

A COMPUTING METHOD OF DOUBLE LINEAR CORRELATION FOR MIRROR IMAGE MATCHING

Izumi ITO

Decision Science and Technology, Tokyo Institute of Technology, Tokyo 152-8551, Japan
Email: ito@nh.cradle.titech.ac.jp

ABSTRACT

A computing method of double linear correlation is proposed in order to estimate the location in which the part of an image matches another image even if one of the images is mirrored across the vertical, horizontal, or both axes. 1-D double linear correlation (DLC) is extended to 2-D DLC. The reduction of the number of zero-padding is considered. The computational complexity of the proposed method is lower than that of an FFT approach by more than 50% the number of operations on real numbers.

Index Terms— Discrete cosine transform, linear correlation, image matching, mirror image.

1. INTRODUCTION

Correlation is a fundamental tool and is widely used for image matching. Discrete cosine transform sign phase correlation (DCT-SPC) is a kind of limited correlation based on the phase difference between two sequences that are extended symmetrically [1][2]. DCT-SPC is calculated using DCT signs, which lowers the computational load, and have an affinity with coded images. However, DCT-SPC function is affected by convolution terms due to its symmetry property.

A computational method of linear convolution using DCTs for linear phase filters is proposed [3]. It was extended to nonlinear phase filters [4][5], which means that the computing method for linear convolution can be used for linear correlation. The author developed a method obtaining linear convolution and linear correlation simultaneously using DCTs and used the method for forward and reverse matching [6].

In the present paper, we propose a computing method of double linear correlation (DLC) for the new concept of a mirror image matching that is to determine whether an image matches another image even if one of the image is mirrored across the vertical, horizontal, or both axes. The mirror pattern and its location can be estimated by the proposed method with low computational load. Two properties of DLC and DCT-SPC in combination enables to lower the computational

load on DLC and to remove the effect of convolutions terms on DCT-SPC.

2. PRELIMINARIES

Let $x(n)$ and $h(n)$ be a sequence of the length M and L , respectively.

2.1. Symmetrically extended sequence (SES) and the relation between DCT type 2 (DCT-2) and DFT

Let $\hat{f}_{2N}(n)$ of length $2N$ be an SES of $f_N(n)$ of length N , i.e.,

$$\hat{f}_{2N}(n) = f_{2N}(n) + f_{2N}(-n - 1) \quad (1)$$

where

$$f_{2N}(n) = \begin{cases} f_N(n), & 0 \leq n \leq N - 1 \\ 0, & N \leq n \leq 2N - 1 \end{cases} \quad (2)$$

The relation between DCT-2 and DFT is given as for $k = 0, 1, \dots, N - 1$

$$\hat{F}(k) = (1/C_k)F_C(k)W_{2N}^{-k/2} \quad (3)$$

where $\hat{F}(k)$ is $2N$ -point DFT coefficient of $\hat{f}_{2N}(n)$, $F_C(k)$ of $f_N(n)$ is N -point DCT-2 coefficients of $f_N(n)$, and $W_N = \exp(-j2\pi/N)$ [7]. The DCT-2 coefficients are given as

$$F_C(k) = 2C_k \sum_{n=0}^{N-1} f_N(n) \cos\left(\frac{\pi(n+1/2)k}{N}\right) \quad (4)$$

$$C_k = \begin{cases} 1/\sqrt{2}, & k = 0 \text{ or } N \\ 1, & \text{otherwise} \end{cases} \quad (5)$$

2.2. Circular convolution between SESs using DCTs

Circular convolution between SESs is calculated using DCTs without making SESs.

The circular convolution, $\hat{y}_{2N}(n)$, between $\hat{x}_{2N}(n)$ and $\hat{h}_{2N}(n)$ is calculated by

$$\hat{y}_{2N}(n - 1) = \frac{1}{N} \sum_{k=0}^{N-1} C_k^2 X_C(k) H_C(k) \cos\left(\frac{\pi nk}{N}\right) \quad (6)$$

where $X_C(k)$ and $H_C(k)$ are the DCT-2 coefficients of $x_N(n)$ and $h_N(n)$, respectively, according to (4).

This work was supported by JSPS KAKENHI Grant Number 24560442.

2.3. DCT sign phase correlation (DCT-SPC)

The DCT sign is defined in terms of $F_C(k)$ and the absolute value, $|F_C(k)|$, as

$$\sigma_F(k) = F_C(k)/|F_C(k)|. \quad (7)$$

When $|F_C(k)|$ is zero, $\sigma_F(k)$ is replaced by zero. From the relation between DCT signs and DFT phase term, i.e.,

$$\sigma_F(k) = W_{2N}^{k/2} \hat{\phi}_F(k) \quad (8)$$

where $\hat{\phi}(k) = \hat{F}(k)/|\hat{F}(k)|$, the DCT sign phase correlation, $r(n)$, between $x_N(n)$ and $h_N(n)$ is given as

$$r(n) = \frac{1}{N} \sum_{k=0}^{N-1} C_k^2 \sigma_X(k) \sigma_H(k) \cos\left(\frac{\pi nk}{N}\right) \quad (9)$$

where $\sigma_X(k)$ and $\sigma_H(k)$ are DCT signs of $X_C(k)$ and $H_C(k)$, respectively, according to (7). The translational displacement is expressed by the location n of the maximum value of $r(n)$.

3. DOUBLE LINEAR CORRELATION

3.1. Linear convolution between SESs

Let $x_N(n)$ and $h_N(n)$ be

$$x_N(n) = \begin{cases} 0, & 0 \leq n \leq z_1 - 1 \\ x(n - z_1), & z_1 \leq n \leq z_1 + M - 1 \\ 0, & z_1 + M \leq n \leq N - 1 \end{cases} \quad (10)$$

$$h_N(n) = \begin{cases} 0, & 0 \leq n \leq z_2 - 1 \\ h(n - z_2), & z_2 \leq n \leq z_2 + L - 1 \\ 0, & z_2 + L \leq n \leq N - 1 \end{cases} \quad (11)$$

where $0 \leq z_1 < N$ and $0 \leq z_2 < N$, ($z_1, z_2 \in \mathbb{Z}$) are the number of zeros to be padded before $x(n)$ and $h(n)$, respectively, and let $\hat{x}_{2N}(n)$ and $\hat{h}_{2N}(n)$ be SESs of $x_N(n)$ and $h_N(n)$, respectively, according to (1).

The linear convolution, $\hat{y}(n) = \hat{x}_{2N}(n) * \hat{h}_{2N}(n)$ can be expressed by the superposition of four linear convolutions as

$$\hat{y}(n) = y^{(1)}(n) + y^{(2)}(n) + y^{(3)}(n) + y^{(4)}(n) \quad (12)$$

where

$$y^{(1)}(n) = \begin{cases} x(n) * h(n), & l_1 \leq n \leq l_1 + P - 1 \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

$$y^{(2)}(n) = \begin{cases} x(n) * h(-n - 1), & l_2 \leq n \leq l_2 + P - 1 \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

$$y^{(3)}(n) = \begin{cases} x(-n - 1) * h(n), & l_3 \leq n \leq l_3 + P - 1 \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

$$y^{(4)}(n) = \begin{cases} x(-n - 1) * h(-n - 1), & l_4 \leq n \leq l_4 + P - 1 \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

and

$$l_1 = z_1 + z_2, \quad (17)$$

$$l_2 = z_1 + 2N - L - z_2, \quad (18)$$

$$l_3 = 2N - M - z_1 + z_2, \quad (19)$$

$$l_4 = 2N - M - z_1 + 2N - L - z_2. \quad (20)$$

That is, $y^{(2)}(n)$ is the linear correlation between $x(n)$ and $h(n)$ and $y^{(1)}(n)$ is the linear correlation between $x(n)$ and $h(n)$ in reverse order. In addition, they are related to as

$$y^{(1)}(n) = y^{(4)}(-n - 1), \quad (21)$$

$$y^{(2)}(n) = y^{(3)}(-n - 1). \quad (22)$$

3.2. 2-D double linear correlation (2-D DLC)

We consider the linear convolution between an $M \times M$ image, $x(n_1, n_2)$, and an $L \times L$ image, $h(n_1, n_2)$.

$2N \times 2N$ images $x_{2N}(n_1, n_2)$ and $h_{2N}(n_1, n_2)$ are defined as

$$x_{2N}(n_1, n_2) = \begin{cases} x(n_1 - z_1, n_2 - z_1), & R_x \\ 0, & \text{otherwise} \end{cases} \quad (23)$$

$$h_{2N}(n_1, n_2) = \begin{cases} h(n_1 - z_2, n_2 - z_2), & R_h \\ 0, & \text{otherwise} \end{cases} \quad (24)$$

where z_1 and z_2 are the number of zeros that are padded before images $x(n_1, n_2)$ and $h(n_1, n_2)$, respectively, and

$$R_x = \begin{cases} z_1 \leq n_1 \leq z_1 + M - 1 \\ z_1 \leq n_2 \leq z_1 + M - 1 \end{cases}, \quad (25)$$

$$R_h = \begin{cases} z_2 \leq n_1 \leq z_2 + L - 1 \\ z_2 \leq n_2 \leq z_2 + L - 1 \end{cases}. \quad (26)$$

The SESs $\hat{x}_{2N}(n_1, n_2)$ and $\hat{h}_{2N}(n_1, n_2)$ are expressed as

$$\hat{x}_{2N}(n_1, n_2) = x_{2N}(n_1, n_2) + x_{2N,2N}(-n_1, n_2) + x_{2N}(n_1, -n_2) + x_{2N}(-n_1, -n_2) \quad (27)$$

$$\hat{h}_{2N}(n_1, n_2) = h_{2N}(n_1, n_2) + h_{2N,2N}(-n_1, n_2) + h_{2N}(n_1, -n_2) + h_{2N}(-n_1, -n_2) \quad (28)$$

respectively.

The linear convolution, $\hat{y}(n_1, n_2) = \hat{x}_{2N}(n_1, n_2) * \hat{h}_{2N}(n_1, n_2)$ can be expressed by the superposition of 16 linear convolutions, each of those linear convolutions is denoted as

$$y^{(i,j)}(n_1, n_2) = \begin{cases} y^{(i)}(n), & \text{with respect to } n_1 \\ y^{(j)}(n), & \text{with respect to } n_2 \end{cases} \quad (29)$$

according to (13) through (16). For example, $y^{(2,1)}(n_1, n_2)$ includes $x(n_1, n_2) * h(-n_1 - 1, n_2)$ over ($l_2 \leq n_1 \leq l_2 + P - 1$ and $l_1 \leq n_2 \leq l_2 + P - 1$).

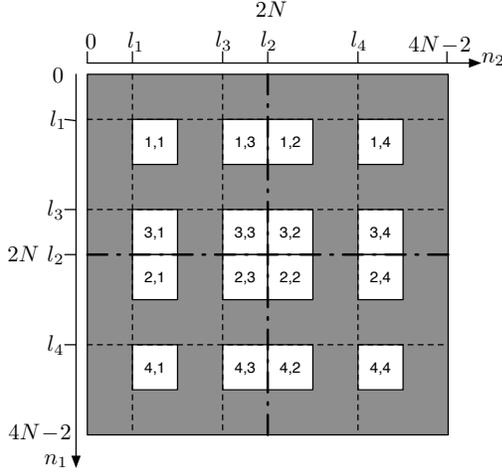


Fig. 1. Location of 16 linear convolutions in linear convolution between SESs. The region in white denotes the region of each linear convolution where only the superscript i, j of $y^{(i,j)}(n_1, n_2)$ is described and the region in gray denotes zero values.

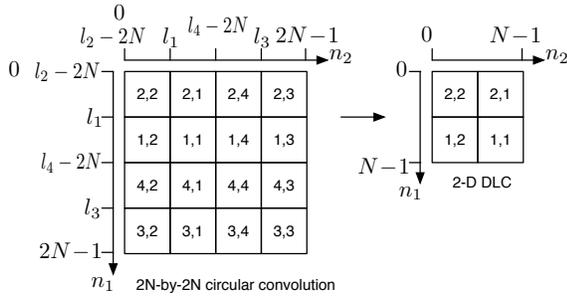


Fig. 2. Location of 16 linear convolutions in the circular convolution between SESs. Only the superscript i, j of $y^{(i,j)}(n_1, n_2)$ is described.

From (13) through (16), the condition for which the 16 linear convolutions are isolated is given as

$$z_1 \geq z_2 + L, \quad (30)$$

$$z_2 > (M - 2)/2, \quad (31)$$

$$N \geq z_1 + z_2 + P + 1. \quad (32)$$

Under the condition, the 16 linear convolutions are located without being superimposed as illustrated in Fig.1.

In the circular convolution between SESs, the output samples over $2N \times 2N$ are wrapped around, which changes the location of the 16 linear convolutions as illustrated in Fig. 2. Note that the left-upper quarter of $\hat{y}_{2N}(n_1, n_2)$ can be obtained using DCTs according to (6).

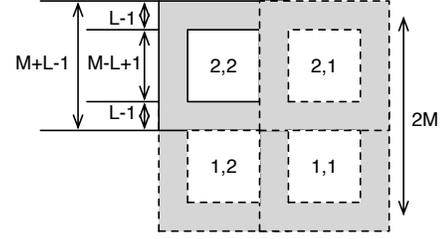


Fig. 3. Limited 2-D DLC. The part in white expresses those parts of the convolution that are computed without the zero-padded edges. Only the superscript i, j of $y^{(i,j)}(n_1, n_2)$ is described.

3.3. Limited double linear correlation

If $M > L$, the upper-left location of $h(n_1, n_2)$ is limited over $(0, 0)$ through $(M - L, M - L)$ in $x(n_1, n_2)$. The number of zero-padding can be reduced by superimposing those parts of the convolution that are computed with the zero-padded edges as illustrated in Fig. 3. The condition for which those parts are superimposed is given from (13) through (16) as

$$z_1 \geq M/2, \quad (33)$$

$$z_2 \geq (M - L)/2, \quad (34)$$

$$N \geq 2M. \quad (35)$$

As a result, the number of zero-padding for DLC is reduced from $2(M + L - 1)$ to $2M$, which is effective especially when L is significantly less than M .

DCT-SPC can be applied to DLC straightforwardly according to (9), which reduces the computational load on DLC and removes the effect of the convolution terms on DCT-SPC.

4. SIMULATIONS

4.1. Mirror image matching

We performed the proposed method for mirror image matching between an $M \times M$ image and an $L \times L$ image.

Figure 4(a) shows the result when one of the images mirrored across the horizontal axis and Fig. 4(b) shows the result when one of the images mirrored across the vertical axis where $M = 160$ and $L = 80$. A peak appears in the region that expresses mirror pattern, and the location in $M \times M$ image was estimated correctly from the location of the peak.

4.2. Computational complexity

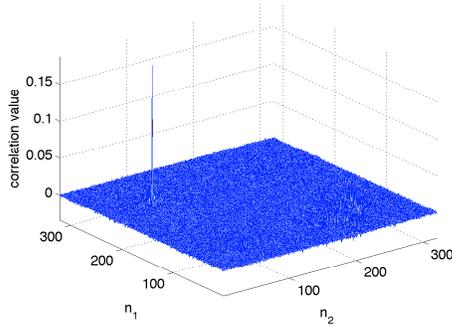
We evaluated the computational complexity comparing with

- FFT approach (the circular correlation between $h(n_1, n_2)$ and the mirror image that is generated from $x(n_1, n_2)$)
- 2-D DLC

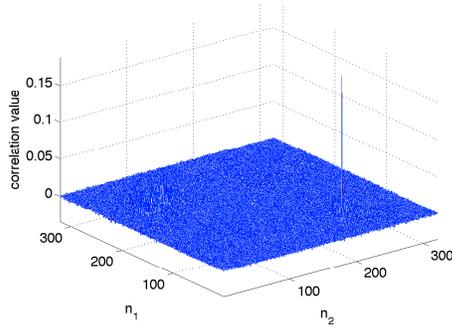
Table 1. Computational complexity for mirror image matching between an $M \times M$ image and an $L \times L$ image.

		a) FFT approach (on complex numbers)	b) 2-D DLC (on real numbers)	c) 2-D DLC using DCT signs (on real numbers)
transform	mu.	$2 \times Q^2 \log_2 Q$	$2 \times (N^2 \log_2 N + 2N)$	$2 \times (N^2 \log_2 N + 2N)$
	ad.	$2 \times 2Q^2 \log_2 Q$	$2 \times (3N^2 \log_2 N - 2N^2 + 2N)$	$2 \times (3N^2 \log_2 N - 2N^2 + 2N)$
product	mu.	Q^2	N^2	-
inverse transform	mu.	$1 \times Q^2 \log_2 Q$	$1 \times (N^2 \log_2 N + 2N)$	1
	ad.	$1 \times 2Q^2 \log_2 Q$	$1 \times (3N^2 \log_2 N - 2N^2 + 2N)$	$N^2 - 1$

$Q = 2M + L - 1$ and $N = 2(M + L - 1)$ for full version, and $Q = 2M$ and $N = 2M$ for limited version.



(a) Mirrored across the horizontal axis

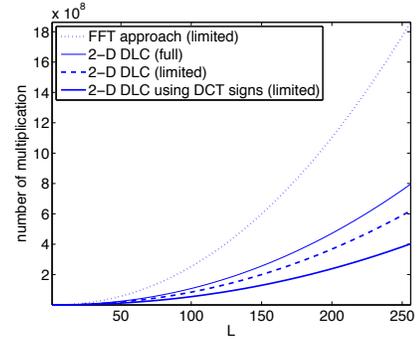


(b) Mirrored across the vertical axis

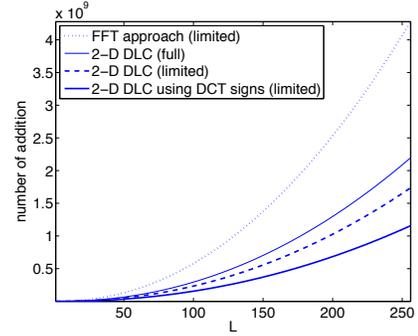
Fig. 4. 2-D DLC using DCT signs. A peak appears in the region that expresses the mirror pattern.

c) 2-D DLC using DCT signs (DCT-SPC)

Table 1 summarizes the computational complexity for a) to c). 2-D FFT and Fast 2-D DCT use row/column approaches with 1-D FFT [8] and Fast 1-D DCT [9][10] algorithms, respectively. Figures 5(a) and 5(b) show the computational complexity on real numbers for a) to c) where $M = 8L$ in which one multiplication calculation on complex numbers is converted to three multiplication calculations and three addition calculations on real numbers by Nakayama's method [11], and one addition calculation on complex numbers is converted to two addition calculations on real numbers. We



(a) multiplication



(b) addition

Fig. 5. Computational complexity on real numbers for mirror image matching where $M = 8L$.

can confirm that the computational complexity of 2-D DLC using DCT signs limited version is less than 25 % of that of the FFT approach limited version on the number of multiplication calculations and is roughly 25% of that of the FFT approach on the number of addition calculations.

5. CONCLUSION

Low computational load linear correlation method was proposed for mirror image matching. This was achieved by combining DLC and DCT-SPC and by obtaining only those parts of the linear convolution that are computed without zero-padded edges.

6. REFERENCES

- [1] I. Ito and H. Kiya, "DCT sign-only correlation with application to image matching and the relationship with phase-only correlation," ICASSP 2007 pp.I-1237–I-1240.
- [2] I. Ito and H. Kiya, "Multiple peak model fitting function for DCT sign phase correlation with non-integer shift precision," ICASSP 2009 pp.449-452.
- [3] S. A. Martucci, "Symmetric convolution and the discrete sine and cosine transforms," *IEEE Trans. Signal Process.*, vol.42, no.5, May 1994.
- [4] I. Ito and H. Kiya, "A computing method for linear convolution in the DCT domain," 19th European Signal Processing Conference (EUSIPCO 2011), pp.323-327, Aug. 2011.
- [5] I. Ito, "A computing method for linear convolution and linear correlation in the DCT domain," *IEICE Trans. A*, Vol.E96-A, No.7, pp. 1518-1525, Jul. 2013.
- [6] I. Ito and K. Egiazarian, "Double linear correlation using DCT and its application to forward and reverse matching," Proc. International symposium on image and signal processing and analysis, pp.409-413, 2013.
- [7] A. V. Oppenheim, R. W. Schaffer, and J. R. Buck, *Discrete-time Signal Processing*, Prentice Hall, 1999.
- [8] H. J. Nussbaumer, *Fast Fourier transform and convolution algorithms*, Springer-Verlog, 1982.
- [9] Z. Wang, "Fast algorithms for the discrete W transform and for the discrete Fourier transform," *IEEE Trans. Acoust., Speech, and Signal Process.*, vol. 32, no. 4, pp.803–816, 1984.
- [10] Z. Wang, "On computing the discrete Fourier and cosine transforms," *IEEE Trans. Acoust., Speech, Signal Process.*, vol.33, no.4, pp.1341-1344, 1985.
- [11] K. Nakayama, "A new discrete Fourier transform algorithm using butterfly structure fast convolution," *IEEE Trans. Acoust. Speech Signal Process.*, vol.ASSP-33, no.4, pp.1197–1208, Oct. 1985.