$
(3)

where  $\widetilde{\cdot}$  is the normalization of the PSD. In (3), m(t) can be seen as a 1st-order trend (time-varying mean), a(t) is a 2ndorder trend (time-varying variance),  $\widetilde{PSD}(f)$  is possibly the PSD of the stationary component (or, if the signal is nonstationary, it is the frequency marginal of the TF spectrum) and  $\epsilon(t, f)$  are the fluctuations in the TF domain that might code for other types of nonstationarities, as frequency modulations [13]. We are interested here only in 2nd-order trend (non)stationarities, where m and a evolve possibly in time, but changes in frequency as coded in  $\epsilon(t, f)$  are not relevant.

The types of nonstationarity we are interested in reduce to test for the presence of time evolving m or a. The time marginal of (3) can be obtained by summing S(t, f) over the frequencies, and it reduces to:  $y(t) = m(t)^2 + a(t)$  as  $\sum_f \epsilon(t, f) \simeq 0$ , as there are no changes in frequency. The case is not that simple, however, since we cannot specify a*priori* a model for y(t) in general situations. On the other hand, we can use trend estimation to first extract trends, before testing its statistical relevance.

We reduce the study of trend stationarity to the study of the time marginal y(t) of the TF representation. To obtain y(t), we first estimate the TF spectrum by means of (1), and then we sum  $\hat{S}_K(t, f)$  over the frequencies. For dealing with any type of signal, we estimate the trend in y(t) by using Empirical Mode Decomposition (EMD), which is a model-free and nonparametric method. The EMD-based technique to estimate trends is given in [15]. The definition of trend adopted here is that of a smooth additive component that carries information about global changes in the time series [17]. We represent the trend by c(t), the stationary (detrended) fluctuation by r(t), and the trended time marginal series by y(t). Then, we consider the following additive model to hold: y(t) = c(t) + r(t). Testing for 2nd-order trend stationarity can be done by first estimating a trend c(t) from the time marginal y(t) (see Section 2.2), then by proposing a way to quantify the relevance of c(t) (see Section 3). In the rest of the paper, we discuss stationarity in regard to trend stationarity only.

# 2.2. Estimating the trend

EMD is a data-adaptative algorithm that decomposes a signal into oscillatory terms known as intrinsic mode functions (IMFs) [18]. The IMFs are computed iteratively and must satisfy two conditions regarding the number of zero crossing versus extrema and the mean values of local envelopes [19]. The algorithm for extracting the IMFs (known as *sifting process* [18]) can be described as follows [20]:

- i) identify all the local extrema in the time series y(t),
- ii) interpolate all the local maxima and minima by a cubic spline to produce an upper envelope  $e_{up}(t)$  and a lower  $e_{lo}(t)$  envelope, respectively,
- iii) compute the mean  $\rho(t) = [e_{up}(t) e_{lo}(t)]/2$ ,
- iv) compute the detail  $m(t) = x(t) \rho(t)$ ,
- v) repeat steps i to iv until the detail m(t) can be considered to be a zero-mean signal according to some stopping criterion [18][20]. If so, m(t) is called an IMF and the procedure continues by iterating on the residual  $\rho(t)$ .

The sifting process stops when the slowly-varying residual function has no more oscillations. We represent this last residual by  $\rho^{I}(t)$ . The result of the sifting algorithm is a collection of I IMFs  $\{m^{(i)}(t), i = 1, ..., I\}$  plus  $\rho^{I}(t)$ . For the time marginal y(t), the EMD gives the following representation:

$$y(t) = \sum_{i=1}^{I} m^{(i)}(t) + \rho^{I}(t).$$
(4)

The IMFs  $m^{(1)}(t), ..., m^{(I)}(t)$  represent oscillations that go from the shortest period to the longest one [15]. The IMFs have two key properties that we further recall in this work:

- I) each IMF is a zero-mean function by construction [19],
- II) for all practical purposes, the consecutive IMFs can be considered to be locally orthogonal to each other [18].

We represent the trend as the superposition of the last few IMFs and the residual  $\rho^{I}(t)$  [15]. Thus, estimating the trend is equivalent to estimating the index  $i = i^*$  in (4) that gives the best approximation of c(t):

$$c(t) = \sum_{i=i^*}^{I} m^{(i)}(t) + \rho^I(t).$$
(5)

To obtain  $i^*$  we choose the smallest common index of two independent methods, namely the *Ratio* and the *Energy* approaches. The former considers the expected ratio of zero crossing of successive IMFs. The latter considers that the energies of the IMFs generally increase for *i* near to  $i^*$  [15].



**Fig. 2**. Estimated trend importance for the stationary Gaussian process (a), and the nonstationary one (b). The histograms over 1000 realizations are shown in (c) and (d).

## 3. TESTING FOR TREND RELEVANCE

#### 3.1. Quantifying the importance of the trend

Existence of an additive trend in y(t) conveys a breakdown in the trend stationarity we are interested in. Here, we propose the following measure to quantify the importance of the trend:

$$\theta_{\mathrm{TI}} = \frac{\mathbb{V}\mathrm{ar}\{y(t)\}}{\mathbb{V}\mathrm{ar}\{y_{\mathrm{dt}}(t)\}} = \frac{\mathbb{V}\mathrm{ar}\{y(t)\}}{\mathbb{V}\mathrm{ar}\{y(t) - c(t)\}},\tag{6}$$

where  $y_{dt}(t) = y(t) - c(t)$  is the detrended time marginal series. In (6) we evaluate the fraction of the original variance of y(t) accounted for by the approximated trend [21]. The estimation of (6) is carried out by the trend importance estimator:

$$\widehat{\theta}_{\mathrm{TI}} = \frac{\sum_{t=1}^{n} \left[ y(t) - \widehat{\mu}_{\mathrm{y}} \right]^{2}}{\sum_{t=1}^{n} \left\{ \left[ y(t) - \widehat{\mu}_{\mathrm{y}} \right] - \left[ c(t) - \widehat{\mu}_{\mathrm{c}} \right] \right\}^{2}}, \tag{7}$$

where  $\hat{\mu}_{y}$ ,  $\hat{\mu}_{dt}$  and  $\hat{\mu}_{c}$  are the sample mean of y(t),  $y_{dt}(t)$  and c(t), respectively. In Fig. 2 we use (7) to evaluate the trends in the time marginals of the signals shown in Fig. 1. Notice that  $\hat{\theta}_{TI}$  takes a higher value for the nonstationary signal  $x_{2}(t)$ . We also show the results for 1000 realizations of each process.

## 3.2. Criterion for rejecting trend relevance

We have observed that for a *trendless* time marginal (which indicates stationarity), we have  $\hat{\theta}_{\text{TI}} \approx 1$ , and the trend c(t) is approximated only by  $\rho^{I}(t)$ . Conversely, for a *trended* time marginal (which indicates nonstationarity), we have  $\hat{\theta}_{\text{TI}} \geq 1$ , and the trend c(t) is given by (5). For the latter case, we can rewrite y(t) and c(t) in (7) using the EMD representation:

$$\widehat{\theta}_{\mathrm{TI}} = \frac{\sum_{t=1}^{n} \left[ \sum_{i=1}^{I} m^{(i)}(t) + \rho^{I}(t) - \widehat{\mu}_{\mathrm{y}} \right]^{2}}{\sum_{t=1}^{n} \left\{ \left[ \sum_{i=1}^{I} m^{(i)}(t) + \rho^{I}(t) - \widehat{\mu}_{\mathrm{y}} \right] - \left[ \sum_{i=i^{*}}^{I} m^{(i)}(t) + \rho^{I}(t) - \widehat{\mu}_{\mathrm{c}} \right] \right\}^{2}}.$$
(8)

Considering the zero-mean property of the IMFs (see property I in Section 2.2), we have  $\hat{\mu}_y \approx \hat{\mu}_\rho$  and  $\hat{\mu}_c \approx \hat{\mu}_\rho$ , where  $\hat{\mu}_\rho$  is the sample mean of  $\rho^I(t)$ . Now, if we define the slightly shifted  $\rho'^I(t) = \rho^I(t) - \hat{\mu}_\rho$ , and if we rearrange the squares, we can rewrite (8) as:

$$\widehat{\theta}_{\mathrm{II}} = \sum_{\underline{t=1}}^{n} \left\{ \sum_{i=1}^{I} \left[ m^{(i)}(t) \right]^{2} + \rho^{\prime I}(t)^{2} + 2 \left[ \sum_{i < j}^{I} m^{(i)}(t) m^{(j)}(t) + \rho^{\prime I}(t) \sum_{i=1}^{I} m^{(i)}(t) \right] \right\} \\ \sum_{t=1}^{n} \left\{ \sum_{i=1}^{i^{*}-1} \left[ m^{(i)}(t) \right]^{2} + 2 \sum_{i < j}^{i^{*}-1} m^{(i)}(t) m^{(j)}(t) \right\} \tag{9}$$

The terms  $\sum_{i < j}^{i^*-1} m^{(i)}(t) m^{(j)}(t)$  in (9) should be approximately equal to zero, since the IMFs are locally orthogonal to each other (see property II in Section 2.2). After rearranging the IMFs of the trended and detrended parts, the following expression is obtained:

$$\widehat{\theta}_{\mathrm{TI}} \cong \frac{\sum_{i=1}^{n} \sum_{i=i^{*}}^{I} \left[ m^{(i)}(t) \right]^{2} + \sum_{t=1}^{n} \left[ \rho^{\prime I}(t) \right]^{2}}{\sum_{t=1}^{n} \sum_{i=1}^{i^{*}-1} \left[ m^{(i)}(t) \right]^{2}} + \frac{2 \sum_{t=1}^{n} \left[ \rho^{\prime I}(t) \sum_{i=1}^{I} m^{(i)}(t) \right]}{\sum_{t=1}^{n} \sum_{i=1}^{i^{*}-1} \left[ m^{(i)}(t) \right]^{2}}.$$
(10)

If we define the energies of the *i*<sup>th</sup> IMF and  $\rho'^{I}(t)$  as:

$$\sum_{t=1}^{n} \left[ m^{(i)}(t) \right]^2 = E_i \quad \text{and} \quad \sum_{t=1}^{n} \left[ \rho'^I(t) \right]^2 = E_{\rho'}, \tag{11}$$

we can verify that (10) is expressed as function of the energies of the IMFs and  $\rho^{I}(t)$ . Moreover, the third term in (10) is the sum of the scalar products between  $\rho^{I}(t)$  and the IMFs. Thus, by considering (11), we can rewrite (10) as follows:

$$\frac{\hat{\theta}_{\mathrm{TI}} - 1}{E_{1^{\ast} + \dots + E_{I} + E_{\rho'}}} + 2 \frac{\sqrt{E_{\rho'}}\sqrt{E_{1}}\cos\alpha_{1} + \dots + \sqrt{E_{\rho'}}\sqrt{E_{I}}\cos\alpha_{I}}{E_{1} + \dots + E_{i^{\ast} - 1}}$$
(12)

where  $\alpha_i$  is the angle of the *i*<sup>th</sup> scalar product. The terms  $\cos \alpha_i$  are *approximately equal to zero* due to the slowlyvarying behavior of  $\rho^{I}(t)$ . In this regard, note that in the third term of (10) (from where (12) was derived), we are estimating the covariance between the zero-mean IMFs and the residual  $\rho^{I}(t)$  (which is nearly constant in comparison to other IMFs). Thus, in a limit case,  $\mathbb{C}ov\{m^{(i)}(t), a\} = 0$  if a is a constant.

It can be also noted in (12) that the energies of the IMFs of the trend component  $(E_{i^*}, ..., E_I)$  and the residual  $(E_{\rho'})$ , are being divided by those of the detrended part  $(E_1, ..., E_{i^*-1})$ . However, in case of trend, the energies are generally increasing for IMFs with index  $i \ge i^*$  [15]. In this case, if the two following conditions are attended:

$$\frac{E_{i^*} + \ldots + E_I}{E_1 + \ldots + E_{i^* - 1}} \ge 1 \quad \text{and} \quad \cos \alpha_i \approx 0, \ i = 1, ..., I,$$

then we have  $\hat{\theta}_{TI} \geq 2$  in (12) as a borderline case for a trended time marginal, or nonstationarity. An empirical study has shown that, in general, we can reject stationarity for  $\hat{\theta}_{TI} \geq 2$ . The results cannot be reproduced here due to space limitations, but it can be seen in Fig. 2 that  $\hat{\theta}_{TI} \approx 2$  is indeed a reasonable lower bound for nonstationarity. Having presented the proposed criterion for rejecting stationarity, we show in the following section the experimental study.

#### 4. APPLYING THE TREND STATIONARITY TEST

The experimental study was carried on two groups of signals: nonstationary Gaussian processes and stationary AR(1) processes. The time series have length N = 450 and N = 1050. The nonstationary signals follow the *smooth-change* model:

$$\theta_{i} = \begin{cases} \xi_{1}, & i = 1, ..., \kappa_{1}, \\ \xi_{1} + \frac{(i - \kappa_{1})(\xi_{2} - \xi_{1})}{(\kappa_{2} - \kappa_{1})}, & i = \kappa_{1} + 1, ..., \kappa_{2} - 1, \\ \xi_{2}, & i = \kappa_{2}, ..., N, \end{cases}$$
(13)

where  $\theta_i$  is either the mean or the variance, and  $\xi_1$  and  $\xi_2$  are the parameters values before and after change, respectively. The model in (13) represents a gradual transition from the initial parameter value to the final one [22].

Two cases were considered for (13):  $\kappa_1 = 1, \kappa_2 = N$ and  $\kappa_1 = N/2, \kappa_2 = N$  (models with  $\kappa_2 = N$  are also known as *onset-of-trend*). These configurations represent a nonstationarity starting at the *beginning* or at the *middle* of the signal, respectively. Also,  $\xi_1$  and  $\xi_2$  were chosen in order to provide a good trade-off between test sensitivity and slowlyvarying behavior. Realizations of the nonstationary signals are shown in Fig. 3. The first row have signals with a *fixed variance* ( $\sigma^2 = 1$ ) and a *varying mean* that ranges from  $\xi_1 =$ 0 to  $\xi_2 = 4$ . The second row refers to the signals with a *fixed mean* ( $\mu = 0$ ) and a *varying variance* that ranges from  $\xi_1 = 1$ to  $\xi_2 = 4$ . The third row have signals with a varying mean *and* variance with the same values of the previous cases.

We have compared the approach proposed in this work with the classical KPSS test [23], which is commonly used for testing trends and slowly-varying nonstationarities. The test has been applied to 5000 realizations of the nonstationary signals shown in Fig. 3 and AR(1) processes. The results are presented in Table 1. For the new method ("New"), the results are given as percentage of cases where  $\hat{\theta}_{TI} \ge 2$ , which is the defined lower bound for nonstationarity. For the KPSS method ("KPSS"), the results are given as the number of observed "nonstationary" outcomes.



Fig. 3. Realizations of the nonstationary signals with length N = 450 and N = 1050 following the model shown in (13).

Note in Table 1 that the KPSS technique could not detect properly nonstationarities starting at the beginning of the time series and a nonstationary variance. The latter would be expected, since the KPSS test is not well suited to detect non-stationarities in the variance. The new method, therefore, appears as an interesting alternative, as the overall performance was good. For the different types of nonstationarity we have obtained an acceptable percentage of the cases for which  $\hat{\theta}_{TI}$  is greater than the lower bound for nonstationarity.

**Table 1**. Results of applying the proposed method ("New") and the KPSS test ("KPSS") to 5000 realizations of the processes shown in Fig. 3 and to AR(1) processes.

Length		N = 450		N = 1050	
Model		Change point		Change point	
Smooth-		Beginning	Middle	Beginning	Middle
change		$\kappa_1 = 1$	$\kappa_1 = N/2$	$\kappa_1 = 1$	$\kappa_1 = N/2$
Mean	KPSS	6.50%	100%	7.50%	100%
	New	49.0%	61.5%	75.0%	71.0%
Variance	KPSS	6.00%	9.00%	7.00%	4.50%
	New	89.0%	98.0%	71.5%	98.0%
Mean and	KPSS	5.50%	100%	6.00%	100%
variance	New	84.0%	99.5%	65.0%	100%
Model		Stationary process		Stationary process	
AR(1)	KPSS	4%		6%	
	New	5%		0.5%	

#### 5. CONCLUSIONS

In this paper we have proposed a new nonparametric method for testing nonstationarities that are observable as a trend or an evolution in the local energy of the signal. The method consists in estimating the trend importance in the time marginal distribution of the TF representation. To estimate the trend, we have used a recent technique based on EMD. In comparison to classical methods, the new approach allows for an overall better detection of slowly-varying nonstationarities.

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