DETECTION OF CORRELATED TIME SERIES IN A NETWORK OF SENSOR ARRAYS

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ABSTRACT

This paper considers the problem of testing for the independence among multiple random vectors with each random vector representing a time series captured at one sensor. Implementing the Generalized Likelihood Ratio Test involves testing the null hypothesis that the composite covariance matrix of the channels is block-diagonal through the use of a generalized Hadamard ratio. These results are then extended to the problem of detecting the presence of correlated time series when several observers each employ an array of sensors. Assuming wide-sense stationary processes in both time and space, results on large block-Toeplitz matrices suggest the use of a broadband integral of a frequency-wavenumber dependent Hadamard ratio as an alternative test statistic.

Index Terms— Broadband Coherence, Cross-Spectral Matrix, Generalized Likelihood Ratio Test, Multichannel Signal Detection

1. INTRODUCTION

Non-parametric detection of a common but unknown signal among two or more data channels [1], [2] is a problem that finds its uses in many applications including collaborative sensor networks [3], geological monitoring of seismic activity [4], as well as radar [5] and sonar [6]. In [7], the authors consider detection with multiple temporally correlated Gaussian time series by constructing a Generalized Likelihood Ratio Test (GLRT) that tests whether or not a space-time covariance matrix is block-diagonal. Under this scenario, the test statistic becomes a *generalized* Hadamard ratio involving an estimate of the space-time covariance matrix computed over multiple realizations. Assuming temporally wide-sense stationary (WSS) random processes and allowing the length of each time series to grow large, the test statistic is then written as a broadband integral of the log Hadamard ratio of an estimated cross spectral matrix, a broadband coherence.

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In this paper, we consider the detection problem addressed in [7] for the purpose of detecting the presence of spatially correlated time series using a network of sensor arrays. Assuming WSS processes in both time and space, the likelihood ratio is shown to converge to a broadband integral of a log Hadamard ratio of a cross frequency-wavenumber spectrum, when the length of each time series and the number of sensors in each array grows large. Although this result is asymptotic, it suggests frequency/wavenumber implementations of the GLRT detector even for low space and time support for measured signals. The proposed detector is finally compared to a similar technique using simulated space-time fields.

2. REVIEW OF THE LIKELIHOOD RATIO

The problem considered here is testing for the independence among L random vectors $\{\mathbf{x}_i\}_{i=1}^{L}$ with each vector $\mathbf{x}_i = [x_i[0] \cdots x_i[N-1]]^T$ representing a length N time series captured at sensor i. Assuming this collection of random vectors to be zero mean, the composite vector $\mathbf{z} = \left[\mathbf{x}_1^T \cdots \mathbf{x}_L^T\right]^T \in \mathbb{C}^{LN}$ has space-time covariance matrix

$$R = E\left[\mathbf{z}\mathbf{z}^H\right] = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1L} \\ R_{12}^H & R_{22} & \cdots & R_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ R_{1L}^H & R_{2L}^H & \cdots & R_{LL} \end{bmatrix} \quad (1)$$

with $R_{ij} = R_{ji}^H = E\left[\mathbf{x}_i\mathbf{x}_j^H\right] \in \mathbb{C}^{N\times N}$ a temporal cross-covariance matrix.

If the set of random vectors $\{\mathbf{x}_i\}_{i=1}^{L}$ is jointly proper complex normal, testing for independence becomes a test of whether or not the covariance matrix R is block-diagonal. Casting this problem into the standard inference framework, we consider the following hypothesis test

$$\begin{aligned} \mathcal{H}_0 &: R \in \mathcal{R}_0 \\ \mathcal{H}_1 &: R \in \mathcal{R} \end{aligned}$$

with $\mathcal{R}$ denoting the set of all positive-definite Hermitian matrices and $\mathcal{R}_0$ denoting the set of all matrices in $\mathcal{R}$ which are additionally block-diagonal.

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We now assume we are given an experiment producing M *iid* realizations, $\{\mathbf{z}[m]\}_{m=1}^{M}$, of the composite vector $\mathbf{z}$. The collection of random vectors

$$\mathcal{Z} = [\mathbf{z}[1] \ \cdots \ \mathbf{z}[M]] \in \mathbb{C}^{LN \times M}$$

has probability density function (PDF)

$$f(\mathcal{Z}; R) = \frac{1}{\pi^{LNM} \det(R)^M} \exp\left\{-M \mathrm{tr}\left(R^{-1}\hat{R}\right)\right\}$$

with $\hat{R}$ being an estimated composite covariance matrix

$$\hat{R} = \frac{1}{M}\sum_{m=1}^{M} \mathbf{z}[m]\mathbf{z}^H[m] = \begin{bmatrix} \hat{R}_{11} & \hat{R}_{12} & \cdots & \hat{R}_{1L} \\ \hat{R}_{12}^H & \hat{R}_{22} & \cdots & \hat{R}_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{R}_{1L}^H & \hat{R}_{2L}^H & \cdots & \hat{R}_{LL} \end{bmatrix}$$

and $\hat{R}_{ij}$ being an M sample estimate of the matrix R_{ij}. The GLRT for this problem involves computing the likelihood ratio [7]

$$\Lambda = \left(\frac{\max\limits_{R \in \mathcal{R}_0} f(\mathcal{Z}; R)}{\max\limits_{R \in \mathcal{R}} f(\mathcal{Z}; R)}\right)^{\frac{1}{M}} = \frac{\det\left(\hat{R}\right)}{\prod_{i=1}^{L} \det\left(\hat{R}_{ii}\right)} = \det\left(\hat{C}\right) \quad (2)$$

where the matrix $\hat{C}$, with $N \times N$ blocks of the form

$$\left\{\hat{C}\right\}_{i,k} = \hat{C}_{ik} = \hat{R}_{ii}^{-1/2}\hat{R}_{ik}\hat{R}_{kk}^{-H/2},$$

is commonly referred to as the coherence matrix. The likelihood ratio given in (2) is a generalized Hadamard ratio and is a statistic that remains invariant to invertible block-diagonal matrices [7].

3. EXTENSIONS TO VECTOR-VALUED TIME SERIES

In certain examples of multi-channel detection applications, one may have the opportunity to observe multiple time series from each channel. One such example is a situation where several platforms or nodes each employ an array of sensors such as in wireless acoustic sensor networks [8]. As before, the problem considered here is to test for the independence among L random vectors, but we now assume that the random vector from each channel

$$\mathbf{x}_i = \left[\mathbf{x}_i^T[0] \ \cdots \ \mathbf{x}_i^T[N-1]\right]^T \in \mathbb{C}^{dN}$$

contains N samples from a d-dimensional vector-valued time series

$$\mathbf{x}_i[n] = [x_{i,0}[n] \ \cdots \ x_{i,d-1}[n]]^T \in \mathbb{C}^d$$

The generalized Hadamard ratio in (2) is still the appropriate test statistic in this situation, the only difference being that each block of the composite covariance matrix, R_{ik}, given in (1) is now $dN \times dN$ rather than $N \times N$.

As described in [7], the extension of the GLRT to the frequency domain can be accomplished by first independently applying the linear transformation $T = F_N \otimes I_d$, with F_N denoting an $N \times N$ DFT matrix, to the data from each channel. Define the length-N DFT vector

$$\mathbf{f}_N(e^{j\theta_l}) = \frac{1}{\sqrt{N}}\left[1 \ e^{j\theta_l} \ \cdots \ e^{j(N-1)\theta_l}\right]^T$$

and the matrix $F_N(e^{j\theta_l}) = \mathbf{f}_N(e^{j\theta_l}) \otimes I_d \in \mathbb{C}^{dN \times d}$. Then the linear transformation

$$F_N^H(e^{j\theta_l})\mathbf{x}_i = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} e^{-j\theta_l n}\mathbf{x}_i[n]$$

simply corresponds to a unitary DFT analysis of the i^{th} channel at frequency $\theta_l = \frac{2\pi}{N}l$, $l = 0, \ldots, N-1$. Recalling that the generalized Hadamard ratio is invariant to invertible block-diagonal transformations, it follows that both sets of signals, $\{\mathbf{x}_i\}$ and $\{T\mathbf{x}_i\}$, share the same likelihood ratio so that (2) can be written

$$\Lambda = \det\left((I_L \otimes T)\hat{C}(I_L \otimes T)^H\right)$$

Introducing a permutation to the rows and columns of the matrix inside this determinant, the GLRT can be written as $\Lambda = \det\tilde{C}$ where

$$\tilde{C} = \begin{bmatrix} \hat{C}(e^{j\theta_0}) & \cdots & \hat{C}(e^{j\theta_0}, e^{j\theta_{N-1}}) \\ \vdots & \ddots & \vdots \\ \hat{C}(e^{j\theta_{N-1}}, e^{j\theta_0}) & \cdots & \hat{C}(e^{j\theta_{N-1}}) \end{bmatrix}$$

is a matrix that not only captures the second-order information between channels but also between frequencies. The matrix $\hat{C}(e^{j\theta_l}, e^{j\theta_m}) \in \mathbb{C}^{dL \times dL}$ is an $L \times L$ block matrix consisting of $d \times d$ blocks of the form

$$\left\{\hat{C}(e^{j\theta_l}, e^{j\theta_m})\right\}_{i,k} = F_N^H(e^{j\theta_l})\hat{C}_{ik}F_N(e^{j\theta_m})$$

and we have used the convention $\hat{C}(e^{j\theta_l}) = \hat{C}(e^{j\theta_l}, e^{j\theta_l})$.

We now assume that all channels are temporally WSS in the sense that, for any pair of channels $\mathbf{x}_i$ and $\mathbf{x}_k$, there exists a matrix-valued sequence, $\{\Gamma_{ik}[l]\}$, such that

$$E\left[\mathbf{x}_i[n]\mathbf{x}_k^H[n+l]\right] = \Gamma_{ik}[l] \in \mathbb{C}^{d \times d}$$

Results on large block-Toeplitz matrices [9], [10] show that matrix $\tilde{C}$ is asymptotically equivalent to the block-diagonal matrix

$$\mathrm{blkdiag}\left\{\hat{C}(e^{j\theta_0}), \ldots, \hat{C}(e^{j\theta_{N-1}})\right\}$$

so that as N and M grow large, but d and L remain fixed, the GLRT becomes

$$\Lambda^{1/N} \overset{N\to\infty}{\longrightarrow} \exp\left\{\int_{-\pi}^{\pi} \ln\det\hat{C}(e^{j\theta})\frac{d\theta}{2\pi}\right\} = \exp\left\{\int_{-\pi}^{\pi} \ln\frac{\det\hat{S}(e^{j\theta})}{\prod_{i=1}^{L}\det\hat{S}_{ii}(e^{j\theta})}\frac{d\theta}{2\pi}\right\} \quad (3)$$

Again, the matrix $\hat{S}(e^{j\theta}) \in \mathbb{C}^{dL\times dL}$ is an $L \times L$ block-structured matrix consisting of $d \times d$ submatrices of the form

$$\left\{\hat{S}(e^{j\theta})\right\}_{i,k} = F_N^H(e^{j\theta})\hat{R}_{ik}F_N(e^{j\theta})$$

which is a quadratic estimate of the cross power spectral density matrix between channels i and k at frequency θ and we use the convention $\hat{S}_{ii}(e^{j\theta}) = \left\{\hat{S}(e^{j\theta})\right\}_{i,i}$.

The likelihood ratio given in (3) is not a particularly novel result and is a direct extension of the results in [7] to account for the situation being considered here. Although this result is perfectly general in that nothing has been assumed about these vector-valued time series other than that they are temporally WSS, we proceed under the context of multiple-array detection in which case a notion of space can be ascribed to the time series of each channel.

To take advantage of the spatiotemporal properties of the problem, we now consider independently applying the linear transformation $T = F_N \otimes F_d$ to each channel (as opposed to the matrix $T = F_N \otimes I_d$ considered earlier) with F_d denoting a $d \times d$ DFT matrix. Note that pre-multiplying the vector $\mathbf{x}_i$ by the matrix T simply corresponds to the application of a 2-dimensional DFT, one applied temporally and the other spatially. For any frequency θ, we can then introduce a permutation of the rows and columns of the previously defined matrix $\hat{C}(e^{j\theta})$ so that

$$\det \hat{C}(e^{j\theta}) = \det \tilde{C}(e^{j\theta})$$

where

$$\tilde{C}(e^{j\theta}) = \begin{bmatrix} \hat{C}(e^{j\theta}, e^{j\phi_0}) & \cdots & \hat{C}(e^{j\theta}, e^{j\phi_0}, e^{j\phi_{N-1}}) \\ \vdots & \ddots & \vdots \\ \hat{C}(e^{j\theta}, e^{j\phi_{N-1}}, e^{j\phi_0}) & \cdots & \hat{C}(e^{j\theta}, e^{j\phi_{N-1}}) \end{bmatrix}$$

and $\hat{C}(e^{j\theta}, e^{j\phi_l}) = \hat{C}(e^{j\theta}, e^{j\phi_l}, e^{j\phi_l})$. Define the length-$d$ DFT vector at frequency $\phi_l = \frac{2\pi}{d}l$ for $l = 0, \ldots, d-1$ as follows

$$\mathbf{f}_d(e^{j\phi_l}) = \frac{1}{\sqrt{d}}\left[1\ e^{j\phi_l}\ \cdots\ e^{j(d-1)\phi_l}\right]^T.$$

Then the matrix $\hat{C}(e^{j\theta}, e^{j\phi_l}, e^{j\phi_m}) \in \mathbb{C}^{L\times L}$ has entries of the form

$$\left[\hat{C}(e^{j\theta}, e^{j\phi_l}, e^{j\phi_m})\right]_{i,k} = \mathbf{f}_d^H(e^{j\phi_l})F_N^H(e^{j\theta})\hat{C}_{ik}F_N(e^{j\theta})\mathbf{f}_d(e^{j\phi_m})$$

When the entries of $\mathbf{x}_i[n]$ correspond to time series at different spatial locations, the frequency variable ϕ is often referred to as the *wavenumber* and, to avoid confusion with the variable θ, we will adopt this terminology.

We now impose additional structure on the problem at hand by assuming that all channels are not only temporally WSS but spatially WSS as well so that the multivariate covariance function, $\{\Gamma_{ik}[l]\}$, considered earlier now corresponds to a sequence of Toeplitz matrices. That is, for any pair of channels $\mathbf{x}_i$ and $\mathbf{x}_k$, we now assume that there exists a two-dimensional sequence, $\{\gamma_{ik}[l,m]\}$, such that

$$E\left[x_{i,p}[n]x^*_{k,p+m}[n+l]\right] = \gamma_{ik}[l,m] \in \mathbb{C}$$

with l a temporal lag and m a spatial lag.

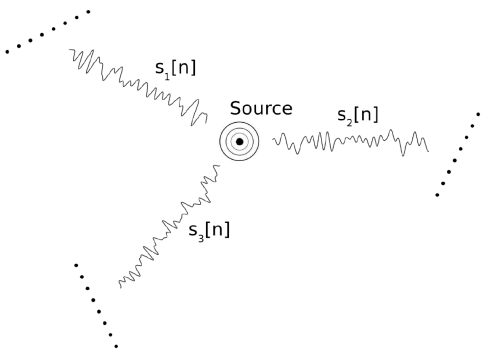

Fig. 1. Detection of a Source using Multiple Linear Arrays.

Again invoking results on large block-Toeplitz matrices, it follows that the matrix $\tilde{C}(e^{j\theta})$ is asymptotically equivalent with the block-diagonal matrix

$$\text{blkdiag}\left\{\hat{C}(e^{j\theta}, e^{j\phi_0}), \ldots, \hat{C}(e^{j\theta}, e^{j\phi_{d-1}})\right\}$$

so that as M, N, and d grow large but L remains fixed the GLRT becomes

$$\begin{aligned}\Lambda^{\frac{1}{dN}} &= \det\left((I_L \otimes T)\hat{C}(I_L \otimes T)^H\right)^{\frac{1}{dN}} = \det\left(\tilde{C}\right)^{\frac{1}{dN}} \\ &\overset{N\to\infty}{\to} \exp\left\{\int_{-\pi}^{\pi} \ln\det\left(\hat{C}(e^{j\theta})\right)^{\frac{1}{d}}\frac{d\theta}{2\pi}\right\} \\ &\overset{d\to\infty}{\to} \exp\left\{\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \ln\det\hat{C}(e^{j\theta}, e^{j\phi})\frac{d\theta d\phi}{4\pi^2}\right\} \\ &= \exp\left\{\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \ln\frac{\det\hat{S}(e^{j\theta}, e^{j\phi})}{\prod_{i=1}^{L}\hat{S}_{ii}(e^{j\theta}, e^{j\phi})}\frac{d\theta d\phi}{4\pi^2}\right\} \quad (4)\end{aligned}$$

The matrix $\hat{S}(e^{j\theta}, e^{j\phi}) \in \mathbb{C}^{L\times L}$ has elements

$$\left[\hat{S}(e^{j\theta}, e^{j\phi})\right]_{i,k=1}^{L} = \mathbf{f}_d^H(e^{j\phi})F_N^H(e^{j\theta})\hat{R}_{ik}F_N(e^{j\theta})\mathbf{f}_d(e^{j\phi})$$

which is a quadratic estimate of the cross power spectral density between channels i and k in the frequency/wavenumber domain. Thus, we see that the GLRT involves the computation of a Hadamard ratio at each frequency/wavenumber pair (θ, ϕ), followed by broadband integration of its logarithm, a broadband-broadwavenumber coherence.

4. SIMULATION RESULTS

In this section we consider a situation where $L = 3$ spatially separated uniform linear arrays (ULAs), each consisting of $d = 16$ sensor elements, receive distinct wideband

signals from a single point source as depicted in Figure 1. The signals received by the elements of each array, $\mathbf{s}[n] = [s_1[n]\ s_2[n]\ s_3[n]]^T$, are assumed to be generated from the following multivariate autoregressive model [11]

$$\mathbf{s}[n] = \Phi\mathbf{s}[n-1] + \mathbf{w}_s[n];\ n = 0, \ldots, MN-1$$

where the driving process $\mathbf{w}_s[n]$ is taken to be a zero-mean proper complex normal random vector with covariance matrix

$$E\left[\mathbf{w}_s[n]\mathbf{w}_s^H[n+l]\right] = \delta_l\sigma_s^2 I_L$$

and Φ is an arbitrarily chosen $L \times L$ matrix. These signals have the following frequency-dependent spectral density matrix

$$\begin{aligned} S_{ss}(e^{j\theta}) &= \begin{bmatrix} \sigma_{11}^2(e^{j\theta}) & \sigma_{12}^2(e^{j\theta}) & \sigma_{13}^2(e^{j\theta}) \\ \sigma_{12}^2(e^{j\theta})^* & \sigma_{22}^2(e^{j\theta}) & \sigma_{23}^2(e^{j\theta}) \\ \sigma_{13}^2(e^{j\theta})^* & \sigma_{23}^2(e^{j\theta})^* & \sigma_{33}^2(e^{j\theta}) \end{bmatrix} \\ &= \sigma_s^2\left(I_L - e^{-j\theta}\Phi\right)^{-1}\left(I_L - e^{-j\theta}\Phi\right)^{-T} \end{aligned}$$

The signal received at each array is then propagated as a planewave among its elements. At each d-element array a spatially correlated, temporally white noise is added independently of all other arrays. Consequently, each $d \times d$ block of the frequency-dependent spectral density matrix of the composite observation can be written as follows

$$\left\{S(e^{j\theta})\right\}_{i,k} = \begin{cases} \sigma_{ii}^2(e^{j\theta})\mathbf{a}_i(e^{j\theta})\mathbf{a}_i^H(e^{j\theta}) + R_{n_i} & i = k \\ \sigma_{ik}^2(e^{j\theta})\mathbf{a}_i(e^{j\theta})\mathbf{a}_k^H(e^{j\theta}) & i \neq k \end{cases}$$

Here, the vector $\mathbf{a}_i(e^{j\theta})$ denotes the array response or steering vector for the i^{th} ULA

$$\mathbf{a}_i(e^{j\theta}) = \left[1\ e^{-j\theta\tau_i}\ \cdots\ e^{-j(d-1)\theta\tau_i}\right]^T$$

with τ_i representing the propagation delay among the sensor elements. Also, the matrix R_{n_i} denotes the banded Toeplitz matrix associated with passing unit-variance white noise through a 5^{th}-order FIR filter with arbitrarily chosen weights.

Upon collecting all MN measurements at each sensor element, the data record is temporally partitioned into M non-overlapping copies of a time series of length $N = 24$. The likelihood ratio given in (4) (denoted "Frequency/Wavenumber Domain GLRT") is then used to discriminate situations where a source is present from those in which each sensor array observes its own correlated noise field only. The performance of this detector is compared to the frequency domain version of the GLRT given in (3) (denoted "Frequency Domain GLRT").

With $M = 250$ and a -3 dB source, Figure 2 displays the Receiver Operating Characteristic (ROC) curves for these two detection methods. Likewise, Figure 3 displays the same with $M = 50$ and a 6 dB source. From these ROC curves it is clear that the version of the GLRT given in (4) provides a significant improvement in detection performance compared to its alternative in (3). This is likely due to the fact that the likelihood ratio given in (4) is better matched to the (spatially) WSS case versus its alternative given in (3) which, while more generally applicable, does not fully exploit wide-sense stationarity.

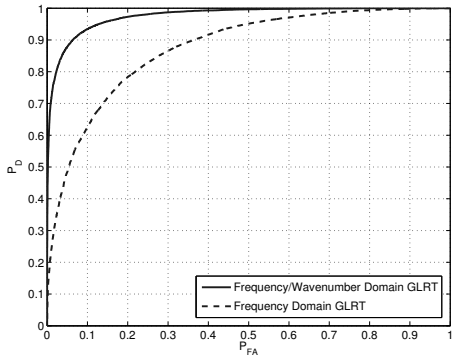

Fig. 2. Detection Performance with $M = 250$ and $SNR = -3$ dB.

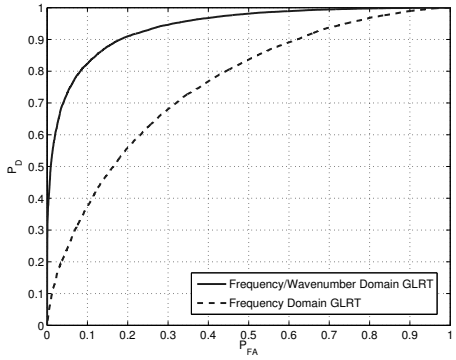

Fig. 3. Detection Performance with $M = 50$ and $SNR = 6$ dB.

5. CONCLUSIONS

Detecting the presence of common characteristics among two or more data channels is a problem that finds its uses in a wide range of applications. One possible solution to this problem is the use of a GLRT that tests whether or not a composite covariance matrix is block-diagonal through the use of a generalized Hadamard ratio. In the case of multiple sensor arrays, wide-sense stationarity in both time and space suggests the use of a broadband-broadwavenumber implementation of the GLRT. Simulations demonstrate that the proposed technique presents an appealing version of the GLRT that can provide improved detection performance for such an application.

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