ANTENNA SUBSET SELECTION OPTIMIZATION FOR LARGE-SCALE MISO CONSTANT ENVELOPE PRECODING

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ABSTRACT

This paper considers robust constant envelope (CE) precoding with antenna-subset selection (AS) in a large-scale MISO downlink scenario where only imperfect channel state information at the transmitter (CSIT) is available. CE precoding is a recently proposed transmission scheme that enables the use of cheap but highly power-efficient power amplifiers, while AS is a well-known approach for reducing the number of power amplifiers. The combination of these two techniques can significantly cut down costs in hardware implementations. We formulate a power minimization problem for AS CE precoding where the worst-case symbol error rate is constrained to be less than a given threshold. The formulation utilizes our recent results on signal characterization of CE precoding. The formulated power minimization optimization problem turns out to be a zero-one linear program. We show that this problem is NP-hard in general. Then, we propose an efficient approximation by Lagrangian dual relaxation and greedy knapsack approximation. Simulation results show that the proposed algorithm can achieve near-optimal performance, and the average number of active antennas accounts for only $19-53\%$ of the total transmit antennas.

Index Terms— Large-scale MIMO, antenna subset selection, constant envelope, power minimization, robust design.

1. INTRODUCTION

Large-scale multiple-input multiple-output (MIMO) systems have recently caught much attention from both academia and industry. The massive number of antennas in large-scale MIMO systems has the advantage of high spectral efficiency, low transmit power, as well as simple receive/transmit processing [1]. However, the practical implementation of large-scale MIMO systems can be very expensive as a large number of highly power-efficient power amplifiers are needed. The power amplifiers for traditional beamforming systems may not be an attractive option for large-scale MIMO systems, as they are designed to have a large dynamic region to accommodate the large variation of signal power in beamforming signals. This linearity requirement inevitably leads to high cost and low power efficiency. Constant envelope (CE) precoding has recently been proposed by Mohammed and Larsson in [2, 3] to overcome the cost and power efficiency issues. In CE precoding, the transmitting signals are constrained to be constant envelope and only the phases are used to convey information. This allows us to employ cheap and highly power-efficient power amplifiers [4, 2], which is very appealing to large MIMO systems.

This work is supported by a Direct Grant by the Chinese University of Hong Kong (Project ID: 4055009).

This paper focuses on robust CE precoding design for single-user large-scale MISO downlink channels, where only channel state information at the transmitter (CSIT) is known. Specifically, we consider CE precoding with antenna-subset selection (AS) [5, 6]. AS is a popular technique for reducing the complexity and cost of transmitter implementations, enabling one to use much smaller number of RF chains than the number of transmit antennas.

The goal of our design is to minimize the total transmission power while ensuring that the symbol error rate performance of the user under the worst-case channel uncertainty is upper bounded by a given threshold. The design problem is based on our recent results [7] on the complete characterization of the so-called doughnut region and the exact recovery of the phases of transmitting signals. As it turns out, the formulated optimization problem for AS CE precoding is a zero-one linear program. We establish the NP-hardness of the AS CE problem by reducing the knapsack problem to the AS CE problem. We then propose an efficient approximate algorithm for the AS CE problem. There are two ingredients of our proposed algorithm. The first one is the Lagrangian dual method, where we tackle the dual problem of the AS CE problem. The dual problem only has one optimization variable, which can be easily handled by the bisection method. The second ingredient is that we rewrite the optimization problem associated with the dual function as a series of knapsack problems which have efficient approximate solutions due to Dantzig [8].

Simulation results reveal that the proposed algorithm yields a power performance very close to a performance lower bound. It is also shown that AS can significantly reduce the number of RF chains; the average number of active antennas can be about $19-53\%$ of the total number of transmitting antennas. Comparing AS CE with beamforming design, AS CE shows a $3.5-4.6$dB performance loss. But it should be noted that AS CE transmission has the advantages of CE transmission, cheap implementation, and fewer RF chains.

We should briefly describe the relation of the present work to the prior works. Our work is based on the recently proposed CE precoding approach in [2, 3, 9]; also [7] for our very recent endeavor. The previous CE precoding works do not consider AS, while the theme of this paper is on AS designs for CE precoding.

2. SYSTEM MODEL

We consider an MISO downlink scenario with imperfect CSIT:

$$y = (\bar{\boldsymbol{h}} + \Delta \boldsymbol{h})^T \boldsymbol{x} + \nu \tag{1}$$

where $y \in \mathbb{C}$ is the receive signal; $\boldsymbol{x} \in \mathbb{C}^N$ the transmitting signal; $\nu \in \mathbb{C}$ the noise whose distribution is $\mathcal{CN}(0, 1)$; and $\bar{\boldsymbol{h}} \in \mathbb{C}^N$

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and $\Delta \boldsymbol{h} \in \mathbb{C}^N$ the estimated channel and the unknown channel uncertainty, respectively. The channel uncertainty is assumed to be deterministic and lies in the following region

$$|\Delta \boldsymbol{h}| \leq \boldsymbol{\epsilon} \tag{2}$$

where $|\cdot|$ denotes the element-wise absolute value and $\boldsymbol{\epsilon} \geq 0$ is the uncertainty upper bound with ϵ_i being the upper bound on $|\Delta h_i|$.

The traditional beamforming (BF) signal takes the following form

$$\boldsymbol{x}_{\rm BF} = \boldsymbol{w} s \tag{3}$$

where $\boldsymbol{w} \in \mathbb{C}^N$ is the beamformer, $s \in \mathcal{S}$ is the information symbol, and $\mathcal{S}$ is the constellation with unit power $\mathbb{E}_{s\in\mathcal{S}}|s|^2 = 1$. As the beamformer $\boldsymbol{w}$ depends on $\bar{\boldsymbol{h}}$ generally, the instantaneous per-antenna power $|x_i|^2 = |w_i s|^2$ may vary significantly due to the information symbol s and the channel $\bar{\boldsymbol{h}}$. Thus the power amplifiers for BF must have a wide linear region, which leads to higher hardware costs and lower power efficiency [4, 2].

Constant envelope precoding has been recently proposed by [2, 3, 9] to alleviate the cost and power efficiency issues in power amplifier. In CE precoding, the transmitting signal $\boldsymbol{x}$ takes the form of

$$x_i = \sqrt{P_{\rm PA}} e^{\boldsymbol{j}\theta_i}, \quad i = 1, \ldots, N \tag{4}$$

where $P_{\rm PA}$ is a fixed per-antenna power and θ_i is the phase of the transmitting signal. It can be seen that the per-antenna power of the CE signal is always constant irrespective of the information symbol as well as the channel $\bar{\boldsymbol{h}}$. Due to this salient feature, the power amplifiers for CE signals can be cheaply implemented and the power efficiency can be much higher than that of power amplifiers for BF [4, 2].

In this paper, we consider CE precoding aided by antenna-subset selection (AS) [5, 6], which can further reduce the implementation costs. The transmitting signal takes the form of

$$x_i = \sqrt{P_{\rm PA}} a_i e^{\boldsymbol{j}\theta_i}, \quad i = 1, \ldots, N \tag{5}$$

where $a_i \in \{0, 1\}$ denotes the on-off state of antenna i.

3. PROBLEM FORMULATION

The goal of this paper is to develop an optimized design for AS-aided CE precoding. In particular, we will be interested in minimizing the total transmission power, while ensuring the user's target quality of service (QoS) being satisfied.

Let us begin with the problem formulation. From (1) and (5), the receive signal can be written as

$$y = \left(\sum_{i=1}^{N} \sqrt{P_{\rm PA}} \bar{h}_i a_i e^{\boldsymbol{j}\theta_i} \right) + \left(\sum_{i=1}^{N} \sqrt{P_{\rm PA}} \Delta h_i a_i e^{\boldsymbol{j}\theta_i} \right) + \nu. \tag{6}$$

To transmit an information symbol s drawn from a constellation $\mathcal{S}$, the transmitter tries to perform CE precoding to find a phase vector $\boldsymbol{\theta}$ such that

$$\alpha s = \sum_{i=1}^{N} \sqrt{P_{\rm PA}} \bar{h}_i a_i e^{\boldsymbol{j}\theta_i}, \tag{7}$$

where $\alpha > 0$ is some scalar. The phase optimization problem in (7) (or the precoder problem) can be solved in closed form [7]; we omit the detail here due to space limit. Also, by [7] (see also [2]), there exists a $\boldsymbol{\theta}$ satisfying (7) for every $s \in \mathcal{S}$ if and only if

$$\alpha \mathcal{S} \subset \mathcal{D}(\sqrt{P_{\rm PA}} \bar{\boldsymbol{h}} \odot \boldsymbol{a}) \tag{8}$$

where $\odot$ is the Hadamard product, $\boldsymbol{a} = [a_1, \ldots, a_N]^T$, and $\mathcal{D}$ is the so-called doughnut region defined as

$$\mathcal{D}(\boldsymbol{g}) = \{ d \in \mathbb{C} \mid 2\|\boldsymbol{g}\|_\infty - \|\boldsymbol{g}\|_1 \leq |d| \leq \|\boldsymbol{g}\|_1 \}. \tag{9}$$

Then the receive signal can be rewritten as

$$y = \alpha s + \left(\sum_{i=1}^{N} \sqrt{P_{\rm PA}} \Delta h_i a_i e^{\boldsymbol{j}\theta_i} \right) + \nu. \tag{10}$$

We are interested in providing a worst-case symbol error rate (SER) guarantee, that is

$$\max_{|\Delta \boldsymbol{h}| \leq \boldsymbol{\epsilon}} \Pr(s \neq \hat{s}\, ; \Delta \boldsymbol{h}) \leq b_o, \tag{11}$$

where $\hat{s}$ is the receiver detection obtained by applying symbol decision on y/α, and b_o is the predefined SER threshold.

Under the aforementioned problem setup, we formulate the AS-aided CE precoding design as a power minimization problem

$$
\begin{aligned}
P_{\rm T} = \min_{\alpha \in \mathbb{R}_+, \boldsymbol{a} \in \mathbb{R}^N} \quad & P_{\rm PA} \boldsymbol{a}^T \boldsymbol{1} && \text{(12a)} \\
\text{s.t.} \quad & \alpha \mathcal{S} \subset \mathcal{D}(\sqrt{P_{\rm PA}} \bar{\boldsymbol{h}} \odot \boldsymbol{a}), && \text{(12b)} \\
& \max_{|\Delta \boldsymbol{h}| \leq \boldsymbol{\epsilon}} \Pr(s \neq \hat{s}\, ; \Delta \boldsymbol{h}) \leq b_o, && \text{(12c)} \\
& \boldsymbol{a} \in \{0, 1\}, && \text{(12d)}
\end{aligned}
$$

where we seek to find an antenna activation pattern that satisfies a worst-case SER guarantee (specified by b_0) and uses the least total transmission power. We proceed to reformulate problem (12) into a more convenient form. Let $|s|_{\max} = \max_{s\in\mathcal{S}} |s|$ and $|s|_{\min} = \min_{s\in\mathcal{S}} |s|$ denote the largest and smallest amplitudes of all constellation points, respectively. Using the definition of $\mathcal{D}$ in (9), the constraint (12b) can be equivalently written as

$$
\begin{aligned}
& \alpha |s|_{\min} \geq 2\sqrt{P_{\rm PA}} \|\bar{\boldsymbol{h}} \odot \boldsymbol{a}\|_\infty - \sqrt{P_{\rm PA}} \|\bar{\boldsymbol{h}} \odot \boldsymbol{a}\|_1 \\
& \alpha |s|_{\max} \leq \sqrt{P_{\rm PA}} \|\bar{\boldsymbol{h}} \odot \boldsymbol{a}\|_1 .
\end{aligned} \tag{13}
$$

To handle the SER constraint (12c), we use the commonly adopted union bound constraint

$$\max_{\substack{|\Delta \boldsymbol{h}| \leq \boldsymbol{\epsilon} \\ s, s' \in \mathcal{S}, s \neq s'}} \Pr(|y - \alpha s| > |y - \alpha s'|; \Delta \boldsymbol{h}) \leq \frac{b_o}{|\mathcal{S}| - 1}, \tag{14}$$

where $|\mathcal{S}|$ is the size of the constellation $\mathcal{S}$. Note that (14) provides a safe guarantee for the SER constraint (12c). Using classical results of detection in SISO channels [10], Eq. (14) can be rewritten as

$$\sqrt{2}\rho + 2\sqrt{P_{\rm PA}} \boldsymbol{\epsilon}^T \boldsymbol{a} \leq \alpha \eta \tag{15}$$

where $\eta = \min_{s, s' \in \mathcal{S}, s \neq s'} |s - s'|$ is the minimum distance of the constellation $\mathcal{S}$, $\rho = Q^{-1}(b_o/(|\mathcal{S}| - 1))$, and Q^{-1} is the inverse Q function. Plugging (13) and (15) into problem (12), we reformulate problem (12) as

$$
\begin{aligned}
\min_{\alpha \in \mathbb{R}_+, \boldsymbol{a} \in \mathbb{R}^N} \quad & \boldsymbol{a}^T \boldsymbol{1} && \text{(16a)} \\
\text{s.t.} \quad & \alpha \leq \tfrac{\sqrt{P_{\rm PA}}}{|s|_{\max}} \|\bar{\boldsymbol{h}} \odot \boldsymbol{a}\|_1, && \text{(16b)} \\
& \tfrac{\sqrt{P_{\rm PA}}}{|s|_{\min}} (2\|\bar{\boldsymbol{h}} \odot \boldsymbol{a}\|_\infty - \|\bar{\boldsymbol{h}} \odot \boldsymbol{a}\|_1) \leq \alpha, && \text{(16c)} \\
& \tfrac{1}{\eta}(\sqrt{2}\rho + 2\sqrt{P_{\rm PA}} \boldsymbol{\epsilon}^T \boldsymbol{a}) \leq \alpha, && \text{(16d)} \\
& \boldsymbol{a} \in \{0, 1\}^N . && \text{(16e)}
\end{aligned}
$$

We eliminate the variable α by combining the constraint (16b) – (16d); the resultant problem is

$$
\begin{aligned}
\min_{\boldsymbol{a}\in\mathbb{R}^N} \quad & \boldsymbol{a}^T\mathbf{1} && (17a)\\
\text{s.t.} \quad & \tfrac{\sqrt{2}|s|_{\max}}{\eta\sqrt{P_{\text{PA}}}}\rho + \tfrac{2|s|_{\max}}{\eta}\boldsymbol{\epsilon}^T\boldsymbol{a} \le \|\bar{\boldsymbol{h}}\odot\boldsymbol{a}\|_1, && (17b)\\
& \tfrac{2|s|_{\max}}{|s|_{\min}+|s|_{\max}}\|\bar{\boldsymbol{h}}\odot\boldsymbol{a}\|_\infty \le \|\bar{\boldsymbol{h}}\odot\boldsymbol{a}\|_1, && (17c)\\
& \boldsymbol{a}\in\{0,1\}^N, && (17d)
\end{aligned}
$$

where (17b) is obtained by combining the right-hand side (RHS) of (16b) and the left-hand side (LHS) of (16d), and (17c) by the RHS of (16b) and the LHS of (16c). Denoting $\boldsymbol{g} = |\bar{\boldsymbol{h}}|$, we have a further simplified problem

$$
\begin{aligned}
\text{(AS CE)} \quad \min_{\boldsymbol{a}\in\mathbb{R}^N} \quad & \boldsymbol{a}^T\mathbf{1} && (18a)\\
\text{s.t.} \quad & \tfrac{\sqrt{2}|s|_{\max}}{\eta\sqrt{P_{\text{PA}}}}\rho + 2\tfrac{|s|_{\max}}{\eta}\boldsymbol{\epsilon}^T\boldsymbol{a} \le \boldsymbol{g}^T\boldsymbol{a} && (18b)\\
& \tfrac{2|s|_{\max}}{|s|_{\min}+|s|_{\max}}\|\boldsymbol{g}\odot\boldsymbol{a}\|_\infty \le \boldsymbol{g}^T\boldsymbol{a} && (18c)\\
& \boldsymbol{a}\in\{0,1\}^N. && (18d)
\end{aligned}
$$

Problem (18) is a zero-one linear program which can be very difficult to solve. Indeed, it is NP-hard, as shown by the following proposition.

Proposition 1. *The AS CE problem* (18) *is NP-hard.*

Proof. The proof is based on reducing the knapsack problem to (18). We omit the proof here due to space limit. The proof can be found in [11]. □

Since problem (18) is NP-hard, in a large antenna array scenario one may want to focus on efficient approximations. One intuitive approach is continuous relaxation and rounding, which means that the zero-one constraint (18d) is relaxed to a convex set $[0,1]^N$, resulting in a convex problem, and the relaxed solution is rounded back to zero or one. However, this simple approach, as we observe in the simulations, frequently fails to generate even a feasible solution to problem (18). In the next section, we will propose an efficient approximation algorithm based on Lagrangian dual and efficient knapsack approximation, which shows near-optimal performance by numerical results.

4. PROPOSED LAGRANGIAN DUAL APPROACH

We propose to handle the AS CE problem via the Lagrangian dual. Let us define a partial Lagrangian function as

$$
\begin{aligned}
\mathcal{L}(\boldsymbol{a},\lambda) &= \boldsymbol{a}^T\mathbf{1} + \lambda\left(\tfrac{2|s|_{\max}}{|s|_{\min}+|s|_{\max}}\|\boldsymbol{g}\odot\boldsymbol{a}\|_\infty - \boldsymbol{g}^T\boldsymbol{a}\right)\\
&= \lambda\tfrac{2|s|_{\max}}{|s|_{\min}+|s|_{\max}}\|\boldsymbol{g}\odot\boldsymbol{a}\|_\infty + (\mathbf{1}-\lambda\boldsymbol{g})^T\boldsymbol{a}.
\end{aligned} \tag{19}
$$

where $\lambda \ge 0$ is the dual variable associated with constraint (18c). The dual function, by definition, is

$$
\begin{aligned}
d(\lambda) = \min_{\boldsymbol{a}} \quad & \mathcal{L}(\boldsymbol{a},\lambda)\\
\text{s.t.} \quad & \tfrac{\sqrt{2}|s|_{\max}}{\eta\sqrt{P_{\text{PA}}}}\rho + \tfrac{2|s|_{\max}}{\eta}\boldsymbol{\epsilon}^T\boldsymbol{a} \le \boldsymbol{g}^T\boldsymbol{a}\\
& \boldsymbol{a}\in\{0,1\}^N.
\end{aligned} \tag{20}
$$

The dual problem is

$$\max_{\lambda\ge 0} \quad d(\lambda). \tag{21}$$

The dual problem (21) involves only one optimization variable which can be easily handled by the bisection method [12]. We omit the details on the bisection method here as it is straightforward. However, in each iteration of the bisection method, problem (20) needs to be solved for the objective value and the solution. Problem (20) is the focus of the remaining development.

Problem (20) again involves zero-one optimization variables, which is difficult to deal with in an optimal way. Alternatively, we rewrite problem (20) as a series of knapsack problems which exhibit efficient approximations. To do this, let us rewrite problem (20) equivalently as a two-stage minimization problem where the infinity norm can be easily handled.

$$
\min_{i=1,\dots,N}
\left(
\begin{aligned}
\min_{\boldsymbol{a}} \quad & \lambda\tfrac{2|s|_{\max}}{|s|_{\min}+|s|_{\max}}\|\boldsymbol{g}\odot\boldsymbol{a}\|_\infty + (1-\lambda\boldsymbol{g})^T\boldsymbol{a}\\
\text{s.t.} \quad & \tfrac{\sqrt{2}|s|_{\max}}{\eta\sqrt{P_{\text{PA}}}}\rho + \tfrac{2|s|_{\max}}{\eta}\boldsymbol{\epsilon}^T\boldsymbol{a} \le \boldsymbol{g}^T\boldsymbol{a}\\
& a_j = 0,\ j<i\\
& a_j = 1,\ j=i\\
& a_j \in\{0,1\},\ j>i.
\end{aligned}
\right) \tag{22}
$$

The above problem can be readily solved by solving the inner minimizations for $i = 1,\dots,N$. Assuming without loss of generality that $\boldsymbol{g}$ is ordered in nonincreasing order $g_1 \ge g_2 \ge \ldots \ge g_N$, the inner minimization problem is rewritten as

$$
\begin{aligned}
\min_{\{a_j\}_{j>i}} \quad & \left(\lambda\tfrac{2|s|_{\max}}{|s|_{\min}+|s|_{\max}}g_i + (1-\lambda g_i)\right) + \sum_{j>i}(1-\lambda g_j)a_j\\
\text{s.t.} \quad & \tfrac{\sqrt{2}|s|_{\max}}{\eta\sqrt{P_{\text{PA}}}}\rho + \tfrac{2|s|_{\max}}{\eta}\epsilon_i - g_i \le \sum_{j>i}\left(g_j - \tfrac{2|s|_{\max}}{\eta}\epsilon_j\right)a_j\\
& a_j\in\{0,1\},\ j>i.
\end{aligned} \tag{23}
$$

For convenience of the following development, let us define $\boldsymbol{z} = [a_j]_{j>i}$, $\boldsymbol{c} = [(1-\lambda g_j)]_{j>i}$, $\boldsymbol{b} = [g_j - \frac{2|s|_{\max}}{\eta}\epsilon_j]_{j>i}$, $d = \frac{\sqrt{2}|s|_{\max}}{\eta\sqrt{P_{\text{PA}}}}\rho + \frac{2|s|_{\max}}{\eta}\epsilon_i - g_i$, and $L = N - i$. Then problem (23) can be written as

$$
\begin{aligned}
\min_{\boldsymbol{z}} \quad & \boldsymbol{c}^T\boldsymbol{z}\\
\text{s.t.} \quad & d \le \boldsymbol{b}^T\boldsymbol{z}\\
& \boldsymbol{z}\in\{0,1\}^L.
\end{aligned} \tag{24}
$$

First, it can be easily seen that if $c_l \ge 0$ and $b_l \le 0$, then the optimal z_l must take zero; similarly, if $c_l \le 0$ and $b_l \ge 0$, then the optimal z_l must take one. Thus, we assume that for all l it holds that $c_l \ge 0$ and $b_l \ge 0$, or $c_l \le 0$ and $b_l \le 0$. By the change of variable $\tilde{\boldsymbol{z}}$ defined as

$$
\tilde{z}_l = \begin{cases} z_l, & \text{if } c_l \ge 0 \text{ and } b_l \ge 0,\\ 1 - z_l, & \text{if } c_l \le 0 \text{ and } b_l \le 0,\end{cases} \tag{25}
$$

problem (24) can be rewritten as

$$
\begin{aligned}
\min_{\tilde{\boldsymbol{z}}} \quad & |\boldsymbol{c}|^T\tilde{\boldsymbol{z}}\\
\text{s.t.} \quad & d - \sum_{c_l\le 0, b_l\le 0} b_l \le |\boldsymbol{b}|^T\tilde{\boldsymbol{z}},\\
& \tilde{\boldsymbol{z}}\in\{0,1\}^L.
\end{aligned} \tag{26}
$$

Problem (26) is a variant of the knapsack problem, where $|c_l|$ and $|b_l|$ represent the weight and value of item l respectively, and $d - \sum_{c_l\le 0, b_l\le 0} b_l$ is the target total value. The goal of problem (26) is to select the items for minimizing the total weight while ensuring the target total value is achieved. To handle problem (26), we use the classic efficient greedy algorithm by Dantzig [8]. We first order the variables such that

$$\frac{|b_1|}{|c_1|} \geq \frac{|b_2|}{|c_2|} \geq \ldots \geq \frac{|b_L|}{|c_L|}. \qquad (27)$$

Eq.(27) means that item l has the lth largest value-weight ratio. The greedy method is to select from item 1 to L until the total target value is achieved. Specifically, we first find the smallest index $M \in \{0, \ldots, L\}$ such that

$$d - \sum_{c_l \leq 0, b_l \leq 0} b_l \leq \sum_{l=1}^{M} |b_l|, \qquad (28)$$

and then set the approximate solution $\boldsymbol{a}^\star$ as

$$a_l^\star = \begin{cases} 1, & \text{for } l \leq M, \\ 0, & \text{for } l > M. \end{cases} \qquad (29)$$

The description of the proposed algorithm is complete. We analyze the computational complexity as follows. The approximation to problem (26) (and thus problem (23)) is dominated by the sorting operation in (27), which has complexity $\mathcal{O}(N \log N)$. Problem (22), which involves handling problem (23) for N times, has complexity $\mathcal{O}(N^2 \log N)$. Finally, the complexity of the whole algorithm for problem (21) is $\mathcal{O}(KN^2 \log N)$ where K denotes the number of iterations of the bisection method.

5. SIMULATION RESULTS

In this section, we use simulations to demonstrate the performance of the proposed algorithm. The number of transmitting antennas is $N = 128$. The channel estimates $\bar{\boldsymbol{h}}$ are generated following an i.i.d. circularly-symmetric complex Gaussian distribution with zero mean and unit variance, and the channel uncertainty upper bound $\boldsymbol{\epsilon}$ is elementwise uniform i.i.d. distributed on $[0, 0.2]$. The constellation $\mathcal{S}$ is 16-QAM. The per-antenna power is $P_{\text{PA}} = 0.1$. The results are averaged over 10^4 realizations. We use the beamforming power minimization problem[1] as a benchmark.

Fig. 1 presents the total transmission power of the proposed AS CE and beamforming. In the figure, AS CE - lower bound denotes the objective value obtained by relaxing problem (18) continuously, which forms a performance lower bound of AS CE. We can see that the performance of the proposed AS CE method is almost the same as the lower bound, suggesting that the proposed algorithm yields near-optimal solution to the power minimization problem for AS CE. It can be further observed that beamforming is better than AS CE; the power gap ranges from 3.5dB at $b_o = 10^{-8}$ to 4.6dB at $b_o = 10^{-2}$. But it should be noted that this advantage of beamforming comes at prices of higher PAPR, higher costs, and lower power efficiency of the power amplifiers.

Table 1 shows the average number of active antennas. It can be seen that beamforming activates 103 antennas on average irrespective of b_o. In contrast, AS CE uses much fewer antennas. The number of active antennas ranges from 24.7 at $b_o = 10^{-2}$ to 68.2 at $b_o = 10^{-8}$, which are only $19 - 53\%$ of total 128 transmitting antennas. This demonstrates the substantial reduction in the number of RF chains used in large-scale MIMO systems.

In Table 2, we present the feasibility rates of beamforming, AS CE by the proposed algorithm, and the previously mentioned AS CE by continuous relaxation and rounding. It can be seen that beamforming shows 100% feasibility rate, while AS CE has a slight degradation when the b_o is at a very stringent level of 10^{-8}. This result is not surprising as AS CE is actually power limited due to fixed per-antenna power and finite number of transmitting antennas. When the target b_o is very small, AS CE does not have enough power to support the stringent target b_o. It can be further seen that the simple continuous relaxation and rounding approach to the AS CE problem, denoted by AS CE - CR & rounding in the table, only yields a feasibility rate of about 50%, which may not acceptable in practical applications.

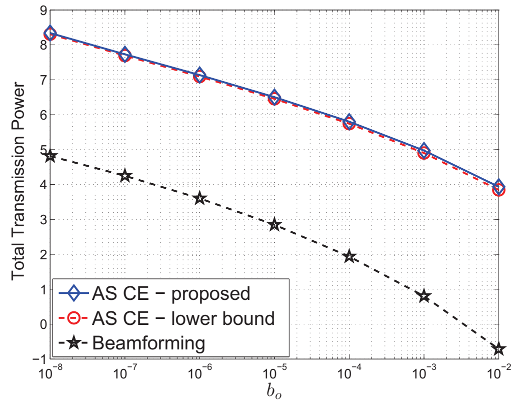

Fig. 1. Total transmission power.

	Target symbol error rate b_o			
	10^{-8}	10^{-6}	10^{-4}	10^{-2}
Beamforming	103.3	103.3	103.3	103.3
AS CE - proposed	68.2	51.7	38.0	24.7

Table 1. Average number of active antennas.

	Target symbol error rate b_o			
	10^{-8}	10^{-6}	10^{-4}	10^{-2}
Beamforming	1	1	1	1
AS CE - proposed	0.923	1	1	1
AS CE - CR & rounding	0.463	0.506	0.507	0.502

Table 2. Feasibility rates

6. CONCLUSION

In this paper, we formulated a robust power minimization problem for CE precoding with AS. We showed the NP-hardness of the formulated optimization problem and proposed an efficient approximation algorithm based on Lagrangian dual relaxation and greedy knapsack approximation. Simulations showed promising results with the proposed algorithm.

[1]In the power minimization problem, we minimize the transmission power subject to the constraint that the worst-case SER is less than b_o. Using the union bound on SER, the problem can be turned to $\min_{\boldsymbol{w} \in \mathbb{C}^N} \|\boldsymbol{w}\|_2^2$ subject to constraint $|\bar{\boldsymbol{h}}^T \boldsymbol{w}|\eta - 2|s|_{\max} \boldsymbol{\epsilon}^T |\boldsymbol{w}| \geq \sqrt{2}\rho$, where $\boldsymbol{w}$ is the beamformer. It can be easily seen that this problem has a closed-form solution. We omit the detail as it is straightforward.

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