TIME DELAY ESTIMATION IN THE PRESENCE OF CLOCK FREQUENCY ERROR

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ABSTRACT

The time difference localization suffers from the performance deterioration of time delay estimation (TDE) due to the presence of clock frequency error in incoherent systems. Based on the modified signal model of TDE, the joint maximum-likelihood (ML) estimation of time delay and system clock frequency error is proposed. Then, the Cramér-Rao lower bounds (CRLBs) of time delay and clock frequency error estimations are given. The performance of time delay estimation may be significantly improved to approach CRLB by proposed method. Further, the accuracy of proposed time delay estimator is unaffected by performance of system clock in moderate condition, that is verified by simulation results.

Index Terms— Time delay estimation (TDE), Local oscillator (LO), Time difference of arrival (TDOA).

1. INTRODUCTION

Time delay estimation (TDE) has many applications such as target localization and tracking in sonar or radar systems. In the past few decades, lots of TDE methods are proposed, such as [1–7]. Most of these methods explicitly or implicitly assume that the time delay relationship between observed signals (usually Intermediate Frequency (IF) band or baseband signals) is identical to the time delay relationship between radio frequency (RF) signals at antennas of two spatially separated receivers. The above assumption is usually satisfied when the observed signals are coherently received, which would require high-cost synchronization equipped in the receivers. In practice, in order to reduce costs, the incoherent systems unequipped accurate synchronization are usually employed to measure time delay.

In the incoherent system, the independent local clocks are equipped at the different receivers respectively, as shown in Fig. 1. Due to the imperfect consistency of frequency and phase among local clocks, time delay relation (in IF band or baseband) is distorted between observed signals after mixing and sampling. Due to the similarity of problem in IF band or baseband time delay estimation, the baseband delay estimation is only discussed herein. In baseband time delay estimation, the frequency and phase offsets between the received observed signal and the reference signal is inevitable, that is due to the independence of two mixing local oscillators (LOs) respectively at a receiver and other receiver. Further, time stretch is also caused by independent operating sampling clocks at a receiver and other receiver, respectively.

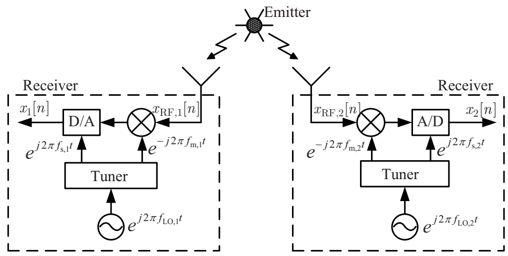

Fig. 1. Incoherent reception in a typical passive system

The contributions of this paper include:

1) An improved signal model of time delay estimation for incoherent reception is developed. The frequency offset and time stretch effect incurred by imperfect mixing and sampling are jointly considered.

2) A joint maximum likelihood (ML) estimation of time delay and frequency error of system clock is devised.

3) The Cramér-Rao lower bounds (CRLBs) of the proposed joint time delay and clock frequency error estimations are obtained in the incoherent reception system. It is verified that the CRLB of time delay estimation is a tighter bound than that of the coherent reception system.

The following notations are used. Bold face upper and lower case letter denote matrix and vector respectively. The superscripts ^{T} and ^{H} denote transpose and Hermitian transpose respectively. The notations $\|\cdot\|$, $\det\{\cdot\}$, $\mathrm{angle}\{\cdot\}$, $\Re\{\cdot\}$ and $\Im\{\cdot\}$ stand for, respectively, 2-norm, determinant, phase angle, real-part and imaginary-part.

2. SIGNAL MODEL

In this paper, the following assumptions are adopted.

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Assumption 1. The frequencies of different system clocks are assumed to be different constants during the observation interval, respectively.

Assumption 2. The clock's frequency of the reference receiver is assumed to equal the nominal frequency. At the other receiver, the relative frequency deviation η always exists with respect to nominal value, and it remains unchanged in frequency multiplication or demultiplication for obtain mixing frequency and sampling frequency. Herein, $\eta \triangleq \frac{f_{\mathrm{op}}-f_{\mathrm{n}}}{f_{\mathrm{n}}}$ with operating frequency f_{op} and nominal frequency f_{n}.

Assumption 3. The noise-free complex envelope signal $s(t)$ is assumed to be unknown and deterministic. Moreover, the additive noise is assumed to be zero-mean complex white Gaussian, and it is uncorrelated to the envelope signal.

The noise-free RF signals at the two receivers, respectively, may be given by $x_{\mathrm{RF},1}(t) = s(t)e^{j2\pi f_{\mathrm{c}}t}$ and $x_{\mathrm{RF},2}(t) = as(t-\tau)e^{j(2\pi f_{\mathrm{c}}t+\varphi)}$, where $0 \leq t \leq T$, τ and φ are unknown differential delay and phase respectively. Herein, differential delay is usually known as time difference of arrival (TDOA) in the passive system. Differential phase is composed by $-j2\pi f_{\mathrm{c}}\tau$ and differential phase-shift which arises when one or both of the RF signals are reflected from some boundary [8]. According to previous assumptions, the operating mixing frequencies equal to f_{m} and $(1+\eta)f_{\mathrm{m}}$ at the reference receiver and auxiliary receiver respectively, and $f_{\mathrm{m}} = f_{\mathrm{c}}$. Thus the baseband signals may be given by

$$\begin{aligned} x_1(t) &= s(t) \\ x_2(t) &= as(t-\tau)e^{j(-2\pi\eta f_{\mathrm{m}}t+\varphi)}. \end{aligned} \tag{1}$$

From the sampling operating frequency assumption, the discrete-time baseband signals are given by

$$\begin{aligned} x_1[n]|_{T_{\mathrm{s}}} &= s[n]|_{T_{\mathrm{s}}} \\ x_2[n]|_{T_{\mathrm{ops}}} &= ae^{j(-2\pi\eta f_m nT_{\mathrm{ops}}+\varphi)}\ s[n-\tau/T_{\mathrm{ops}}]|_{T_{\mathrm{ops}}} \end{aligned} \tag{2}$$

where $x[n]|_{T_{\mathrm{ops}}} \triangleq x(nT_{\mathrm{ops}})$, and T_{ops} and T_{s} denote the sampling operating period and sampling nominal period, respectively. Let $\beta \triangleq \frac{\eta}{1+\eta}$, $D \triangleq \tau/T_{\mathrm{s}}$. According to the definition of η in Assumption 2, it is known that $\beta = \frac{f_{\mathrm{op}}-f_{\mathrm{n}}}{f_{\mathrm{op}}}$. Hence, β may be viewed as the relative frequency deviation between the operating frequency and nominal frequency with respect to normalized frequency f_{op}. And D is the time delay normalized by the sampling period. Under the identical sampling period T_{s}, with the additive noise considered, the baseband observed signals may be respectively rewritten as

$$\begin{aligned} x_1[n]|_{T_{\mathrm{s}}} &= s[n]|_{T_{\mathrm{s}}} + q_1[n]|_{T_{\mathrm{s}}} \\ x_2[n]|_{T_{\mathrm{ops}}} &= ae^{j(\varphi-2\pi\beta f_{\mathrm{m}}nT_{\mathrm{s}})}\ s[(1-\beta)n-D]|_{T_{\mathrm{s}}} + q_2[n]|_{T_{\mathrm{s}}}. \end{aligned} \tag{3}$$

After adopting identical sampling frequency, for the sake of simplicity, the subscripts about sampling are omitted. Thus, the observed signal model may be simplified as

$$\begin{aligned} x_1[n] &= s[n] + q_1[n] \\ x_2[n] &= ae^{j(\varphi-2\pi\beta f_{\mathrm{m}}nT_{\mathrm{s}})}s\left[(1-\beta)n-D\right] + q_2[n]. \end{aligned} \tag{4}$$

From the above derivation, it is known that the frequency and phase offsets are caused by imperfect mixing while the time stretch is caused by imperfect sampling. Due to the presence together of the frequency offset and time stretch that originate from identical frequency error of system clock, the proposed algorithm is different from the joint estimation of TDOA and frequency difference of arrival (FDOA) [8–11]. In the form of the vector, the signal model can be given as

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{s} + \mathbf{q}_1 \\ \mathbf{x}_2 &= ae^{j\varphi}\mathbf{\Phi}\mathbf{s}_{D,\beta} + \mathbf{q}_2 \end{aligned} \tag{5}$$

where

$$\begin{aligned} \mathbf{x}_i &\triangleq [x_i[0], x_i[1], \cdots, x_i[N-1]]^T, i = 1,2 \\ \mathbf{q}_i &\triangleq [q_i[0], q_i[1], \cdots, q_i[N-1]]^T, i = 1,2 \\ \mathbf{s}_{D,\beta} &\triangleq [s[(1-\beta)\cdot 0 - D], \cdots, s[(1-\beta)\cdot(N-1)-D]]^T \\ \mathbf{\Phi} &\triangleq \mathrm{diag}\{[e^{-j2\pi\beta f_{\mathrm{m}}T_{\mathrm{s}}\cdot 0}\cdots, e^{-j2\pi\beta f_{\mathrm{m}}T_{\mathrm{s}}\cdot(N-1)}]^T\}. \end{aligned} \tag{6}$$

Herein, $\mathrm{diag}\{\mathbf{x}\}$ denotes a diagonal matrix with vector $\mathbf{x}$ on the main diagonal. Note that, $\mathbf{s}_{D,\beta}$ is the time stretched and delayed replica of the emitted signal vector

$$\mathbf{s} \triangleq \mathbf{s}_{0,0} = [s[0], s[1], \cdots, s[N-1]]^T. \tag{7}$$

In practice, time stretch and subsample delay may be approximately implemented by a time-varying finite impulse response (FIR) filter with $2K+1$ taps. The tap weight coefficient of the FIR is given by samples of sinc function, i.e., the $(n+1)$th element of $\mathbf{s}_{D,\beta}$ may be approximated as

$$[\mathbf{s}_{D,\beta}]_{n+1} \approx \sum_{k=-K}^{K} \mathrm{sinc}(k-\beta n-D)s[n-k], \quad n = 0, 1, \cdots, N-1 \tag{8}$$

where $\mathrm{sinc}(x) \triangleq \frac{\sin(\pi x)}{\pi x}$.

3. TIME DELAY ESTIMATION

As in passive localization system, the noise-free signal $\mathbf{s}$ is assumed to be unknown. Thus, the unknown real vector, composed of the real-part of $\mathbf{s}$, the imaginary-part of $\mathbf{s}$, and the parameter vector $\boldsymbol{\theta} \triangleq [D, \beta, \varphi, a]^T$ is defined as

$$\boldsymbol{\vartheta} = [\Re\{\mathbf{s}^T\}, \Im\{\mathbf{s}^T\}, \boldsymbol{\theta}^T]^T. \tag{9}$$

Due to the deterministic (unknown) signal assumption and zero-mean complex white Gaussian noise assumption, it is known that the concatenated vector $\mathbf{x} = [\mathbf{x}_1^T, \mathbf{x}_2^T]^T$ is also a white Gaussian vector, i.e., $\mathbf{x} \sim \mathcal{CN}(\mathbf{u}, \mathbf{C})$ with mean and covariance matrix respectively given by

$$\mathbf{u}(\boldsymbol{\vartheta}) = [\mathbf{s}^T, [ae^{j\varphi}\boldsymbol{\Phi}\mathbf{s}_{D,\beta}]^T]^T \quad (10)$$

$$\mathbf{C} = \begin{bmatrix} \sigma_1^2\mathbf{I} & \\ & \sigma_2^2\mathbf{I} \end{bmatrix} \quad (11)$$

where σ_1^2 and σ_2^2 denote the variances of $\mathbf{q}_1$ and $\mathbf{q}_2$ respectively. Therefore, the maximum likelihood (ML) probability density function is given by

$$p(\mathbf{x};\boldsymbol{\vartheta}) = \frac{1}{\pi^N \det\{\mathbf{C}\}} \cdot \exp\left(-(\mathbf{x}-\mathbf{u}(\boldsymbol{\vartheta}))^H\mathbf{C}^{-1}(\mathbf{x}-\mathbf{u}(\boldsymbol{\vartheta}))\right). \quad (12)$$

To obtain the ML estimate of the unknown parameters, we maximize the log of the likelihood function (12), which may be demonstrated to be equivalent to maximizing

$$L_1(\boldsymbol{\vartheta}) = L_2(\boldsymbol{\theta}) - L_3(\boldsymbol{\vartheta}) \quad (13)$$

where $L_2(\boldsymbol{\theta})$ and $L_3(\boldsymbol{\vartheta})$ are given in (14) and (15) respectively.

$$L_2(\boldsymbol{\theta}) = \frac{1}{\sigma_2^2 + a^2\sigma_1^2}\mathbf{x}_1^H\mathbf{x}_1 + \frac{a^2}{\sigma_2^2 + a^2\sigma_1^2}\bar{\mathbf{x}}_2^H\bar{\mathbf{x}}_2 + \frac{2a}{\sigma_2^2 + a^2\sigma_1^2}\mathrm{Re}\left\{\mathbf{x}_1^H(e^{-j\varphi}\bar{\boldsymbol{\Phi}}\bar{\mathbf{x}}_2)\right\}. \quad (14)$$

The observed data $\bar{\mathbf{x}}_2$ and diagonal matrix $\bar{\boldsymbol{\Phi}}$ are given by

$$[\bar{\mathbf{x}}_2]_i = x_2[(i-1+D)/(1-\beta)], i = 1, 2, \cdots, N, \quad (16)$$

$$[\bar{\boldsymbol{\Phi}}]_{i,i} = e^{-j2\pi\beta f_m T_s(i-1+D)/(1-\beta)}, i = 1, 2, \cdots, N. \quad (17)$$

According to (13), it is known that maximization of $L_1(\boldsymbol{\vartheta})$ is equivalent to maximization of $L_2(\boldsymbol{\theta})$ with minimization of $L_3(\boldsymbol{\vartheta})$. It is obvious that $L_3(\boldsymbol{\vartheta}) \geq 0$ according to (15). Thus, the estimate of the unknown signal $\mathbf{s}$ may be given as

$$\hat{\mathbf{s}}(\boldsymbol{\theta}) = \frac{\sigma_2^2}{\sigma_2^2 + a^2\sigma_1^2}\mathbf{x}_1 + \frac{a\sigma_1^2 e^{-j\varphi}}{\sigma_2^2 + a^2\sigma_1^2}\bar{\boldsymbol{\Phi}}^H\bar{\mathbf{x}}_2 \quad (18)$$

while $L_3(\boldsymbol{\vartheta}) = 0$. It is obvious that the unknown signal estimate is dependent on the unknown parameter $\boldsymbol{\theta}$. In addition, according the assumption that the envelope signal and noise is uncorrelated, $\bar{\mathbf{x}}_2^H\bar{\mathbf{x}}_2$ may be approximated as $a^2\mathbf{s}^H\mathbf{s}+\sigma_2^2$, i.e., $\bar{\mathbf{x}}_2^H\bar{\mathbf{x}}_2$ is a approximately constant. Thus, the maximization of $L_2(\boldsymbol{\theta})$ is equivalent to the maximization of the third term on the right-hand side of the equal sign in (14). Therefore, the joint estimation may be obtained by maximizing

$$L(D,\beta,\varphi) = \Re\left\{e^{-j\varphi}\mathbf{x}_1^H\bar{\boldsymbol{\Phi}}^H\bar{\mathbf{x}}_2\right\} = A(D,\beta,\varphi)\cos(\alpha-\varphi). \quad (19)$$

where

$$A(D,\beta,\varphi) = |e^{-j\varphi}\mathbf{x}_1^H\bar{\boldsymbol{\Phi}}^H\bar{\mathbf{x}}_2| = |\mathbf{x}_1^H\bar{\boldsymbol{\Phi}}^H\bar{\mathbf{x}}_2| \quad (20)$$

$$\alpha = \mathrm{angle}\{\mathbf{x}_1^H\bar{\boldsymbol{\Phi}}^H\bar{\mathbf{x}}_2\}. \quad (21)$$

Thus, maximizing $L(D,\beta,\varphi)$ is reduces to the maximizing $A(D,\beta,\varphi)$ on condition of $\cos(\alpha-\varphi) = 1$. It is obvious that the value of $A(D,\beta,\varphi)$ is only related to D and β. Thus, the differential delay and relative frequency deviation may be obtained via two-dimensional (2D) grid searches, i.e.,

$$[\hat{D}, \hat{\beta}] = \arg\max_{D,\beta}\{|\mathbf{x}_1^H\bar{\boldsymbol{\Phi}}^H\bar{\mathbf{x}}_2|\}. \quad (22)$$

And φ may be estimated by $\hat{\varphi} = \hat{\alpha}$ where $\hat{\alpha}$ may be obtained by substituting $\hat{D}$ and $\hat{\beta}$ into (21). Herein, $\hat{\beta}$ and $\hat{\varphi}$ may be used to compensation clock frequency and phase errors. However, estimation of nuisance parameter a and source signal $\mathbf{s}$ are omitted.

4. CRAMÉR-RAO LOWER BOUND

When the relative frequency deviation is moderate small, the CRLBs on the estimations of D and β in incoherent reception may be obtained with detailed mathematical manipulations omitted,[1]

$$\mathrm{CRLB}_1(D) = \frac{a^2\sigma_1^2 + \sigma_2^2}{2a^2} \cdot \frac{\gamma}{\gamma\dot{\mathbf{s}}^H\dot{\mathbf{s}} - \kappa} \quad (23)$$

$$\mathrm{CRLB}_1(\beta) = \frac{a^2\sigma_1^2 + \sigma_2^2}{2a^2} \cdot \frac{\nu}{\gamma\dot{\mathbf{s}}^H\dot{\mathbf{s}} - \kappa} \quad (24)$$

where γ, ν, and κ are given in (25)-(27) respectively.

$$\gamma \triangleq \left\|(\dot{\mathbf{s}}^H\mathbf{N} - 2\pi f_m T_s \mathbf{s}_{D,\beta}^H\mathbf{N})\right\|^2 \mathbf{s}_{D,\beta}^H\mathbf{s}_{D,\beta} - \left(\Im\{\dot{\mathbf{s}}^H\mathbf{N}\mathbf{s}_{D,\beta}\} - 2\pi f_m T_s \mathbf{s}_{D,\beta}^H\mathbf{N}\mathbf{s}_{D,\beta}\right)^2. \quad (25)$$

$$\nu \triangleq \dot{\mathbf{s}}^H\dot{\mathbf{s}}\mathbf{s}_{D,\beta}^H\mathbf{s}_{D,\beta} - \Im\{\dot{\mathbf{s}}^H\mathbf{s}_{D,\beta}\}\Im\{\dot{\mathbf{s}}^H\mathbf{s}_{D,\beta}\}. \quad (26)$$

Herein, $\mathbf{N} \triangleq \mathrm{diag}\{[0, 1, \cdots, N-1]^T\}$ and $\dot{\mathbf{s}} \triangleq \frac{\partial \mathbf{s}_{D,\beta}}{\partial n}$. The CRLB on time delay estimation in coherent reception is given as [11]

$$\mathrm{CRLB}_2(D) = \frac{a^2\sigma_1^2 + \sigma_2^2}{2a^2}\frac{1}{\dot{\mathbf{s}}^H\dot{\mathbf{s}}}. \quad (28)$$

Thus, the relationship between of the two CRLBs on time delay estimation is given by

$$\mathrm{CRLB}_1(D) = \xi \cdot \mathrm{CRLB}_2(D) \quad (29)$$

where the degradation coefficient ξ is defined as

$$\xi = \frac{\dot{\mathbf{s}}^H\dot{\mathbf{s}}}{\dot{\mathbf{s}}^H\dot{\mathbf{s}} - \kappa/\gamma}. \quad (30)$$

And it may be readily proved that $\xi \geq 1$, i.e., $\mathrm{CRLB}_1(D) \geq \mathrm{CRLB}_2(D)$.[1]

[1] The details is given in the extended paper.

$$L_3(\boldsymbol{\vartheta}) = \frac{\sigma_2^2 + a^2\sigma_1^2}{\sigma_1^2\sigma_2^2} \left\| \left(\frac{\sigma_2^2}{\sigma_2^2 + a^2\sigma_1^2}\mathbf{x}_1 + \frac{a\sigma_1^2 e^{-j\varphi}}{\sigma_2^2 + a^2\sigma_1^2}(\bar{\boldsymbol{\Phi}}^H \bar{\mathbf{x}}_2) \right) - \mathbf{s} \right\|^2. \tag{15}$$

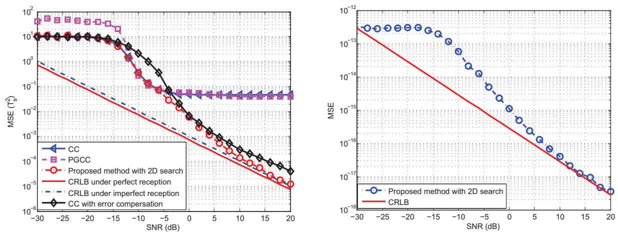
$$\kappa \triangleq \left\| \Im\{\dot{\mathbf{s}}^H \mathbf{s}_{D,\beta}\}(\dot{\mathbf{s}}^H \mathbf{N} - 2\pi f_m T_s \mathbf{s}_{D,\beta}^H \mathbf{N}) - \mathbf{s}_{D,\beta}^H(\dot{\mathbf{s}}^H \mathbf{N}\dot{\mathbf{s}} - 2\pi f_m T_s \Im\{\dot{\mathbf{s}}^H \mathbf{N}\mathbf{s}_{D,\beta}\}) \right\|^2. \tag{27}$$

5. SIMULATION RESULTS

In the following numerical experiments, it is assumed that digital baseband signal is produced by mixing and sampling of FM signal with carrier frequency $f_c = 101.7$MHz, i.e.,

$$s[n] = \exp\left\{ j \cdot \frac{\Delta f \sin(2\pi f_b n T_s)}{f_b} \right\}, n = 0, 1, \cdots, N-1.$$

The sampling period $T_s = 4\mu$s, modulated signal's frequency $f_b = 100$Hz, frequency modulation deviation $\Delta f = 50$kHz, data length $N = 2048$, the actual time delay $D = 3.2T_s$, $\varphi = 1.2$rad. In addition, signal noise ratios (SNRs) of two observed signal are assumed to be identical, with SNR is defined as SNR $\triangleq a^2\sigma_s^2/\sigma_1^2$, where σ_s^2 denotes power of noise-free signal $e^{j\varphi}\boldsymbol{\Phi}\mathbf{s}_{D,\beta}$. And the mean square errors (MSEs) are obtained by 500 independent trials in all simulations.



(a) (b)

Fig. 2. (a) MSE of $\hat{D}$, (b) MSE of $\hat{\beta}$. ($\eta = 2 \times 10^{-7}$)

The MSEs of $\hat{D}$ and $\hat{\beta}$ are illustrated in Fig. 2(a) and Fig. 2(b) respectively. It is obvious that the proposed method significantly outperforms the cross-correlation (CC) [1] and the parametric generalized cross-correlation (PGCC) methods [12]. Moreover, the proposed method behaves even better than CC with clock frequency error compensation.[2] The MSEs of $\hat{D}$ and $\hat{\beta}$ approach the respective CRLBs in moderate SNRs. In addition, under low SNRs environment, the MSEs is almost unchanged due to grid search area that is fixed in simulations.

As illustrated Fig. 3, the delay estimation accuracy of the proposed method is almost invariant along with the variation of relative frequency derivation η, but the delay estimation MSE of the traditional CC method increases with absolute value of relative frequency derivation. Hence, the low cost local clock, which usually provide bad frequency accuracy, may be equipped when proposed method is employed in time delay estimation system.

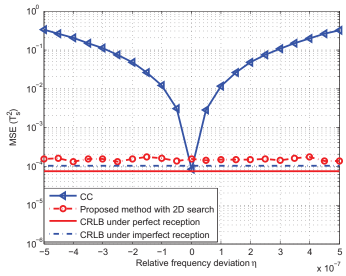


Fig. 3. MSE of $\hat{D}$ with different relative frequency derivation of local clock (SNR=10dB,$D = 3.2T_s$)

6. CONCLUSION

The time delay estimation is investigated in the presence of system clock frequency error, Under the deterministic signal and white gaussian noise assumption, the joint ML estimation of time delay and clock frequency error is proposed. Further, the CRLB on time delay estimation is given under the incoherent reception. The accuracy of time delay estimation is unaffected by the magnitude of clock frequency error when the proposed method is employed in incoherent systems, that may be not achieved with traditional methods. In addition, the improved accuracy of proposed method are also confirmed by simulation results.

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