

JOINT FREQUENCY AND PHASOR ESTIMATION IN UNBALANCED THREE-PHASE POWER SYSTEMS

Tirza Routtenberg and Lang Tong

School of Electrical and Computer Engineering
Cornell University
Ithaca, NY 14850, New York

ABSTRACT

The problem of joint off-nominal frequency and phasor estimation in an unbalanced three-phase power system using a phasor measurement unit (PMU) is considered. Voltage unbalance arises when magnitudes are different from each other or when phases are unequally spaced. A general model for the zero, positive, and negative sequences from a PMU measurement at off-nominal frequencies is presented. Then, the maximum likelihood (ML) estimator is developed under an unbalanced condition for joint symmetrical components phasors and frequency estimation. Finally, a low complexity technique is developed for the estimation of frequency deviation based a new autoregressive (AR) representation of the innovation process for the symmetrical component.

Index Terms— Frequency estimation, phasor measurement unit (PMU), unbalanced power system, symmetrical components, Smart Grid

1. INTRODUCTION

The three-phase power system is designed to operate at a nominal frequency in a near-balanced fashion [1], [2]. In practice, frequency deviation and load imbalance are the norm rather than the exception. In most situations, frequency deviations and minor imbalances can be mitigated by frequency regulation or load compensation techniques [3]. However, situations sometimes arise when frequency deviations and imbalances may be a precursor to more serious contingencies leading to possible blackouts [4], [5]. Under perfectly balanced three-phase operating conditions, the zero and negative sequences are absent; thus, the state-estimation and signal analysis for this case are carried out using only the positive-sequence model [4], [6]. When system imbalance occurs, the zero and negative sequences are nonzero, the PMU's output does not represent the true positive-sequence phasors [5], [7], and the positive-sequence have a non-circular nature, as described in [8], [9]. Therefore, the existing balanced state estimation methods do not represent the optimal state estimation based on the three symmetrical components. It is in this context that modern sensing devices, such as phasor measurement units (PMUs), have the potential to provide rapid detection of contingencies and situation awareness based on the three symmetrical components (see [1] and references therein). To this end, effective algorithms are crucial for estimating and tracking frequency deviations and phasors in the event of system imbalance, that are based on the zero, positive, and negative sequences.

This research was partially supported by the National Science Foundation under Grant no. CNS-1135844

1.1. Summary of results

In this paper, we consider joint frequency and phasor estimation in unbalanced three-phase power systems using the PMU output. We demonstrate that for a perfectly balanced power system operating at off-nominal frequency, the PMU output is a single complex sinusoid so that its frequency and complex amplitude can be estimated by using well-known signal processing methods (e.g. [10], [11], [12]). However, under unbalanced conditions the PMU output is no longer a single frequency signal. Instead, the symmetrical components at the PMU output have two related frequencies and a specific structure. We develop the maximum-likelihood (ML) estimators of the frequency-deviation and three symmetrical components- zero, positive, and negative, phasors - for unbalanced three-phase power systems model. A suboptimal estimation method of the frequency deviation is derived by using the weighted least-squares (WLS) method on the autoregressive (AR) representation of the symmetrical components' innovation processes. Finally, the performance of the proposed estimators is tested via simulations.

1.2. Related works

Currently, many different aspects of state estimation for balanced systems in the context of smart grid are investigated [13], such as distributed and robust estimation [14], the influence of malicious data attack [15], imperfect synchronization [16], and Kirchhoff's Current Law (KCL) [17] on the state estimation based on PMU measurements. The mismatch estimation error caused by using the balanced state estimation under unbalanced conditions is studied in [4]. In the pioneering works of [8] and [9], new methods were derived for frequency-estimation based on non-circular models and the Clarkes transformation. These methods use only the positive-sequence and analyze the measurements in the time-domain, whereas the proposed model uses three symmetrical sequences, measurement in the frequency-domain of the PMU output, and phasor estimation for the three symmetrical components. In [18] a distribution system state estimator suitable for monitoring unbalanced distribution networks is presented. In [19], a generalized likelihood ratio test (GLRT) is developed to detect voltage and phase unbalance based on three-phase time-measurements. The fault detection problem and its corresponding GLRT are discussed in [20].

2. MEASUREMENT MODEL

The system and measurement models considered here are based on existing models (e.g. [1, 7]). In this section we present the model in a statistical signal processing formulation that includes a description of the noise statistics, and is more convenient for developing estimation and detection algorithms [21]. In particular, we describe

the statistical behavior of the PMU output signals, i.e. after the sampling, symmetrical transformation [22], and nominal-frequency discrete Fourier transform (DFT) operation.

The PMU constructs the complex representation phasors of the signals by using a DFT operator over one cycle of the nominal-frequency [1], [7]. That is, the PMU DFT operation on any arbitrary signal $x[n]$ results in the following phasor sequence:

$$X[k] = \frac{\sqrt{2}}{N} \sum_{n=k}^{k+N-1} x[n]e^{-j\gamma n}, \quad k = 0, \dots, K-1, \quad (1)$$

where the index k refers to the beginning of the DFT window.

The voltages in a three-phase power system are assumed to be pure sinusoidal signals of frequency $\omega_0 + \Delta$, where ω_0 is the known nominal-frequency (100π or 120π) and Δ is the frequency-deviation from this nominal value. The magnitudes and phases of the three voltages are denoted by $V_a, V_b, V_c \geq 0$ and $\varphi_a, \varphi_b, \varphi_c \in [0, 2\pi]$, respectively. The three-phase power system is called balanced if $V_a = V_b = V_c$ and $\varphi_a = \varphi_b + \frac{2\pi}{3} = \varphi_c - \frac{2\pi}{3}$. The PMU samples these real signals N times per cycle of the nominal-frequency, ω_0 , to produce the following discrete-time, noisy measurements model:

$$\begin{bmatrix} v_a[n] \\ v_b[n] \\ v_c[n] \end{bmatrix} = \frac{1}{2}e^{j\gamma\frac{\omega_0+\Delta}{\omega_0}n} \mathbf{v} + \frac{1}{2}e^{-j\gamma\frac{\omega_0+\Delta}{\omega_0}n} \mathbf{v}^* + \mathbf{w}_{a,b,c}[n], \quad (2)$$

$\forall n \in \mathbb{R}$, where $\gamma \triangleq \frac{2\pi}{N}$ and $\mathbf{v} \triangleq [V_a e^{j\varphi_a}, V_b e^{j\varphi_b}, V_c e^{j\varphi_c}]^T$. The noise sequence, $\{\mathbf{w}_{a,b,c}[n]\}_{n \in \mathbb{Z}}$, is assumed to be a real white Gaussian noise sequences with a known covariance matrix $\sigma^2 \mathbf{I}_3$, in which \mathbf{I}_3 is the 3×3 identity matrix. The three-phase power system is called balanced or symmetrical if $V_a = V_b = V_c$ and $\varphi_a = \varphi_b + \frac{2\pi}{3} = \varphi_c - \frac{2\pi}{3}$.

By substituting the three sequences, $\{v_a[n]\}$, $\{v_b[n]\}$, and $\{v_c[n]\}$, from (2) in (1) and using the identity [23]:

$$\sum_{n=k}^{k+N-1} e^{j\alpha n} = \frac{\sin(\alpha N/2)}{\sin(\alpha/2)} e^{j\alpha(k + \frac{N-1}{2})}, \quad \forall \alpha \in \mathbb{R},$$

we obtain the following phasor sequences measurements:

$$\begin{bmatrix} V_a[k] \\ V_b[k] \\ V_c[k] \end{bmatrix} = \frac{1}{\sqrt{2}} P e^{j\gamma\frac{\Delta}{\omega_0}k} \mathbf{v} + \frac{1}{\sqrt{2}} Q e^{-j\gamma\frac{2\omega_0+\Delta}{\omega_0}k} \mathbf{v}^* + \frac{\sqrt{2}}{N} \sum_{n=k}^{k+N-1} \mathbf{w}_{a,b,c}[n] e^{-j\gamma n}, \quad (3)$$

$k = 0, \dots, K-1$ where

$$P = \frac{\sin(\gamma N \frac{\Delta}{2\omega_0})}{N \sin(\gamma \frac{\Delta}{2\omega_0})} e^{j\gamma \frac{\Delta}{\omega_0} \frac{N-1}{2}} \quad (4)$$

$$Q = \frac{\sin(\gamma N \frac{2\omega_0+\Delta}{2\omega_0})}{N \sin(\gamma \frac{2\omega_0+\Delta}{2\omega_0})} e^{-j\gamma \frac{2\omega_0+\Delta}{\omega_0} \frac{N-1}{2}}. \quad (5)$$

It is seen, then, that P and Q are functions of the unknown frequency deviation, Δ , but independent of k and the voltages' magnitudes and phases. For the sake of simplicity we omit the dependency of P and Q on Δ .

Finally, the PMU creates the symmetrical component transformation. In particular, the zero, positive, and negative voltage sequences are calculated from three-phase voltages by using the symmetrical component transformation [22]:

$$\begin{bmatrix} V_0[k] \\ V_+[k] \\ V_-[k] \end{bmatrix} = \frac{1}{3} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}}_{\hat{\mathbf{H}}} \begin{bmatrix} V_a[k] \\ V_b[k] \\ V_c[k] \end{bmatrix}, \quad (6)$$

$\forall k = 0, 1, \dots, K-1$, where $V_0[k]$, $V_+[k]$, and $V_-[k]$ are the zero, positive, and negative sequences, respectively, and $\alpha = e^{j2\pi/3}$. By substituting (3) in (6), we obtain

$$V_0[k] = P e^{j\gamma\frac{\Delta}{\omega_0}k} C_0 + Q e^{-j\gamma\frac{2\omega_0+\Delta}{\omega_0}k} C_0^* + W_0[k] \quad (7)$$

$$V_+[k] = P e^{j\gamma\frac{\Delta}{\omega_0}k} C_+ + Q e^{-j\gamma\frac{2\omega_0+\Delta}{\omega_0}k} C_+^* + W_+[k] \quad (8)$$

$$V_-[k] = P e^{j\gamma\frac{\Delta}{\omega_0}k} C_- + Q e^{-j\gamma\frac{2\omega_0+\Delta}{\omega_0}k} C_-^* + W_-[k] \quad (9)$$

$k = 0, \dots, K-1$, where

$$[C_0, C_+, C_-]^T = \frac{\sqrt{2}}{6} \mathbf{H} \mathbf{v}$$

and

$$\begin{bmatrix} W_0[k] \\ W_+[k] \\ W_-[k] \end{bmatrix} = \frac{\sqrt{2}}{3N} \mathbf{H} \sum_{n=k}^{k+N-1} \mathbf{w}_{a,b,c}[n] e^{-j\gamma n},$$

$\forall k = 0, \dots, K-1$. Since $\mathbf{H}\mathbf{H}^H = 3\mathbf{I}_3$, the noise sequences, $\{W_0[k]\}_{k=1}^{K-1}$, $\{W_+[k]\}_{k=1}^{K-1}$, $\{W_-[k]\}_{k=1}^{K-1}$, are independent complex circularly symmetric Gaussian noise sequences where each sequence has a variance of $\frac{2\sigma^2}{3N}$. It can be seen that the PMU output in (7)-(9) includes samples of the zero, positive, and negative sequences at the nominal-frequency bin. However, these "phasors" are different from the true value of the input sequence phasors, C_0 , C_+ , and C_- , and since they do not operate at a constant single frequency, the definition of "phasor" is inherently ambiguous in this case.

In this work, we are interested in the joint estimation of the phasors and frequency-deviation in unbalanced systems based on K measurements of the three symmetrical components from (7)-(9). The models for these K measurements can be written in matrix form as follows:

$$\tilde{\mathbf{v}}_0 = P C_0 \tilde{\mathbf{e}}_1 + Q C_0^* \tilde{\mathbf{e}}_2 + \tilde{\mathbf{w}}_0 \quad (10)$$

$$\tilde{\mathbf{v}}_+ = P C_+ \tilde{\mathbf{e}}_1 + Q C_+^* \tilde{\mathbf{e}}_2 + \tilde{\mathbf{w}}_+ \quad (11)$$

$$\tilde{\mathbf{v}}_- = P C_- \tilde{\mathbf{e}}_1 + Q C_-^* \tilde{\mathbf{e}}_2 + \tilde{\mathbf{w}}_-, \quad (12)$$

where

$$\tilde{\mathbf{e}}_1 = [1, e^{j\gamma\frac{\Delta}{\omega_0}}, \dots, e^{j\gamma\frac{\Delta}{\omega_0}(K-1)}]^T, \quad (13)$$

$$\tilde{\mathbf{e}}_2 = [1, e^{-j\gamma\frac{2\omega_0+\Delta}{\omega_0}}, \dots, e^{-j\gamma\frac{2\omega_0+\Delta}{\omega_0}(K-1)}]^T, \quad (14)$$

$\tilde{\mathbf{v}}_0 = [V_0[0], \dots, V_0[K-1]]^T$, $\tilde{\mathbf{v}}_+ = [V_+[0], \dots, V_+[K-1]]^T$, and $\tilde{\mathbf{v}}_- = [V_-[0], \dots, V_-[K-1]]^T$. It is worth mentioning that the vectors in $\tilde{\mathbf{e}}_1$ and $\tilde{\mathbf{e}}_2$ are identical to the steering vector for a uniform linear array [24]. The noise vectors, $\tilde{\mathbf{w}}_0$, $\tilde{\mathbf{w}}_+$, and $\tilde{\mathbf{w}}_-$, are independent zero-mean complex, circularly symmetric, colored Gaussian noise sequences with covariance matrix \mathbf{R} , where \mathbf{R} is a $K \times K$ matrix with the following k, l th element

$$[\mathbf{R}]_{k,l} = \frac{2\sigma^2}{3N^2} \begin{cases} N - |k-l| & \text{if } -N \leq k-l \leq N \\ 0 & \text{otherwise} \end{cases}$$

Since the error covariance matrix is known, the signals in (10)-(12) can be prewhitened. The whitening operation is performed by left-multiplication of the terms in (10)-(12) by $\mathbf{R}^{-\frac{1}{2}}$:

$$\boldsymbol{\nu}_0 = PC_0\mathbf{e}_1 + QC_0^*\mathbf{e}_2 + \mathbf{w}_0 \quad (15)$$

$$\boldsymbol{\nu}_+ = PC_+\mathbf{e}_1 + QC_+^*\mathbf{e}_2 + \mathbf{w}_+ \quad (16)$$

$$\boldsymbol{\nu}_- = PC_-\mathbf{e}_1 + QC_+^*\mathbf{e}_2 + \mathbf{w}_-, \quad (17)$$

where $\mathbf{e}_m \triangleq \mathbf{R}^{-\frac{1}{2}}\tilde{\mathbf{e}}_m$, $m = 1, 2$, and the whitened vectors are given by $\boldsymbol{\nu}_0 = \mathbf{R}^{-\frac{1}{2}}\tilde{\boldsymbol{\nu}}_0$, $\boldsymbol{\nu}_+ = \mathbf{R}^{-\frac{1}{2}}\tilde{\boldsymbol{\nu}}_+$, $\boldsymbol{\nu}_- = \mathbf{R}^{-\frac{1}{2}}\tilde{\boldsymbol{\nu}}_-$. The modified noise vectors, $\mathbf{w}_0 = \mathbf{R}^{-\frac{1}{2}}\tilde{\mathbf{w}}_0$, $\mathbf{w}_+ = \mathbf{R}^{-\frac{1}{2}}\tilde{\mathbf{w}}_+$, $\mathbf{w}_- = \mathbf{R}^{-\frac{1}{2}}\tilde{\mathbf{w}}_-$ have an identity covariance matrix, \mathbf{I}_K . In the following, the PMU output model in (15)-(17) is used for the ML estimation of the unknown vector $\boldsymbol{\theta} = [C_0, C_+, C_-, \Delta]$.

3. THE ML ESTIMATION

Based on the model described in (15)-(17), the ML estimator of $\boldsymbol{\theta}$ is given by $\hat{\boldsymbol{\theta}} = \max_{\boldsymbol{\theta}} L(\boldsymbol{\theta})$, where the likelihood function is

$$\begin{aligned} L(\boldsymbol{\theta}) = 3K \log \pi & - \|\boldsymbol{\nu}_0 - PC_0\mathbf{e}_1 - QC_0^*\mathbf{e}_2\|^2 \\ & - \|\boldsymbol{\nu}_+ - PC_+\mathbf{e}_1 - QC_+^*\mathbf{e}_2\|^2 \\ & - \|\boldsymbol{\nu}_- - PC_-\mathbf{e}_1 - QC_+^*\mathbf{e}_2\|^2. \end{aligned} \quad (18)$$

3.1. Phasors estimation

For a fixed Δ , by equating the complex derivatives of the right hand side (r.h.s.) of (18) with respect to (w.r.t.) C_0 , C_+ , and C_- , to zero, one obtains

$$\hat{C}_0 = \frac{z_0 - \kappa_2(\hat{C}_0)^*}{\kappa_1} \quad (19)$$

$$\hat{C}_+ = \frac{z_+ - \kappa_2(\hat{C}_-)^*}{\kappa_1}, \quad (20)$$

$$\hat{C}_- = \frac{z_- - (\hat{C}_+)^*\kappa_2}{\kappa_1}, \quad (21)$$

respectively, where $z_0 \triangleq P^*\mathbf{e}_1^H\boldsymbol{\nu}_0 + Q\boldsymbol{\nu}_0^H\mathbf{e}_2$, $z_+ \triangleq P^*\mathbf{e}_1^H\boldsymbol{\nu}_+ + Q\boldsymbol{\nu}_+^H\mathbf{e}_2$, $z_- \triangleq P^*\mathbf{e}_1^H\boldsymbol{\nu}_- + Q\boldsymbol{\nu}_-^H\mathbf{e}_2$, $\kappa_1 \triangleq |P|^2\mathbf{e}_1^H\mathbf{e}_1 + |Q|^2\mathbf{e}_2^H\mathbf{e}_2$, and $\kappa_2 \triangleq 2P^*Q\mathbf{e}_1^H\mathbf{e}_2$. By using mathematical manipulation, the ML estimators in (19)-(21) can be rewritten as:

$$\hat{C}_0 = \frac{\kappa_1 z_0 - \kappa_2 z_0^*}{\kappa_1^2 - |\kappa_2|^2}, \quad (22)$$

$$\hat{C}_+ = \frac{\kappa_1 z_+ - \kappa_2 z_-^*}{\kappa_1^2 - |\kappa_2|^2}, \quad (23)$$

$$\hat{C}_- = \frac{\kappa_1 z_- - \kappa_2 z_+^*}{\kappa_1^2 - |\kappa_2|^2}. \quad (24)$$

3.2. Frequency estimation

By substituting \hat{C}_0 , \hat{C}_+ , and \hat{C}_- in (18) and using (19) and (20), one obtains the following cost

$$\begin{aligned} L(\boldsymbol{\theta}) = 3K \log \pi & - \|\boldsymbol{\nu}_0\|^2 - \|\boldsymbol{\nu}_+\|^2 - \|\boldsymbol{\nu}_-\|^2 \\ & + \kappa_1 \left(|\hat{C}_0|^2 + |\hat{C}_+|^2 + |\hat{C}_-|^2 \right) \\ & + \text{Real} \left\{ \kappa_2^* \left(\hat{C}_0^2 + 2\hat{C}_+\hat{C}_- \right) \right\}. \end{aligned} \quad (25)$$

The left terms in (25) are independent of Δ ; thus, the frequency-deviation estimator is given by:

$$\begin{aligned} \hat{\Delta} = \arg \max_{\Delta} & \left[\kappa_1 \left(|\hat{C}_0|^2 + |\hat{C}_+|^2 + |\hat{C}_-|^2 \right) \right. \\ & \left. + \text{Real} \left\{ \kappa_2^* \left(\hat{C}_0^2 + 2\hat{C}_+\hat{C}_- \right) \right\} \right]. \end{aligned} \quad (26)$$

In order to avoid ambiguities, the estimates of Δ are restricted to $\left[-\frac{N\omega_0}{2}, \frac{N\omega_0}{2} \right)$.

3.3. Special case: perfectly balanced system

For the special case of a perfectly balanced system, the three-phase voltages satisfy $V_a e^{j\varphi_a} = V_b e^{j(\varphi_b + \frac{2\pi}{3})} = V_c e^{j(\varphi_c - \frac{2\pi}{3})}$. Therefore, for this case $C_0 = 0$, $C_- = 0$, and the model in (15)-(17) is reduced to

$$\begin{cases} \boldsymbol{\nu}_0 = \mathbf{w}_0 \\ \boldsymbol{\nu}_+ = PC_+\mathbf{e}_1 + \mathbf{w}_+ \\ \boldsymbol{\nu}_- = QC_+^*\mathbf{e}_2 + \mathbf{w}_- \end{cases}. \quad (27)$$

The model in (27) indicates that for balanced systems the zero-sequence is a noise-sequence and the positive and negative sequences create sinusoidal signals. For the balance case, we only estimate C_+ and the zero and negative phasors are zero. Therefore, (20) implies that the ML estimator of the three phasors for the balanced case are given by

$$\hat{C}_0^{(b)} = 0, \quad \hat{C}_+^{(b)} = \frac{z_+}{\kappa_1}, \quad \hat{C}_-^{(b)} = 0. \quad (28)$$

4. LOW-COMPLEXITY FREQUENCY ESTIMATION

In practice, since the ML estimator of the frequency-deviation in (26) is based on a high complexity maximum-search, many other low-complexity frequency estimation methods are used in power systems (e.g. [1, 17, 8, 25]). In this section, we propose a suboptimal frequency-deviation estimator based on a recursive formulation of the symmetrical components. It is shown that the phasor sequences can be described by an AR model and then, the WLS method is used for frequency estimation. The proposed method does not require phasors estimation.

The positive-sequence phasor measurement observation model in (8) can be rewritten recursively as

$$\begin{aligned} V_+[k] = e^{-j\gamma} a(\Delta) V_+[k-1] & - e^{-j2\gamma} V_+[k-2] \\ & + W_+[k] - e^{-j\gamma} a(\Delta) W_+[k-1] + e^{-j2\gamma} W_+[k-2], \end{aligned} \quad (29)$$

$k = 2, \dots, K-1$, where

$$a(\Delta) = 2 \cos \left(\gamma \frac{\omega_0 + \Delta}{\omega_0} \right). \quad (30)$$

The model in (29) is a second-order complex linear AR model where the complex coefficients are functions of Δ . The recursive model in (29) for K unwhitened phasor measurements of the three symmetrical components can be written in matrix form as follows:

$$\begin{bmatrix} \mathbf{E}_1 \tilde{\boldsymbol{\nu}}_0 \\ \mathbf{E}_1 \tilde{\boldsymbol{\nu}}_+ \\ \mathbf{E}_1 \tilde{\boldsymbol{\nu}}_- \end{bmatrix} = a(\Delta) \begin{bmatrix} \mathbf{E}_2 \tilde{\boldsymbol{\nu}}_0 \\ \mathbf{E}_2 \tilde{\boldsymbol{\nu}}_+ \\ \mathbf{E}_2 \tilde{\boldsymbol{\nu}}_- \end{bmatrix} + \mathbf{w}_{AR}[k], \quad (31)$$

where

$$\mathbf{E}_1 \triangleq \begin{bmatrix} e^{-j2\gamma} & 0 & 1 & 0 & \dots & \dots & 0 \\ 0 & e^{-j2\gamma} & 0 & 1 & 0 & \dots & 0 \\ \dots & \ddots & \ddots & \ddots & \ddots & \dots & 0 \\ 0 & \dots & 0 & e^{-j2\gamma} & 0 & 1 & \dots \end{bmatrix},$$

$$\mathbf{E}_2 \triangleq \begin{bmatrix} 0 & e^{-j\gamma} & 0 & \dots & 0 \\ 0 & 0 & e^{-j\gamma} & 0 & \dots & 0 \\ 0 & \dots & \ddots & \dots & \dots & 0 \\ 0 & \dots & \dots & 0 & e^{-j\gamma} & 0 \end{bmatrix}.$$

The noise term, $\mathbf{w}_{AR}[k]$, $k = 0, \dots, K-1$ is zero-mean Gaussian noise with a covariance matrix

$$\begin{bmatrix} \mathbf{T} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{T} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{T} \end{bmatrix},$$

where $\mathbf{T} \triangleq (\mathbf{E}_1 - a(\Delta)\mathbf{E}_2)\mathbf{R}(\mathbf{E}_1 - a(\Delta)\mathbf{E}_2)^H$. The weighted least-squares (WLS) estimation of $a(\Delta)$ based on the recursive model in (29) and the separability of the problem is given by

$$\begin{aligned} \hat{a}(\Delta) = & \left(\tilde{\mathbf{v}}_0^H \mathbf{E}_2^H \mathbf{T}^{-1} \mathbf{E}_2 \tilde{\mathbf{v}}_0 \right. \\ & \left. + \tilde{\mathbf{v}}_+^H \mathbf{E}_2^H \mathbf{T}^{-1} \mathbf{E}_2 \tilde{\mathbf{v}}_+ + \tilde{\mathbf{v}}_-^H \mathbf{E}_2^H \mathbf{T}^{-1} \mathbf{E}_2 \tilde{\mathbf{v}}_- \right)^{-1} \\ & \times \left(\tilde{\mathbf{v}}_0^H \mathbf{E}_2^H \mathbf{T}^{-1} \mathbf{E}_1 \tilde{\mathbf{v}}_0 + \tilde{\mathbf{v}}_+^H \mathbf{E}_2^H \mathbf{T}^{-1} \mathbf{E}_1 \tilde{\mathbf{v}}_+ \right. \\ & \left. + \tilde{\mathbf{v}}_-^H \mathbf{E}_2^H \mathbf{T}^{-1} \mathbf{E}_1 \tilde{\mathbf{v}}_- \right). \end{aligned} \quad (32)$$

It can be seen that in order to compute the WLS estimator of $a(\Delta)$ in (32) we use the matrix \mathbf{T} , which is a function of $a(\Delta)$. Hence, this estimation is used iteratively by first estimating $a(\Delta)$, then substituting the estimator in \mathbf{T} and vice versa, where the initial estimate can be determined by using state-of-the-art frequency estimation methods or simply set to $\Delta = 0$. Finally, according to (30), we estimate the frequency-deviation by:

$$\hat{\Delta} = \omega_0 / \gamma \cos^{-1}(\hat{a}/2) - \omega_0,$$

where $\cos^{-1}(\cdot)$ is the inverse cosine function. The convergence properties of the proposed iterative method should be done in a future research.

5. SIMULATIONS

We consider a single PMU and sampling rate of $N = 48$ samples per cycle of the nominal grid frequency, $\omega_0 = 2\pi \cdot 60$. The performance is evaluated using 500 Monte-Carlo simulations and $K = 8$ frequency samples. The frequency of the input signal is assumed to have a $\Delta = 2.5 \cdot 2\pi$ offset from the nominal-frequency. The signal-to-noise ratios (SNR) is defined as $\text{SNR} = \frac{V_a^2 + V_b^2 + V_c^2}{\sigma^2}$. The voltage magnitudes and phases are considered as $V_a = V_b = 1$, $V_c = \beta V_a$ p.u. and $\varphi_a = \frac{\pi}{4}$, $\varphi_b = \varphi_a - \frac{2\pi}{3}$, $\varphi_c = \varphi_a + \frac{2\pi}{3} + \epsilon$. Single-phase voltage magnitude and angle unbalance is considered by setting $\beta = 0$ and $\epsilon = 0$. These parameters are suitable for a contingency scenario.

In Fig. 1, we present the MSE of the unbalanced and balanced ML estimators of the zero, positive, and negative phasors, \hat{C}_0 , \hat{C}_+ , \hat{C}_- , $\hat{C}_0^{(b)}$, $\hat{C}_+^{(b)}$, and $\hat{C}_-^{(b)}$ from (22), (23), (24), and (28), respectively, for the aforementioned single-phase unbalanced case. It can

be seen that the MSE of the ML estimators that are taking into account the unbalances, is significantly lower than the MSE of the ML estimators for the balanced case. The MSE of the balanced ML estimators is a constant, since they assumed as zero. Therefore, the proposed estimation method can significantly improve the performance of the proposed estimation method, based on balanced system assumption. The MSE of the ML and low-complexity esti-

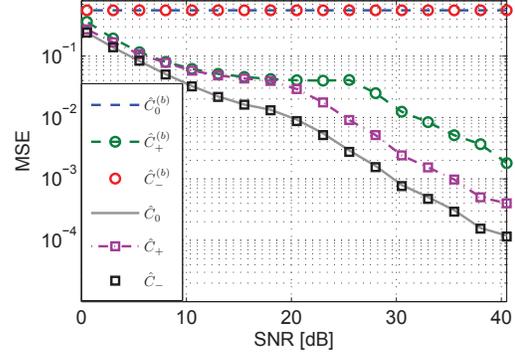


Fig. 1. The MSE of the ML phasors estimators for $K = 8$, $N = 48$, $\Delta = 2.5 \times 2\pi$, and for an unbalanced system.

mators of the normalized frequency-deviation, $\frac{\gamma\Delta}{\omega_0}$, are presented in Fig. 2 and their performance is compared to the MSE of the state-of-the-art frequency-estimation method, which is based on the positive-sequence and given by [7]:

$$\hat{\Delta}_s = \frac{\omega_0}{\gamma} \frac{1}{K} \sum_{k=0}^{K-2} \text{angle}(V_+[k+1]) - \text{angle}(V_+[k]). \quad (33)$$

It can be verified that for low-SNR, the state-of-the-art estimation method performs well, while for high SNR the proposed low complexity method is significantly better. The proposed low-complexity frequency estimation method uses $3(K-1)$ measurements of the PMU output, while the ML uses $3K$ measurements. Therefore, the proposed low-complexity model becomes more accurate when K and/or SNR increases and its performance convergence to the ML performance. Hence, it is demonstrated by these simulations that asymptotically, there is no information loss. The MSE of the ML estimator is the lowest for any SNR but it suffers from high complexity and is affected by the search resolution.

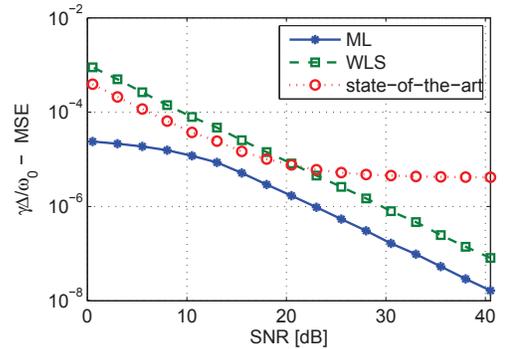


Fig. 2. The MSE of the normalized frequency-deviation, $\gamma \frac{\Delta}{\omega_0}$, estimators for $K = 8$, $N = 48$, $\Delta = 2.5 \times 2\pi$ and for an unbalanced system.

6. REFERENCES

- [1] A. Phadke and J. Thorp, *Synchronized Phasor Measurements and Their Applications*. New York: Springer Science, 2008.
- [2] A. Abur and M. Celik, "Least absolute value state estimation with equality and inequality constraints," *IEEE Trans. Power Systems*, vol. 8, no. 2, pp. 680–686, May 1993.
- [3] A. Ghosh and A. Joshi, "A new approach to load balancing and power factor correction in power distribution system," *IEEE Trans. Power Delivery*, vol. 15, no. 1, pp. 417–422, Jan. 2000.
- [4] S. Zhong and A. Abur, "Effects of nontransposed lines and unbalanced loads on state estimation," in *IEEE Power Engineering Society Winter Meeting*, vol. 2, Jan. 2002, pp. 975–979.
- [5] A. Von Jouanne and B. Banerjee, "Assessment of voltage unbalance," *IEEE Trans. Power Delivery*, vol. 16, no. 4, pp. 782–790, Oct. 2001.
- [6] A. Gomez-Exposito, A. Abur, A. de la Villa Jaen, and C. Gomez-Quiles, "A multilevel state estimation paradigm for smart grids," *Proceedings of the IEEE*, vol. 99, no. 6, pp. 952–976, June 2011.
- [7] A. G. Phadke, J. S. Thorp, and M. G. Adamiak, "A new measurement technique for tracking voltage phasors, local system frequency, and rate of change of frequency," *IEEE Trans. Power Apparatus and Systems*, vol. PAS-102, no. 5, pp. 1025–1038, May 1983.
- [8] Y. Xia, S. Douglas, and D. Mandic, "Adaptive frequency estimation in smart grid applications: Exploiting noncircularity and widely linear adaptive estimators," *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 44–54, Sep. 2012.
- [9] D. Dini, Y. Xia, S. Douglas, and D. Mandic, "Widely linear state space models for frequency estimation in unbalanced three-phase systems," in *SAM 2012*, 2012, pp. 9–12.
- [10] D. Rife and R. Boorstyn, "Single tone parameter estimation from discrete-time observations," *IEEE Trans. Information Theory*, vol. 20, no. 5, pp. 591–598, Sep. 1974.
- [11] S. Kay, "A fast and accurate single frequency estimator," *IEEE Trans. Acoustics, Speech and Signal Processing*, vol. 37, no. 12, pp. 1987–1990, Dec. 1989.
- [12] B. G. Quinn and E. J. Hannan, *The Estimation and Tracking of Frequency*, ser. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge: Cambridge University Press, 2001.
- [13] Y.-F. Huang, S. Werner, J. Huang, N. Kashyap, and V. Gupta, "State estimation in electric power grids: Meeting new challenges presented by the requirements of the future grid," *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 33–43, 2012.
- [14] V. Kekatos and G. Giannakis, "Distributed robust power system state estimation," *IEEE Trans. Power Systems*, vol. 28, no. 2, pp. 1617–1626, 2013.
- [15] L. Jia, R. Thomas, and L. Tong, "Malicious data attack on real-time electricity market," in *ICASSP 2011*, May 2011, pp. 5952–5955.
- [16] P. Yang, Z. Tan, A. Wiesel, and A. Nehorai, "Power system state estimation using PMUs with imperfect synchronization," *IEEE Trans. Power Systems*, vol. 28, no. 4, pp. 4162–4172, 2013.
- [17] T. Routtenberg and L. Tong, "Joint frequency and phasor estimation under the KCL constraint," *IEEE Signal Processing Letters*, vol. 20, no. 6, pp. 575–578, June 2013.
- [18] A. Angioni, C. Muscas, S. Sulis, F. Ponci, and A. Monti, "Impact of heterogeneous measurements in the state estimation of unbalanced distribution networks," in *Instrumentation and Measurement Technology Conference (I2MTC), 2013 IEEE International*, May 2013, pp. 935–939.
- [19] M. Sun, S. Demirtas, and Z. Sahinoglu, "Joint voltage and phase unbalance detector for three phase power systems," *IEEE Signal Processing Letters*, vol. 20, no. 1, pp. 11–14, Jan. 2013.
- [20] C. Wei, A. Wiesel, and R. Blum, "Change detection in smart grids using errors in variables models," in *Proc. Sensor Array and Multichannel Signal Processing Workshop (SAM) 2012*, 2012, pp. 17–20.
- [21] T. Routtenberg, Y. Xie, R. M. Willett, and L. Tong, "Unbalance detection in three-phase power systems based on PMU output," *In preparation*.
- [22] C. Fortescue, "Method of symmetrical coordinates applied to the solution of polyphase networks," *Transactions of AIEE*, vol. 37, pp. 1027–1140, 1918.
- [23] J. Li, X. Xie, J. Xiao, and J. Wu, "The framework and algorithm of a new phasor measurement unit," in *DRPT 2004*, vol. 2, Apr. 2004, pp. 826–831.
- [24] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Englewood Cliffs (N.J.): Prentice Hall PTR, 1998, vol. 2.
- [25] V. Terzija, N. Djuric, and B. Kovacevic, "Voltage phasor and local system frequency estimation using Newton type algorithm," *IEEE Trans. Power Delivery*, vol. 9, no. 3, pp. 1368–1374, July 1994.