MULTICHANNEL DETECTION OF AN UNKNOWN RANK-ONE SIGNAL WITH UNCALIBRATED RECEIVERS

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ABSTRACT

This paper addresses the problem of detecting an unknown rank-one signal using multiple receivers that are uncalibrated in the sense that they each apply an unknown scaling to the received signal, and their respective noise powers are unknown. This problem has been addressed for the case in which the unknown signal can be modeled as a Gaussian random vector. However, that assumption is not applicable to some signal types, such as the constant modulus signals found in radar and communications. For these problems, the signal can be modeled as a deterministic unknown, which is the approach taken here. We derive a generalized likelihood ratio test for this problem under a low signal-to-noise ratio (SNR) assumption. The resulting detector is invariant to relative scalings of the data, and therefore possesses the constant false alarm rate (CFAR) property with respect to the unknown noise powers. Numerical examples show the proposed detector can outperform CFAR detectors derived under the Gaussian assumption.

Index Terms— CFAR Detection, Generalized Likelihood Ratio Test, Multichannel Signal Detection, Rank-One Signal Detection, Noise Power Uncertainty

1. INTRODUCTION

The detection of a common but unknown signal within multiple receiver channels is a problem that arises in many applications, such as passive source localization, spectrum sensing, and passive radar [1–7]. In each case, the measurement model for the j^{th} receiver can be written as

$$\mathbf{s}_j = \mu_j \,\mathbf{u} + \mathbf{n}_j, \tag{1}$$

where $\mathbf{s}_j \in \mathbb{C}^{L\times 1}$ is the observation, μ_j is a complex coefficient representing channel and receiver effects, and $\mathbf{u} \in \mathbb{C}^{L\times 1}$ is the unknown signal. The noise vector $\mathbf{n}_j$ is assumed to be a realization of a zero-mean circular complex Gaussian random process that is independent across receivers. Note that in some applications the model in (1) is obtained only after applying a unitary linear transform to align the received data (e.g., in delay and Doppler) [6, 7].

When the unknown signal can be modeled as a zero-mean circular complex Gaussian random vector, detection can be performed by discriminating between the presence or absence of correlation between measured signals [1–4]. The resulting detectors are defined in terms of the *coherence matrix*, $\widehat{\mathbf{C}} \in \mathbb{C}^{N_r\times N_r}$, where N_r is the number of receivers, and the jk^{th} element of $\widehat{\mathbf{C}}$ is the sample correlation coefficient between the j^{th} and k^{th} measured signals, i.e.,

$$\big[\widehat{\mathbf{C}}\big]_{jk} = \frac{\mathbf{s}_j^H \mathbf{s}_k}{\|\mathbf{s}_j\|\|\mathbf{s}_k\|}. \tag{2}$$

Leshem *et al.* showed that $|\widehat{\mathbf{C}}|$, i.e., the determinant of $\widehat{\mathbf{C}}$, is the generalized likelihood ratio test (GLRT) for detection of temporally-uncorrelated Gaussian signals with arbitrary non-diagonal spatial covariance [2]. We note that Cochran *et al.* had previously used a geometric argument to formulate an equivalent statistic referred to as *generalized coherence* [1]. When the signal subspace is rank-one, the spatial covariance is diagonal-plus-rank-one, and this additional structure can be exploited to improve detection performance. Lopez-Valcarce *et al.* considered this problem under low signal-to-noise ratio (SNR) conditions. The resulting GLRT was shown to be $\|\widehat{\mathbf{C}}\|_2$, i.e., the spectral norm of $\widehat{\mathbf{C}}$ [3]. Ramirez *et al.* considered the same problem under close hypotheses conditions, i.e., low-SNR and low sample support [4]. They showed that, irrespective of signal rank, the locally-most powerful invariant test (LMPIT) is $\|\widehat{\mathbf{C}}\|_F$, i.e., the Frobenius norm of $\widehat{\mathbf{C}}$. All three statistics, i.e., $|\widehat{\mathbf{C}}|$, $\|\widehat{\mathbf{C}}\|_2$, and $\|\widehat{\mathbf{C}}\|_F$, are invariant to arbitrary scaling of each receiver channel. Consequently, they are constant false alarm rate (CFAR) with respect to unknown and possibly unequal receiver noise powers.

The Gaussian signal assumption on which the previous detectors are based is a good assumption for many signal types. However, it is not a suitable approximation for some signal types, such as the constant modulus signals commonly encountered in radar and communications applications. In such cases, it is appropriate to model the signal as a deterministic unknown. Besson *et al.* [5] considered this problem when the noise powers are *equal* but unknown. The result-

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ing GLRT was shown to be $\|\mathbf{G}\|_2/\mathrm{tr}(\mathbf{G})$, where $\mathrm{tr}(\cdot)$ denotes the trace, and $\mathbf{G} \in \mathbb{C}^{N_r \times N_r}$ is the Gram matrix whose jk^{th} element is given by $\left[\mathbf{G}\right]_{jk} = \frac{1}{L}\,\mathbf{s}_j^H \mathbf{s}_k$. No comparable result exists for the case in which the unknown noise powers are *unequal*, which is frequenly encountered in practice.

This paper addresses CFAR detection of an unknown rank-one signal using multiple receivers with unknown-unequal noise powers. The GLRT is derived under a low SNR assumption. Comparison to other known GLRTs suggests that this detector should perform well at all SNRs, and numerical examples support this conclusion. Numerical examples also demonstrate that the proposed detector can outperform other CFAR detectors at detecting constant modulus signals in unknown-unequal noise power scenarios.

2. DERIVATION

Detection may be formulated as a binary hypothesis test between alternative ($\mathcal{H}_1$) and null ($\mathcal{H}_0$) hypotheses:

$$\begin{aligned} \mathcal{H}_1: &\quad \mathbf{s}_j = \mu_j\,\mathbf{u} + \mathbf{n}_j \\ \mathcal{H}_0: &\quad \mathbf{s}_j = \mathbf{n}_j \end{aligned} \tag{3}$$

for $j = 1 \ldots N_r$. Let $\mathbf{s} = [\mathbf{s}_1^T \ldots \mathbf{s}_{N_r}^T]^T \in \mathbb{C}^{N_r L}$ denote the vector of concatenated receiver measurements. Also, let $\boldsymbol{\mu} = [\mu_1 \ldots \mu_{N_r}]^T \in \mathbb{C}^{N_r}$ and $\boldsymbol{\sigma} = [\sigma_1 \ldots \sigma_{N_r}]^T \in \mathbb{R}^{N_r}$, where σ_j^2 is the noise power associated with the j^{th} receiver. The transmit signal $\mathbf{u}$ and channel coefficients $\boldsymbol{\mu}$ are considered deterministic unknowns. Consequently, assuming independence of the receiver noise across receivers, the conditional density of $\mathbf{s}$ under $\mathcal{H}_1$ is

$$p_1(\mathbf{s}|\mathbf{u},\boldsymbol{\mu},\boldsymbol{\sigma}) = \frac{\exp\left\{-\sum_{j=1}^{N_r} \frac{1}{\sigma_j^2}\|\mathbf{s}_j - \mu_j \mathbf{u}\|^2\right\}}{\prod_{j=1}^{N_r}(\pi\sigma_j^2)^L}. \tag{4}$$

The conditional density of $\mathbf{s}$ under $\mathcal{H}_0$, $p_0(\mathbf{s}|\boldsymbol{\sigma})$, is given by (4) for $\boldsymbol{\mu} = \mathbf{0}_{N_r}$, i.e., $p_0(\mathbf{s}|\boldsymbol{\sigma}) = p_1(\mathbf{s}|\mathbf{u},\mathbf{0}_{N_r},\boldsymbol{\sigma})$, where $\mathbf{0}_{N_r}$ denotes the length-N_r zero vector.

Define $\ell_1(\mathbf{u},\boldsymbol{\mu},\boldsymbol{\sigma}|\mathbf{s}) \triangleq \log p_1(\mathbf{s}|\boldsymbol{\mu},\mathbf{u},\boldsymbol{\sigma})$ and $\ell_0(\boldsymbol{\sigma}|\mathbf{s}) \triangleq \log p_0(\mathbf{s}|\boldsymbol{\sigma})$ so that the log generalized likelihood ratio (GLR) can be written as

$$\max_{\{\mathbf{u},\boldsymbol{\mu},\boldsymbol{\sigma}\}} \ell_1(\mathbf{u},\boldsymbol{\mu},\boldsymbol{\sigma}|\mathbf{s}) - \max_{\{\boldsymbol{\sigma}\}} \ell_0(\boldsymbol{\sigma}|\mathbf{s}). \tag{5}$$

To maximize the second term in (5), note that

$$\ell_0(\boldsymbol{\sigma}|\mathbf{s}) = -L\sum_{j=1}^{N_r}\log(\pi\sigma_j^2) - \sum_{j=1}^{N_r}\tfrac{1}{\sigma_j^2}\|\mathbf{s}_j\|^2. \tag{6}$$

Setting the partial derivative with respect to σ_j^2 equal to zero and solving for σ_j^2 yields the maximum likelihood estimate (MLE) of σ_j^2 under $\mathcal{H}_0$, i.e.,

$$\hat{\sigma}_j^2 = \tfrac{1}{L}\|\mathbf{s}_j\|^2. \tag{7}$$

Substituting (7) for σ_j^2 in (6), and simplifying, gives

$$\ell_0(\hat{\boldsymbol{\sigma}}|\mathbf{s}) = -N_r L - L\sum_{j=1}^{N_r}\log\left(\pi\|\mathbf{s}_j\|^2/L\right) \triangleq c_0. \tag{8}$$

To maximize the first term in (5), note that

$$\ell_1(\mathbf{u},\boldsymbol{\mu},\boldsymbol{\sigma}|\mathbf{s}) = -L\sum_{j=1}^{N_r}\log(\pi\sigma_j^2) - \sum_{j=1}^{N_r}\tfrac{1}{\sigma_j^2}\|\mathbf{s}_j - \mu_j\mathbf{u}\|^2. \tag{9}$$

The maximum likelihood estimate of μ_j is the least-squares solution to $\mathbf{u}\mu_j = \mathbf{s}_j$, which is

$$\hat{\mu}_j = \frac{\mathbf{u}^H\mathbf{s}_j}{\|\mathbf{u}\|^2}. \tag{10}$$

Substituting (10) for μ_j in (9), and simplifying, gives

$$\ell_1(\mathbf{u},\hat{\boldsymbol{\mu}},\boldsymbol{\sigma}|\mathbf{s}) = -L\sum_{j=1}^{N_r}\log(\pi\sigma_j^2) - \sum_{j=1}^{N_r}\tfrac{1}{\sigma_j^2}\|\mathbf{P}_{\mathbf{u}}^{\perp}\mathbf{s}_j\|^2, \tag{11}$$

where $\mathbf{P}_{\mathbf{u}}^{\perp} = \mathbf{I} - \frac{\mathbf{u}\mathbf{u}^H}{\|\mathbf{u}\|^2}$ is the projection matrix into the orthogonal complement of $\mathbf{u}$. Setting the partial derivative of (11) with respect to σ_j^2 equal to zero, and solving for σ_j^2, yields the MLE

$$\hat{\sigma}_j^2 = \tfrac{1}{L}\|\mathbf{P}_{\mathbf{u}}^{\perp}\mathbf{s}_j\|^2. \tag{12}$$

Substitution of (12) into (11) leads to an intractable maximization problem over $\mathbf{u}$. However, under low-SNR conditions the majority of the signal energy falls outside the signal subspace, i.e., $\|\mathbf{P}_{\mathbf{u}}^{\perp}\mathbf{s}_j\|^2 \approx \|\mathbf{s}_j\|^2$. In this case, (11) is well approximated by (6), and

$$\hat{\sigma}_j^2 \approx \tfrac{1}{L}\|\mathbf{s}_j\|^2, \tag{13}$$

which is equivalent to the MLE of σ_j^2 under $\mathcal{H}_0$ in (7). Substituting (13) into (11) and simplifying gives

$$\ell_1(\mathbf{u},\hat{\boldsymbol{\mu}},\hat{\boldsymbol{\sigma}}|\mathbf{s}) = c_0 + \frac{\mathbf{u}^H\widehat{\Phi}\widehat{\Phi}^H\mathbf{u}}{\|\mathbf{u}\|^2}, \tag{14}$$

where $\widehat{\Phi} = \Phi\widehat{\boldsymbol{\Sigma}}^{-1} \in \mathbb{C}^{L\times N_r}$, $\Phi = [\mathbf{s}_1 \ldots \mathbf{s}_{N_r}] \in \mathbb{C}^{L\times N_r}$, $\widehat{\boldsymbol{\Sigma}} = \mathrm{diag}(\hat{\boldsymbol{\sigma}}) \in \mathbb{C}^{N_r\times N_r}$ is the diagonal matrix formed from the elements of $\hat{\boldsymbol{\sigma}}$. Let $\boldsymbol{\gamma}_1(\cdot)$ denote the eigenvector associated with $\|\cdot\|_2$. Then, the Rayleigh quotient in (14) achieves its maximum value, $\|\widehat{\Phi}\widehat{\Phi}^H\|_2$, when $\mathbf{u} = \boldsymbol{\gamma}_1\big(\widehat{\Phi}\widehat{\Phi}^H\big)$. Therefore the MLE of $\mathbf{u}$ is $\hat{\mathbf{u}} = \boldsymbol{\gamma}_1\big(\widehat{\Phi}\widehat{\Phi}^H\big)$, and (14) becomes

$$\ell_1(\hat{\mathbf{u}},\hat{\boldsymbol{\mu}},\hat{\boldsymbol{\sigma}}|\mathbf{s}) = c_0 + \|\widehat{\Phi}\widehat{\Phi}^H\|_2. \tag{15}$$

Since $\|\widehat{\Phi}\widehat{\Phi}^H\|_2 = \|\widehat{\Phi}^H\widehat{\Phi}\|_2$, and expanding $\widehat{\Phi}$ as $\Phi\widehat{\boldsymbol{\Sigma}}^{-1}$, (15) may be expressed as

$$\ell_1(\hat{\mathbf{u}},\hat{\boldsymbol{\mu}},\hat{\boldsymbol{\sigma}}|\mathbf{s}) = c_0 + L\,\|\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{G}\widehat{\boldsymbol{\Sigma}}^{-1}\|_2, \tag{16}$$

where $\mathbf{G} = \frac{1}{L}\Phi^H\Phi \in \mathbb{C}^{N_r\times N_r}$. Substituting (8) and (16) into (5), the resulting low-SNR GLRT is

$$\|\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{G}\widehat{\boldsymbol{\Sigma}}^{-1}\|_2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \kappa_1, \tag{17}$$

where κ_1 is a suitably chosen threshold.

3. DISCUSSION

Fig. 1 summarizes existing GLRT detectors in relation to (17) according to their respective noise power assumptions. Note that σ^2 and Σ denote the true noise variance and covariance, respectively. The GLRT for the known-equal case is derived in [8] for the Gaussian signal model, and in [6] for the deterministic unknown signal model. Both can be extended to the known-unequal case by a straightforward application of whitening. The unknown-equal case was also derived in [8] for the Gaussian signal model, and in [5] for the deterministic unknown signal model. The unknown-unequal case is derived for low SNR conditions in [3] for the Gaussian signal model, and in (17) for the deterministic unknown signal model.

The similarity between all four test statistics is immediately apparent, i.e., each test statistic is the spectral norm of a normalized Gram matrix $\mathbf{G}$, where the normalization depends on the noise power assumptions. This normalization has the effect of whitening the received signals to unity noise power. Note that the jk^{th} element of $\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{G}\widehat{\boldsymbol{\Sigma}}^{-1}$ is given by

$$\left[\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{G}\widehat{\boldsymbol{\Sigma}}^{-1}\right]_{jk} = \frac{1}{L}\left(\frac{\mathbf{s}_j}{\hat{\sigma}_j}\right)^H\left(\frac{\mathbf{s}_k}{\hat{\sigma}_k}\right) = \frac{\mathbf{s}_j^H\mathbf{s}_k}{\|\mathbf{s}_j\|\|\mathbf{s}_k\|}. \quad (18)$$

This is equivalent to the jk^{th} element of the coherence matrix $\widehat{\mathbf{C}}$ in (2), which is fundamental to the CFAR detectors derived under the Gaussian signal assumption that were discussed in Sec. 1. The similarity in form between all four detectors suggests that (17) should perform comparatively well under both low and high SNR conditions.

Despite the fact that their test statistics take the same form, there are still significant differences between the GLRTs under deterministic unknown and Gaussian signal formulations. Since the distribution of a statistic depends on the underlying signal model, the distribution of a given statistic will *differ* under $\mathcal{H}_1$ between formulations. More precisely, under $\mathcal{H}_1$ and the zero-mean Gaussian signal model, the matrix $\mathbf{G}$ is a *central complex Wishart* matrix of order L, while under $\mathcal{H}_1$ and the deterministic unknown signal model, the matrix $\mathbf{G}$ is a *non-central complex Wishart* matrix of order L with a rank-one noncentrality. On the other hand, the distribution of a given statistic is the *same* under $\mathcal{H}_0$ for both formulations. More precisely, under $\mathcal{H}_0$ and both signal formulations, $\mathbf{G}$ is a central complex Wishart matrix. This implies the same threshold can be used for both formulations, as the threshold depends only on the statistic distribution under $\mathcal{H}_0$. However, the probability of detection curves will differ between formulations, as these depend on the statistic distribution under $\mathcal{H}_1$.

We also note that the LMPITs under close hypotheses of [4], which were derived assuming vector observations at each receiver and Gaussian received signals, reduce to the statistics shown in Fig. 2 for scalar observations. These are closely related to the CFAR GLRTs in Fig. 1, namely, they are identical except that they are expressed in terms of the Frobenius norm rather than the spectral norm. As discussed in [4], this reflects

	Equal ($\sigma_j=\sigma_k$)	Unequal ($\sigma_j\neq\sigma_k$)
$\boldsymbol{\sigma}$ Known	$\\|\frac{1}{\sigma^2}\mathbf{G}\\|_2$	$\\|\boldsymbol{\Sigma}^{-1}\mathbf{G}\boldsymbol{\Sigma}^{-1}\\|_2$
$\boldsymbol{\sigma}$ Unknown	$\\|\frac{1}{\hat{\sigma}^2}\mathbf{G}\\|_2$	$\\|\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{G}\widehat{\boldsymbol{\Sigma}}^{-1}\\|_2$

Fig. 1. Comparison of GLRT detectors under different noise cases for both the deterministic unknown and Gaussian signal model formulations.

	Equal ($\sigma_j=\sigma_k$)	Unequal ($\sigma_j\neq\sigma_k$)
$\boldsymbol{\sigma}$ Unknown	$\\|\frac{1}{\hat{\sigma}^2}\mathbf{G}\\|_F$	$\\|\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{G}\widehat{\boldsymbol{\Sigma}}^{-1}\\|_F$

Fig. 2. LMPIT detectors under close hypotheses (low-SNR and/or low sample support) for the Gaussian signal model [4].

a distinction in how each detector treats knowledge of the signal subspace rank, i.e., the spectral norm exploits knowledge of the rank-1 signal subspace, while the Frobenius norm ignores this knowledge because of difficulty in estimating the signal subspace under close hypotheses. In the following section, we compare the unknown-unequal LMPIT and GLRT detectors via numerical simulation, and comment on the desirability of utilizing the signal subspace rank when detecting non-Gaussian rank-one signals.

4. SIMULATIONS

This section compares the detection performance of the unknown-unequal GLRT, $\|\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{G}\widehat{\boldsymbol{\Sigma}}^{-1}\|_2$, with the performance of other detectors that are CFAR with respect to unknown-unequal noises and derived under the Gaussian signal assumption, namely, generalized coherence $|\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{G}\widehat{\boldsymbol{\Sigma}}^{-1}|$, and the LMPIT $\|\widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{G}\widehat{\boldsymbol{\Sigma}}^{-1}\|_F$. For compactness, the following discussion uses coherence matrix notation, i.e., $\mathbf{C} = \boldsymbol{\Sigma}^{-1}\mathbf{G}\boldsymbol{\Sigma}^{-1}$ and $\widehat{\mathbf{C}} = \widehat{\boldsymbol{\Sigma}}^{-1}\mathbf{G}\widehat{\boldsymbol{\Sigma}}^{-1}$. A scenario with $N_r = 6$ distributed receivers and varying length received signals is considered. A constant-modulus signal $\mathbf{u}$ is randomly chosen on each $\mathcal{H}_1$ trial according to $\mathbf{u} = \exp\{i\boldsymbol{\theta}\}$, where $\boldsymbol{\theta} \in \mathbb{R}^{L\times 1}$ is a phase vector with i.i.d. elements uniformly distributed on $[0, 2\pi]$, such that $\|\mathbf{u}\|^2 = L$. This signal is chosen because it is non-Gaussian and resembles phase modulated radar and communications waveforms. The coefficient vector $\boldsymbol{\mu}$ is also chosen randomly on each $\mathcal{H}_1$ trial from a circular Gaussian distribution, $\boldsymbol{\mu} \sim \mathcal{CN}(\mathbf{0}_{N_r}, \mathbf{I}_{N_r})$, and scaled to achieve a desired SNR_{avg}, defined by

$$\text{SNR}_{\text{avg}} = \frac{1}{N_r}\sum_{j=1}^{N_r}\text{SNR}_j, \quad (19)$$

where $\text{SNR}_j = |\mu_j|^2/\sigma_j^2$ is the input SNR at the j^{th} receiver. Detection thresholds are set empirically using 10^5 Monte

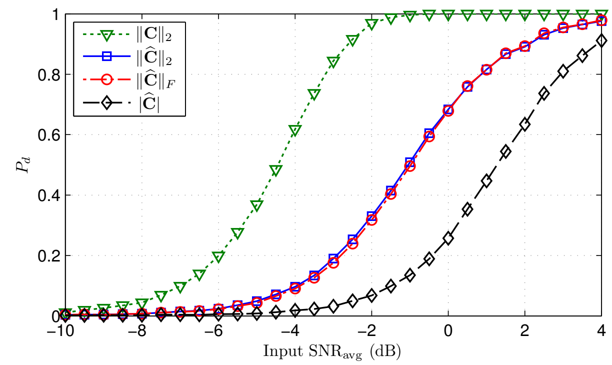
Carlo trials under $\mathcal{H}_0$ to achieve a false alarm probability $P_{fa} = 10^{-3}$. Note that these $\mathcal{H}_0$ Monte Carlo trials may be performed for each CFAR statistic using $\boldsymbol{\sigma} = \mathbf{1}_{N_r}$ due to the invariance of these statistics to independent scaling of each received signal. Probability of detection (P_d) curves are calculated using 10^4 Monte Carlo trials for each value of SNR_{avg}. Finally, unequal receiver noise powers are chosen as $[\sigma_1^2, \sigma_2^2, \ldots, \sigma_6^2] = [1,\ 2,\ 3,\ 4,\ 5,\ 6] \times 10^{-6}$.

Fig. 3 depicts P_d curves for the CFAR detectors as the signal length L is varied over $L = \{10, 100, 1000\}$. For comparison, the performance of the clairvoyant known-unequal GLRT, $\|\mathbf{C}\|_2$, is also depicted. The separation between $\|\mathbf{C}\|_2$ and each CFAR detector represents CFAR loss resulting from noise power uncertainty. This loss decreases with increasing signal length L, which reflects the ability of each detector to better estimate the unknown noise powers at low SNRs with more measurement data. Among CFAR detectors, the unknown-unequal GLRT, $\|\hat{\mathbf{C}}_u\|_2$, is more sensitive than both the Gaussian LMPIT, $\|\hat{\mathbf{C}}_u\|_F$, and generalized coherence, $|\hat{\mathbf{C}}_u|$, for all considered signal lengths, although the relative separation of $|\hat{\mathbf{C}}_u|$ from the other two decreases with increasing signal length. In addition, the sensitivity advantage of $\|\hat{\mathbf{C}}_u\|_2$ over $\|\hat{\mathbf{C}}_u\|_F$, though modest, increases with increasing L.

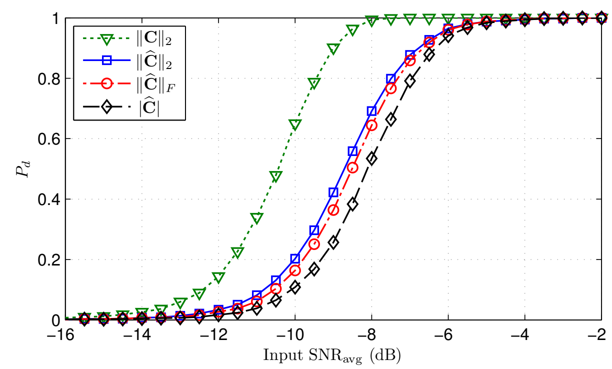
These results are interesting in light of the fact that only $\|\hat{\mathbf{C}}_u\|_2$ exploits knowledge of the underlying rank-one signal subspace. Consequently, we conclude that exploiting this knowledge is beneficial in the detection of rank-one deterministic unknown signals under all evaluated signal lengths and all SNRs that have non-negligible P_d. Nonetheless, the relative performance of $\|\hat{\mathbf{C}}_u\|_2$ and $\|\hat{\mathbf{C}}_u\|_F$ is still consistent with the conjecture that $\|\hat{\mathbf{C}}_u\|_F$ is also the LMPIT under close hypotheses for deterministic unknown signals. Indeed, this conjectures seems plausible because of the mirror relationship between GLRTs derived under both signal model formulations, and the fact that the distinction between distributions under $\mathcal{H}_1$ diminishes with decreasing signal strength.

5. CONCLUSION

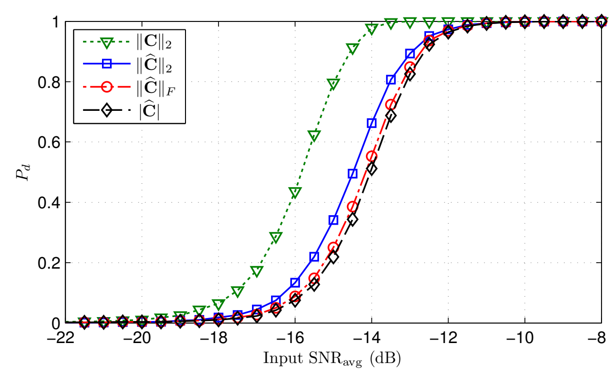
In this paper, we proposed a CFAR detector for the detection of a deterministic unknown signal using multiple uncalibrated receivers with unknown-unequal receiver noise powers. This detector is the GLRT under low-SNR conditions, and has a form that is consistent with GLRTs derived under other noise power formulations. We also identified the relationship between GLRTs derived under the deterministic unknown and Gaussian signal assumptions, which have identical forms but different probability distributions (and consequently P_d performance) under the target-present hypothesis. The proposed detector outperformed other CFAR detectors derived for the Gaussian signal case, including the LMPIT, at all practical signal lengths and SNRs for an example scenario. Derivation of a closed form GLRT under all SNR conditions or a LMPIT under close hypotheses for an unknown signal with unknown-unequal receiver noise powers remain open problems.



(a) L = 10



(b) L = 100



(c) L = 1000

Fig. 3. Probability of detection comparisons for different L.

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7. REFERENCES

[1] D. Cochran, H. Gish, and D. Sinno, "A geometric approach to multiple-channel signal detection," *IEEE Trans. Signal Process.*, vol. 43, no. 9, pp. 2049–2057, 1995.

[2] A. Leshem and A. Van der Veen, "Multichannel detection of Gaussian signals with uncalibrated receivers," *IEEE Signal Process. Lett.*, vol. 8, no. 4, pp. 120–122, 2001.

[3] R. Lopez-Valcarce, G. Vazquez-Vilar, and J. Sala, "Multiantenna spectrum sensing for cognitive radio: overcoming noise uncertainty," in *Cognitive Information Processing (CIP), 2010 2nd International Workshop on*, 2010, pp. 310–315.

[4] D. Ramirez, J. Via, I. Santamaria, and L. Scharf, "Locally most powerful invariant tests for correlation and sphericity of Gaussian vectors," *IEEE Trans. Inf. Theory*, vol. 59, no. 4, pp. 2128–2141, 2013.

[5] O. Besson, S. Kraut, and L. Scharf, "Detection of an unknown rank-one component in white noise," *IEEE Trans. Signal Process.*, vol. 54, no. 7, pp. 2835–2839, 2006.

[6] Naresh Vankayalapati and Steven Kay, "Asymptotically optimal detection of low probability of intercept signals using distributed sensors," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 48, no. 1, pp. 737–748, 2012.

[7] D. Hack, L. Patton, B. Himed, and M. Saville, "Centralized passive MIMO radar detection without direct-path reference signals," *IEEE Trans. Signal Process.*, (submitted).

[8] A. Taherpour, M. Nasiri-Kenari, and S. Gazor, "Multiple antenna spectrum sensing in cognitive radios," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 814–823, 2010.