FOURTH-ORDER TENSOR METHOD FOR BLIND SPATIAL SIGNATURE ESTIMATION†

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ABSTRACT

In this paper, we consider a wireless communication scenario where M sources simultaneously transmit towards a base station equipped with an array of K sensors. A new method is proposed to solve the spatial signature estimation problem without resorting to training sequences and without knowledge of sources' covariance structure. By assuming that the sources' amplitudes vary between successive time blocks, a fourth-order tensor decomposition of the multimode spatio-temporal data covariance is proposed, from which an iterative algorithm is formulated to estimate sources' spatial signatures. A distinguishing feature of the proposed tensor method is its efficiency in treating the case where the sources' covariance matrix is non-diagonal and unknown, which generally happens when working with sample data covariances computed from a reduced number of snapshots.

***Index Terms*—** Array processing, spatial signature estimation, tensor decomposition.

1. INTRODUCTION

In the uplink scenario, where mobile users communicate with the base station, the knowledge of spatial signatures is important to the design of space division multiple access (SDMA) techniques [1]. The literature on matrix-based methods for spatial signature estimation is abundant (see e.g. [2] for an overview). The existing solutions can be categorized into different ways depending on assumptions involving (i) the knowledge (or not) of pilot signals, (ii) the use of parametric or nonparametric models for the spatial signatures, (ii) the use of sources' statistical independency or cyclostationarity, to mention a few. In this context, blind methods are of particular interest, as they are more bandwidth-efficient and avoid tight user synchronization [3]-[8].

The large majority of blind estimation methods in general do not take into account the multidimensional structure of the data, which may span several dimensions such as space, time and/or frequency. Note that space may have two dimensions (e.g. azimuth and elevation) while time dimension can be divided into snapshots and frames. In order to deal with such multidimensional nature, tensor decompositions have extensively been applied in recent years to array signal processing problems [6, 7, 9] and also to communication problems [10, 11, 12]. There are significant advantages of using tensor-based signal processing instead of matrix-based signal processing. Among these advantages, we can cite the improved identifiability conditions in a blind setting, which generally come from the essential uniqueness property of tensor decompositions. Tensor-based algorithms also inherits the so-called "tensor gain", which translates into improved accuracy due to the efficient noise rejection capability [13, 14].

In the context of tensor-based methods for spatial signature estimation, previously proposed methods, such as [8], [7], [6], are mainly based on the parallel factor (PARAFAC) analysis [15]. The approach of [8] is of particular interest as it does not require any knowledge on the propagation channel and/or array manifold, while providing performances comparable to (or better than) competing matrix-based methods. However, in these works, especially in [8], the covariance matrix of the source signals is assumed to be perfectly known and diagonal. In practice, it is known that these properties correspond to having (i) uncorrelated signals coming from the different users and (ii) a perfect estimate of the spatial covariance matrix. Although the first assumption may hold in some scenarios, the second only holds asymptotically, and a good approximation requires computing sample covariance over a sufficiently large number of snapshots. Our interest is to devise a method that can provide accurate estimations of the users' spatial signatures without relying on such idealized assumptions.

In this paper, we propose an efficient method to solve the spatial signature estimation problem without resorting to training sequences and without knowledge of sources' covariance structure. By assuming that the sources' amplitudes vary between successive time blocks, a symmetric Tucker decomposition is formulated by mapping a multimode spatio-temporal data covariance into a fourth-order tensor. From the resultant tensor model, an alternating least squares (ALS)-based iterative algorithm is proposed to estimate sources' spatial signatures. In contrast to [8], which relies on the diagonality assumption for the sources' covariance matrix, the proposed tensor method can efficiently handle arbitrary (non-diagonal) and unknown covariance structures. Consequently, our approach is more attractive in practical settings where sample covariances are computed from reduced number of snapshots.

Notation: The superscripts $^{\mathrm{T}}$,$^{\mathrm{H}}$,†, and * represent transpose, Hermitian transpose, pseudo-inverse and complex conjugate, respectively. The operator $\mathrm{diag}(\boldsymbol{a})$ forms a diagonal matrix based on $\boldsymbol{a}$. The r-th column of $\boldsymbol{A} \in \mathbb{C}^{J\times R}$ is denoted by $\boldsymbol{A}(:,r) \in \mathbb{C}^{J\times 1}$. The operator $\mathrm{vec}(\boldsymbol{A})$ converts $\boldsymbol{A}$ to a vector $\boldsymbol{a}$ by stacking its columns on top of each other, while $\mathrm{unvec}_{J\times R}(\boldsymbol{a})$ denotes the inverse vectorization operation that converts $\boldsymbol{a} \in \mathbb{C}^{JR\times 1}$ back to a $J \times R$ matrix. $D_j(\boldsymbol{A})$ is a diagonal matrix constructed from the j-th row of $\boldsymbol{A}$, and $||\cdot||_F$ represents the Frobenius norm of a matrix or tensor, which is defined as the square root of the sum of the squared amplitude of its elements. The Kronecker product and outer product operators are denoted by $\otimes$ and $\circ$, respectively. The Khatri-Rao product between two matrices $\boldsymbol{A} \in \mathbb{C}^{J\times R}$ and $\boldsymbol{B} = [\boldsymbol{b}_1 \ \dots \ \boldsymbol{b}_R] \in \mathbb{C}^{K\times R}$, denoted by $\diamond$, is their column-wise

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Kronecker product

$$\boldsymbol{A} \diamond \boldsymbol{B} \doteq [\boldsymbol{A}(:,1) \otimes \boldsymbol{B}(:,1), \ldots, \boldsymbol{A}(:,R) \otimes \boldsymbol{B}(:,R)]. \quad (1)$$

In this paper, the following property of the Kronecker product is used

$$\text{vec}(\mathbf{ABC}) = (\mathbf{C}^{\mathrm{T}} \otimes \mathbf{A})\text{vec}(\mathbf{B}), \quad (2)$$

where $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are matrices of compatible dimensions. The tensor operations are consistent with [16]: The r-mode unfolding of a tensor $\boldsymbol{\mathcal{T}} \in \mathbb{C}^{J_1 \times J_2 \times \cdots \times J_R}$, symbolized by $[\boldsymbol{\mathcal{T}}]_{(r)} \in \mathbb{C}^{J_r \times (J_1 J_2 \ldots J_{r-1} J_{r+1} \cdots J_R)}$, represents the matrix of r-mode vectors of $\boldsymbol{\mathcal{T}}$. The order of the columns is chosen in accordance with [16]. The r-mode product of $\boldsymbol{\mathcal{T}}$ and a matrix $\boldsymbol{U} \in \mathbb{C}^{K_r \times J_r}$ along the r-th mode is denoted as $\boldsymbol{\mathcal{T}} \times_r \boldsymbol{U} \in \mathbb{C}^{J_1 \times J_2 \cdots \times K_r \cdots \times J_R}$, which is obtained by multiplying the r-mode unfolding of $\boldsymbol{\mathcal{T}}$ from the left hand side (LHS) by $\boldsymbol{U}$.

2. MEASUREMENT MODEL

We consider a wireless communication scenario where M sources simultaneously transmit towards a base station equipped with an array of K sensors. Let $s_m(n)$ denote the signal transmitted by the m-th source that impinges on the antenna array with an angle of arrival θ_m. The discrete-time received signal is given by $\boldsymbol{x}(n) = \boldsymbol{A}\boldsymbol{s}(n) + \boldsymbol{v}(n)$, $n = 1, \ldots, N$, where $\boldsymbol{A} = [\boldsymbol{a}_1, \ldots, \boldsymbol{a}_M] \in \mathbb{C}^{K \times M}$ is the spatial signature matrix of the sources, $\boldsymbol{s}(n) = [s_1(n), \ldots, s_M(n)]^T \in \mathbb{C}^{M \times 1}$ is the source signal vector associated with the n-th snapshot, and $\boldsymbol{v}(n) \in \mathbb{C}^{M \times 1}$ is the additive white Gaussian noise term. Assuming that $\boldsymbol{A}$ is constant during the observation interval of N snapshots, we have

$$\boldsymbol{X} = \boldsymbol{A}\boldsymbol{S} + \boldsymbol{V}, \quad (3)$$

where $\boldsymbol{X} = [\boldsymbol{x}(1), \ldots, \boldsymbol{x}(N)] \in \mathbb{C}^{K \times N}$ is a matrix collecting the received data, $\boldsymbol{S} = [\boldsymbol{s}(1), \ldots, \boldsymbol{s}(N)] \in \mathbb{C}^{M \times N}$ is the transmitted symbol matrix and $\boldsymbol{V} = [\boldsymbol{v}(1), \ldots, \boldsymbol{v}(N)] \in \mathbb{C}^{K \times N}$ is the noise matrix. The additive noise is assumed to be uncorrelated with respect to the sources. The sample spatial covariance matrix $\boldsymbol{R} \in \mathbb{C}^{K \times K}$ of the signals received at the antenna array is given by

$$\boldsymbol{R} \triangleq \frac{1}{N}\boldsymbol{X}\boldsymbol{X}^{\mathrm{H}} = \boldsymbol{A}\left(\frac{1}{N}\boldsymbol{S}\boldsymbol{S}^{\mathrm{H}}\right)\boldsymbol{A}^{\mathrm{H}} + \boldsymbol{E}, \quad (4)$$

where $\boldsymbol{E} = \frac{1}{N}\left(2\text{Re}\{\boldsymbol{A}\boldsymbol{S}\boldsymbol{V}^{\mathrm{H}}\} + \boldsymbol{V}\boldsymbol{V}^{\mathrm{H}}\right)$ represents the covariance of the additive noise in conjunction with the cross-covariance between the source signals and noise. Note that, for $N \to \infty$, we have $\frac{1}{N}\boldsymbol{X}\boldsymbol{X}^{\mathrm{H}} \to E\{\boldsymbol{x}(n)\boldsymbol{x}^H(n)\}$, $\frac{1}{N}\boldsymbol{S}\boldsymbol{S}^{\mathrm{H}} \to E\{\boldsymbol{s}(n)\boldsymbol{s}^H(n)\} = \text{diag}(\alpha_1, \ldots, \alpha_K)$, where α_k is the k-th source variance, and $\frac{1}{N}\boldsymbol{V}\boldsymbol{V}^{\mathrm{H}} = \sigma_v^2\boldsymbol{I}$, where σ_v^2 is the noise variance. We call attention to the fact that $\alpha_1, \ldots, \alpha_K$ are unknown and not necessarily equal. Equation (3) is the classical model for spatial signature estimation, for which several matrix based techniques have been developed. Among them, we can refer to traditional algorithms such a multiple signal classification (MUSIC) and estimation of signal parameters by rotational invariance techniques (ESPRIT) [2].

3. PROPOSED METHOD

Let us consider that the fixed number N of snapshots are divided into P time blocks, each consisting of $N_s = \frac{N}{P}$ snapshots. At every time block, the m-th source transmits a sequence $\boldsymbol{s}_m \in \mathbb{C}^{N_s \times 1}$, $m = 1, \ldots, M$, which is not known to the receiver. These The mean

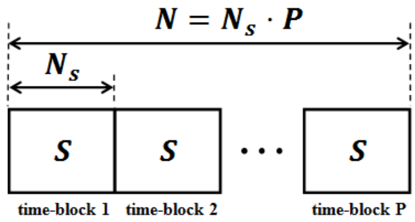

Fig. 1. Data transmission structure.

power of each transmitted source waveform is guaranteed to vary across these P time blocks. Such power fluctuations may naturally occur in block fading channels (under the reasonable assumption that source spatial signatures/directions of arrival vary much slowly than the fading channel), or it may be artificially induced by means of a time-varying power loading scheme in mobile communications, where the mean power of each mobile user is subject to small variations that can be done on top of usual power control [8]. Figure 2 illustrates the data transmission structure. Let us define the power loading matrix $\boldsymbol{W} = [\boldsymbol{w}_1, \ldots, \boldsymbol{w}_M] \in \mathbb{R}^{P \times M}$, the m-th column of which contains the set of coefficients for the m-th source, while $\boldsymbol{S} = [\boldsymbol{s}_1, \ldots, \boldsymbol{s}_M]^{\mathrm{T}} \in \mathbb{C}^{M \times N_s}$ concatenates the source sequences. The data received during the p-th time block is given by

$$\boldsymbol{X}_p = \boldsymbol{A}D_p(\boldsymbol{W})\boldsymbol{S} + \boldsymbol{V}_p \in \mathbb{C}^{K \times N_s}. \quad (5)$$

Collecting the data received during the P time blocks, we obtain

$$\underbrace{\begin{bmatrix} \boldsymbol{X}_1 \\ \boldsymbol{X}_2 \\ \vdots \\ \boldsymbol{X}_P \end{bmatrix}}_{\boldsymbol{X}} = \begin{bmatrix} \boldsymbol{A}D_1(\boldsymbol{W}) \\ \boldsymbol{A}D_2(\boldsymbol{W}) \\ \vdots \\ \boldsymbol{A}D_P(\boldsymbol{W}) \end{bmatrix} \boldsymbol{S} + \underbrace{\begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_2 \\ \vdots \\ \boldsymbol{V}_P \end{bmatrix}}_{\boldsymbol{V}} \in \mathbb{C}^{PK \times N_s} \quad (6)$$

or, equivalently,

$$\boldsymbol{X} = (\boldsymbol{W} \diamond \boldsymbol{A})\boldsymbol{S} + \boldsymbol{V} \in \mathbb{C}^{PK \times N_s}. \quad (7)$$

Let us introduce the spatio-temporal multimode sample covariance matrix

$$\boldsymbol{R}_{\text{mm}} \triangleq \frac{1}{N_s}\boldsymbol{X}\boldsymbol{X}^{\mathrm{H}} \in \mathbb{C}^{PK \times PK}. \quad (8)$$

By first subtracting the estimated noise power from $\boldsymbol{R}_{\text{mm}}$, we get $\overline{\boldsymbol{R}}_{\text{mm}} = \boldsymbol{R}_{\text{mm}} - \sigma_v^2\boldsymbol{I}$. Then, from (7), we have

$$\overline{\boldsymbol{R}}_{\text{mm}} = (\boldsymbol{W} \diamond \boldsymbol{A})\boldsymbol{R}_s(\boldsymbol{W} \diamond \boldsymbol{A})^{\mathrm{H}} + \boldsymbol{E} \in \mathbb{C}^{PK \times PK}, \quad (9)$$

where $\boldsymbol{R}_s \triangleq \frac{1}{N_s}\boldsymbol{S}\boldsymbol{S}^{\mathrm{H}}$ is the sample covariance matrix of the source waveforms. The noiseless multimode matrix (9) can be viewed as a symmetrical multimode unfolding of a fourth-order tensor $\boldsymbol{\mathcal{R}} \in \mathbb{C}^{K \times P \times K \times P}$ given by

$$\boldsymbol{\mathcal{R}}_x = \boldsymbol{\mathcal{R}}_s \times_1 \boldsymbol{A} \times_2 \boldsymbol{W} \times_3 \boldsymbol{A}^* \times_4 \boldsymbol{W}^* + \boldsymbol{E}, \quad (10)$$

where $\boldsymbol{\mathcal{R}}_s \in \mathbb{C}^{M \times M \times M \times M}$ is the sources covariance tensor. The fourth-order tensor decomposition (10) is commonly known as the "Tucker4" decomposition [18] of $\boldsymbol{\mathcal{R}}_x$, where the associated core tensor is given by $\boldsymbol{\mathcal{R}}_s$. In our case, due to the conjugate symmetry of the spatio-temporal covariance matrix, the resulting Tucker4 decomposition exhibits a partial symmetry. Let $[\boldsymbol{\mathcal{R}}_x]_{(1,2);(3,4)} \in \mathbb{C}^{PK \times PK}$ denote the multimode matrix unfolding of the spatial-temporal covariance tensor $\boldsymbol{\mathcal{R}}_x$ that merges its first and second modes row-wise and the third and second modes column-wise. We have

$$[\boldsymbol{\mathcal{R}}_x]_{(1,2);(3,4)} = \overline{\boldsymbol{R}}_{\text{mm}}, \tag{11}$$

$$\boldsymbol{\mathcal{R}}_s(:,m,:,m') = \begin{cases} \boldsymbol{R}_s, & m = m' \\ 0, & \text{otherwise} \end{cases} \tag{12}$$
$$\{m, m'\} = 1, \ldots, M,$$

where $\boldsymbol{\mathcal{R}}_s(:,m,:,m') \in \mathbb{C}^{M\times M}$ denotes a matrix slice obtained by slicing the covariance tensor $\boldsymbol{\mathcal{R}}_s$ along the plane (m, m'), $\{m, m'\} = 1, \ldots, M$. Due to the partial symmetry of the spatial-temporal covariance tensor $\boldsymbol{\mathcal{R}}_x$, we also have $[\boldsymbol{\mathcal{R}}_x]_{(1,4);(3,2)} = \overline{\boldsymbol{R}}_{\text{mm}}$.

The Tucker4 decomposition (10) naturally captures any structure for the sources covariance into the core tensor $\boldsymbol{\mathcal{R}}_s$, meaning that the assumption of uncorrelated source signals is not a restriction of the proposed decomposition. Moreover, assuming that the covariances are computed over a finite (possibly small) number N_s of snapshots, the diagonality of the sample covariance does not hold, i.e. $\boldsymbol{R}_s = \frac{1}{N}\boldsymbol{S}\boldsymbol{S}^{\text{H}} \neq \text{diag}(\alpha_1, \ldots, \alpha_K)$, even for sources that are uncorrelated in practice. In fact, assuming that the sources covariance matrix is diagonal only holds asymptotically. Working with arbitrary and unknown sources' covariance structure is the main motivation for the proposed approach. As a special case, under the optimistic assumption of perfectly uncorrelated sources with known unit variances, the Tucker4 decomposition simplifies to

$$\boldsymbol{\mathcal{R}}_x = \boldsymbol{\mathcal{I}}_{4,M} \times_1 \boldsymbol{A} \times_2 \boldsymbol{W} \times_3 \boldsymbol{A}^* \times_4 \boldsymbol{W}^* + \boldsymbol{E}, \tag{13}$$

where $\boldsymbol{\mathcal{I}}_{4,M}$ is the fourth-order "identity" tensor whose elements are equal to one when all indices are equal and zero elsewhere. In this case, the spatial-temporal data covariance follows a fourth-order PARAFAC decomposition [18].

Our approach consists in estimating the spatial signature matrix $\boldsymbol{A}$ without knowledge of the source covariance tensor $\boldsymbol{\mathcal{R}}_s$ from the spatial-temporal data covariance tensor $\boldsymbol{\mathcal{R}}_x$. This can be done without knowing the power loading matrix $\boldsymbol{W}$, although its knowledge may be assumed in mobile communications under some level of coordination. More specifically, we propose to minimize the following cost function:

$$f(\hat{\boldsymbol{\mathcal{R}}}_s, \hat{\boldsymbol{A}}, \hat{\boldsymbol{W}}) = \|\boldsymbol{\mathcal{R}}_x - \hat{\boldsymbol{\mathcal{R}}}_x\|_F^2 = \|\boldsymbol{E}\|_F^2 \tag{14}$$

where

$$\hat{\boldsymbol{\mathcal{R}}}_x = \hat{\boldsymbol{\mathcal{R}}}_s \times_1 \hat{\boldsymbol{A}} \times_2 \hat{\boldsymbol{W}} \times_3 \hat{\boldsymbol{A}}^* \times_4 \hat{\boldsymbol{W}}^* \tag{15}$$

Let $[\boldsymbol{\mathcal{R}}_s]_{(1)} \in \mathbb{C}^{M\times M^3}, [\boldsymbol{\mathcal{R}}_s]_{(2)} \in \mathbb{C}^{M\times M^3}, [\boldsymbol{\mathcal{R}}_s]_{(3)} \in \mathbb{C}^{M\times M^3}$ e $[\boldsymbol{\mathcal{R}}_s]_{(4)} \in \mathbb{C}^{M\times M^3}$ be the unimodal unfoldings of the sources' covariance tensor $\boldsymbol{\mathcal{R}}_s \in \mathbb{C}^{M\times M\times M\times M}$. Likewise, $[\boldsymbol{\mathcal{R}}_x]_{(1)} \in \mathbb{C}^{K\times P^2K}$, $[\boldsymbol{\mathcal{R}}_x]_{(2)} \in \mathbb{C}^{P\times PK^2}$, $[\boldsymbol{\mathcal{R}}_x]_{(3)} \in \mathbb{C}^{K\times P^2K}$, $[\boldsymbol{\mathcal{R}}_x]_{(4)} \in \mathbb{C}^{P\times PK^2}$ are the unimodal unfoldings of the spatio-temporal data covriance tensor. We have

$$[\boldsymbol{\mathcal{R}}_x]_{(1)} = \boldsymbol{A}[\boldsymbol{\mathcal{R}}_s]_{(1)}(\boldsymbol{W}^* \otimes \boldsymbol{A}^* \otimes \boldsymbol{W})^T, \tag{16}$$

$$[\boldsymbol{\mathcal{R}}_x]_{(2)} = \boldsymbol{W}[\boldsymbol{\mathcal{R}}_s]_{(2)}(\boldsymbol{W}^* \otimes \boldsymbol{A}^* \otimes \boldsymbol{A})^T, \tag{17}$$

$$[\boldsymbol{\mathcal{R}}_x]_{(3)} = \boldsymbol{A}^*[\boldsymbol{\mathcal{R}}_s]_{(3)}(\boldsymbol{W}^* \otimes \boldsymbol{W} \otimes \boldsymbol{A})^T, \tag{18}$$

$$[\boldsymbol{\mathcal{R}}_x]_{(4)} = \boldsymbol{W}^*[\boldsymbol{\mathcal{R}}_s]_{(4)}(\boldsymbol{A}^* \otimes \boldsymbol{W} \otimes \boldsymbol{A})^T. \tag{19}$$

From these matrix unfoldings, four LS estimation steps can be derived to estimate $\boldsymbol{A}$, $\boldsymbol{W}$, $\boldsymbol{A}^*$ and $\boldsymbol{W}^*$, respectively. Note, however, that the sources' covariance matrix $\boldsymbol{R}_s$ is assumed to be unknown, and should also be estimated. This can be done from the multimode spatio-temporal covariance matrix. Applying the property (2) to (9) we get

$$\text{vec}(\overline{\boldsymbol{R}}_{\text{mm}}) = \Big[(\boldsymbol{W} \diamond \boldsymbol{A})^* \otimes (\boldsymbol{W} \diamond \boldsymbol{A})\Big]\text{vec}(\boldsymbol{R}_s) \in \mathbb{C}^{P^2K^2\times 1}. \tag{20}$$

From (20), an estimate of $\text{vec}(\overline{\boldsymbol{R}}_{\text{mm}})$ can be extracted in the LS sense. We propose to estimate all the unknown quantities in a alternating way. The common solution relies on the alternating least squares (ALS) procedure [17, 18], which allows to iteratively solve these LS estimation steps. The proposed algorithm consists in estimating at each time, a given factor matrix by fixing the other matrices to their values obtained at previous estimation steps. The so-called ALS-Tucker4 algorithm proposed to solve the blind spatial signature estimation problem is detailed as follows:

ALS-Tucker4 algorithm for blind spatial signature estimation

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1. Set $i = 0$; Randomly initialize $\hat{\boldsymbol{\mathcal{R}}}_s^{(i=0)}$, $\hat{\boldsymbol{A}}_{(i=0)}, \hat{\boldsymbol{B}}_{(i=0)}$, $\hat{\boldsymbol{C}}_{(i=0)} = \hat{\boldsymbol{A}}^*_{(i=0)}, \hat{\boldsymbol{D}}_{(i=0)} = \hat{\boldsymbol{B}}^*_{(i=0)}$;
2. From $\hat{\boldsymbol{\mathcal{R}}}_s^{(i=0)}$ construct the unimodal matrix unfoldings $[\hat{\boldsymbol{\mathcal{R}}}_s^{(i=0)}]_{(1)}, [\hat{\boldsymbol{\mathcal{R}}}_s^{(i=0)}]_{(2)}, [\hat{\boldsymbol{\mathcal{R}}}_s^{(i=0)}]_{(3)}, [\hat{\boldsymbol{\mathcal{R}}}_s^{(i=0)}]_{(4)}$;
3. $i = i + 1$;
4. Using $[\boldsymbol{\mathcal{R}}_x]_{(1)}$, find an LS estimate of $\hat{\boldsymbol{A}}_{(i)}$:
   $\hat{\boldsymbol{A}}_{(i)} = [\boldsymbol{\mathcal{R}}_x]_{(1)} \left[[\hat{\boldsymbol{\mathcal{R}}}_s^{(i-1)}]_{(1)}(\hat{\boldsymbol{D}}_{(i-1)} \otimes \hat{\boldsymbol{C}}_{(i-1)} \otimes \hat{\boldsymbol{B}}_{(i-1)})^T \right]^\dagger$;
5. Using $[\boldsymbol{\mathcal{R}}_x]_{(2)}$, find an LS estimate of $\hat{\boldsymbol{W}}_{(i)}$:
   $\hat{\boldsymbol{B}}_{(i)} = [\boldsymbol{\mathcal{R}}_x]_{(2)} \left[[\hat{\boldsymbol{\mathcal{R}}}_s^{(i-1)}]_{(2)}(\hat{\boldsymbol{D}}_{(i-1)} \otimes \hat{\boldsymbol{C}}_{(i-1)} \otimes \hat{\boldsymbol{A}}_{(i)})^T \right]^\dagger$;
6. Using $[\boldsymbol{\mathcal{R}}_x]_{(3)}$, find an LS estimate of $\hat{\boldsymbol{C}}_{(i)}$:
   $\hat{\boldsymbol{C}}_{(i)} = [\boldsymbol{\mathcal{R}}_x]_{(3)} \left[[\hat{\boldsymbol{\mathcal{R}}}_s^{(i-1)}]_{(3)}(\hat{\boldsymbol{D}}_{(i-1)} \otimes \hat{\boldsymbol{B}}_{(i)} \otimes \hat{\boldsymbol{A}}_{(i)})^T \right]^\dagger$;
7. Using $[\boldsymbol{\mathcal{R}}_x]_{(4)}$, find an LS estimate of $\hat{\boldsymbol{D}}_{(i)}$:
   $\hat{\boldsymbol{D}}_{(i)} = [\boldsymbol{\mathcal{R}}_x]_{(4)} \left[[\hat{\boldsymbol{\mathcal{R}}}_s^{(i-1)}]_{(4)}(\hat{\boldsymbol{C}}_{(i)} \otimes \hat{\boldsymbol{B}}_{(i)} \otimes \hat{\boldsymbol{A}}_{(i)})^T \right]^\dagger$;
8. Construct the following Khatri-Rao products:
   $\hat{\boldsymbol{Z}}_{1(i)} = \hat{\boldsymbol{B}}_{(i)} \diamond \hat{\boldsymbol{A}}_{(i)}$; $\hat{\boldsymbol{Z}}_{2(i)} = \hat{\boldsymbol{D}}_{(i)} \diamond \hat{\boldsymbol{C}}_{(i)}$;
9. From the multimode covariance matrix $\overline{\boldsymbol{R}}_{\text{mm}}$ in (9), find an estimate of $\hat{\boldsymbol{\mathcal{R}}}_s^{(i)}$ from the following steps:
   $\hat{\boldsymbol{q}}_{(i)} = \left[\hat{\boldsymbol{Z}}_{2(i)} \otimes \hat{\boldsymbol{Z}}_{1(i)} \right]^\dagger \text{vec}(\overline{\boldsymbol{R}}_{\text{mm}})$;
   $\hat{\boldsymbol{Q}}_{(i)} = \text{unvec}_{PK\times PK}(\hat{\boldsymbol{q}}_{(i)})$;
   for $m = 1 : M$
   $\hat{\boldsymbol{\mathcal{R}}}_s^{(i)}(:,m,:,m) = \text{diag}(\hat{\boldsymbol{Q}}_{(i)}(:,m))$;
   end
10. From $\hat{\boldsymbol{\mathcal{R}}}_s^{(i)}$ obtain the unfolded matrices $[\hat{\boldsymbol{\mathcal{R}}}_s^{(i)}]_{(1)}$, $[\hat{\boldsymbol{\mathcal{R}}}_s^{(i)}]_{(2)}, [\hat{\boldsymbol{\mathcal{R}}}_s^{(i)}]_{(3)}, [\hat{\boldsymbol{\mathcal{R}}}_s^{(i)}]_{(4)}$;
11. Repeat steps 3-10 until convergence;

Let $e_{(i)} \triangleq \|\boldsymbol{\mathcal{R}}_x - \hat{\boldsymbol{\mathcal{R}}}_x^{(i)}\|_F^2$ be the residual error at the i-th iteration, obtained from the original and reconstructed covariance tensors. Convergence of the algorithm at the i-th iteration is declared when $|e_{(i)} - e_{(i-1)}| \leq 10^{-6}$. After convergence, the final estimate of the spatial signature matrix is given by averaging over the 1-mode and 3-mode factor matrices:

$$\hat{\boldsymbol{A}}_{\text{final}} = \frac{\hat{\boldsymbol{A}} + \hat{\boldsymbol{C}}^*}{2}.$$

Remark: Note that $\hat{\boldsymbol{A}}$ can be obtained with good accuracy (comparable to the proposed method) by fitting a PARAFAC model to the received data tensor according to (5). However, for a large number N of snapshots the proposed method is preferable since its computational complexity is independent of N.

4. NUMERICAL RESULTS

We present some computer simulation results for performance evaluation. First, we compare the performance of the proposed method using the ALS-Tucker4 algorithm with that of the ALS-PARAFAC algorithm proposed in [8]. Note that the latter method imposes a (known) diagonal structure on sources' covariance matrices. Our aim is to show the effectiveness of the proposed tensor method to work with actual sample covariance matrices computed over a finite (sometimes small) number of snapshots. Comparisons with classical MUSIC and ESPRIT methods, and with the Cramér-Rao lower bound (CRLB) of the classical model [19] are also shown as a reference. We assume BPSK modulation for all sources and the results represent an average over 1000 Monte Carlo runs. The normalized mean square error (NMSE) of the estimated spatial signature matrix is shown as a function of the signal to noise ratio (SNR). A uniform linear array (ULA) is considered at the receiver.

Figure 2 depicts the NMSE performance for different values of N_s, for a scenario with $M = 4$, $K = 3$ and $P = 25$. The sources angle of arrival are $\theta_1 = 21.2°$, $\theta_2 = 37.5°$, $\theta_3 = 56.3°$ and $\theta_4 = 77.4°$. In this experiment, the power loading matrix is assumed to be known. It can be seen that the ALS-Tucker4 method outperforms the ALS-PARAFAC one in all cases. The floor exhibited by the ALS-PARAFAC method is directly related to the modeling errors due to the assumption of a perfectly diagonal sources' covariance matrix. Therefore, when working with actual sample covariances the proposed method is preferable. Figure 3 shows the root mean square error (RMSE) of the estimated directions of arrival (extracted from the spatial signature estimates), for a scenario with $M = 2$, $K = 6$, $N_s = 40$ and $P = 25$. The sources angle of arrival are $\theta_1 = 28.2°$, $\theta_2 = 54.3°$. Note that the proposed method presents an improved performance over MUSIC and ESPRIT methods working under the same conditions, while being closer to the CRLB.

In Figure 4, we consider a more challenging scenario with a smaller number of sensors ($K = 4$), compared to the previous experiment. Additionally, we also show the results obtained when the power loading matrix is assumed to be unknown. Some degradation is observed when the power loading matrix is unknown, especially for medium-to-high SNR values in the simulated range. Nevertheless, our approach still presents the best performance, being more attractive than the competing methods in the low SNR range.

5. CONCLUSIONS

A fourth-order tensor-based method has been proposed for the blind estimation of spatial signatures, by exploiting a Tucker4 decomposition of the spatial-temporal covariance tensor. The proposed method does not require any knowledge on the propagation channel, array manifold and sources' covariance structure. Although the first two properties are also shared by existing blind methods, the distinguishing feature of our approach is its efficiency in treating the case where the sources' covariance matrix is unknown and possibly non-diagonal, which is the case when working with actual sample data covariances computed from a finite number of snapshots. Compared with classical matrix methods (MUSIC, ESPRIT), the gains of the proposed tensor method are more evidenced when working with a smaller number of sensors and under lower SNRs.

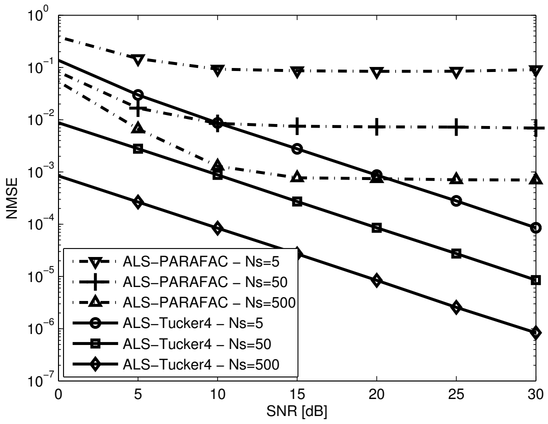

Fig. 2. NMSE of $\hat{\boldsymbol{A}}_{\text{final}}$ vs. SNR for $M = 4$, $K = 3$ and $P = 25$.

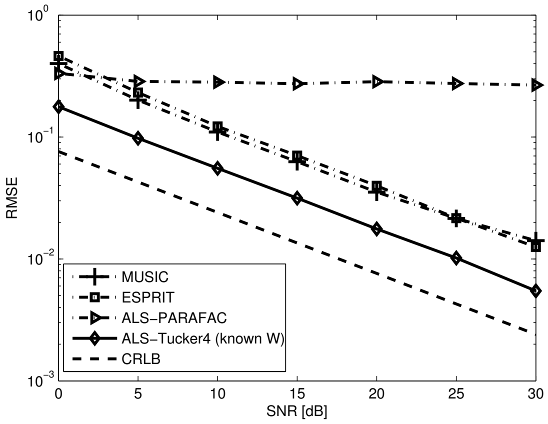

Fig. 3. RMSE vs. SNR for $M = 2$, $K = 6$, $N_s = 40$ and $P = 25$.

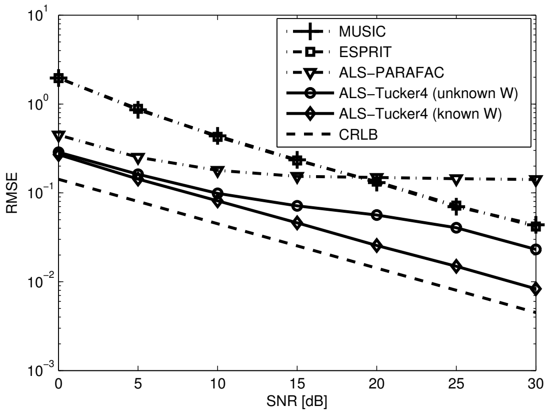

Fig. 4. RMSE vs. SNR for $M = 2$, $K = 4$, $N_s = 40$ and $P = 20$.

6. REFERENCES

[1] J. H. Winters, "Smart antennas for wireless systems," *IEEE Personal Communications*, vol. 5, no. 1, pp. 23–27, Feb. 1998.

[2] H. Krim and M. Viberg, "Two decades of array signal processing research: The parametric approach," *IEEE Sig. Process. Mag.*, vol. 13, no. 4, pp. 67–94, July 1996.

[3] J. -F. Cardoso and A. Souloumiac, "Blind beamforming for non-Gaussian signals," *Proc. Inst. Elect. Eng. F*, vol. 140, no. 6, p. 362–370, Dec. 1993.

[4] A. Belouchrani, K. Abed-Meraim, J. -F. Cardoso and E. Moulines, "A blind source separation technique using second-order statistics," *IEEE Trans. Signal Process.*, vol. 45, no. 2, p. 434–444, Feb. 1997.

[5] D. Astèly, A. L. Swindlehurst and B. Ottersten, "Spatial signature estimation for uniform linear arrays with unknown receiver gains and phases," *IEEE Trans. Signal Process.*, vol. 47, no. 8, p. 2128–2138, Aug. 1999.

[6] N. D. Sidiropoulos, R. Bro, and G. B. Giannakis, "Parallel factor analysis in sensor array processing," *IEEE Trans. Signal Process.*, vol. 48, no. 8, p. 2377–2388, Aug. 2000.

[7] N.D. Sidiropoulos, X. Liu, "Identifiability results for blind beamforming in incoherent multipath with small delay spread,", *IEEE Trans. Signal Process.* vol. 49, no. 1, pp. 228–236, Jan. 2001.

[8] Y. Rong, S. A. Vorobyov, A. B. Gershman, and N. D. Sidiropoulos, "Blind spatial signature estimation via time-varying user power loading and parallel factor analysis," *IEEE Trans. Signal Process.*, vol. 53, no. 5, pp. 1697–1710, May 2005.

[9] J. P. C. L. da Costa, F. Roemer, M. Weis, and M. Haardt, "Robust R-D parameter estimation via Closed-Form PARAFAC," In Proc. ITG Workshop on Smart Antennas (WSA'10), Feb. 2010, Bremen, Germany.

[10] A. L. F. de Almeida, G. Favier, and J. C. M. Mota, "Parafac-based unified tensor modeling for wireless communication systems with application to blind multiuser equalization," *Signal Processing*, vol. 87, pp. 337–351, 2007.

[11] A. L. F. Almeida, G. Favier, and L. R. Ximenes, "Space-time-frequency (STF) MIMO communication systems with blind receiver based on a generalized PARATUCK2 model," *IEEE Trans. Signal Process.*, vol. 61, no. 8, pp. 1895–1909, Apr. 2013.

[12] K. Liu, J. P. C. L. da Costa, A. L. F. de Almeida, and H. C. So, "Semi-blind receivers for joint symbol and channel estimation in space-time-frequency MIMO-OFDM systems," *IEEE Trans. Signal Process.*, vol. 61, no. 21, pp.5444–5457, Nov. 2013.

[13] J. P. C. L. da Costa, F. Roemer, M. Haardt, and R. T. de Souza Jr., "Multi-dimensional model order selection," *EURASIP Journal on Advances in Signal Processing*, vol. 2011, no. 1, pp. 1–13, 2011.

[14] F. Roemer, *Advanced algebraic concepts for eficient multi-channel signal processing*. PhD thesis, Ilmenau University of Technology, Ilmenau, 2013.

[15] R. Harshman, "Foundations of the parafac procedure: models and conditions for an "explanatory" multimodal factor analysis," *UCLA Working Papers in Phonetics*, vol. 16, no. 1, 1970.

[16] L. de Lathauwer and B. de Moor and J. Vanderwalle, *A multilinear singular value decomposition*. SIAM J. Matrix Anal. Appl., vol. 21, no. 4, pp. 1253–1278, 2000.

[17] R. Bro, *Multi-way analysis in the food industry: models, algorithms and applications*. PhD thesis, University Denmark, 1998.

[18] T. G. Kolda and B. W. Bader, "Tensor decompositions and applications," *Society for Industrial and Applied Mathematics*, vol. 51, no. 3, p. 45, 2009.

[19] S. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall, New Jersey, 1993.