A Parametric Approach to Optimal Soft Signal Relaying in Wireless Parallel-Relay Systems

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Abstract—This paper proposes an optimal estimate-forward strategy, termed Z-forwarding, for 2-hop parallel-relay systems. The previous tanh-forwarding strategy, which is optimized for the single-relay system is shown to be no longer optimal for a parallel-relay system. Instead, a new, parametrically-optimized Z-forwarding strategy is proposed, where the relay re-transmits a nonlinear but piece-wise linear function of the log-likelihood ratio (LLR) of the source signal. By analytically formulating the end-to-end bit error rate (BER), optimal thresholds that minimize the BER are computed as a function of all the source-relay and relay-destination channels. Maximum likelihood (ML) detector is also developed for the destination to recoup all the diversity gains from the multiple relays. It is shown that Z-forwarding strategy delivers a performance comparable to tanh-forwarding in a single relay system, but considerably better than tanh-forwarding (as well as amplify-forward and decode-forward) in a parallel-relay system.

I. INTRODUCTION

Both regenerative and non-regenerative signal relaying strategies have been extensively studied in the literature of user cooperation. Notable examples include *amplify-forward* (AF) for the non-regenerative strategies and *decode-forward* (DF) for the regenerative strategies [1]. Geometry-inclusive performance analysis [2] and channel signal-to-noise ratio (SNR) based analysis [3] for single relay systems show that AF and DF each has its advantages in different scenarios. In addition to the pure regenerative and the pure non-regenerative, there is also the proposal of compute-forward (CF), estimate-forward (EF) strategies (e.g. [3] [4]), and soft-decoding-forward (SDF) [5] [6] such as decode-amplify-forward (DAF) [7] and the soft mutual information forwarding protocols [8]. These strategies generalize the conventional DF practice by allowing the relay to generate a function, rather than a mere duplication, of the source signal, where the function may either reflect a level of reliability estimate of the signal, or a transformation of the signal in some signal or codeword space. Such generalization promises additional gains in many scenarios. For example, it is shown in [7] that in the low SNR region, DAF can double the capacity of AF (which is already the better choice compared to DF). To combine all the merits of previous forwarding strategies, opportunistic selection between AF and DF is proposed, with several switching criterion based on SNR and CRC (cyclic redundancy check) being studied [9] [10]. There is also the study of selective relay strategies based on SNR [11] [12] or log-likelihood ratio (LLR) [13] [14].

Early works of relaying strategies concentrated on a single active relay. Recent studies have shifted the focus to multipoint cooperation with a set of relays. The involvement of multiple relays have opened up new opportunities for efficient inter-relay cooperative strategies. At the same time, however, it also poses considerable complexity in the system design, sometimes rendering non-convex problems that are analytically intractable.

This paper considers a 2-hop parallel relaying model as shown in Fig. 1, where the system pulls together the joint effort of multiple relays for an enhanced spatial diversity. Rather than consider selection among the relays, here we are primarily interested in how the active relays should cooperate to perform estimate-forward. It is shown in the studies of Jafar [3] and Abou-Faycal *et al* [4] that in a 2-hop single-relay system, having the relay forward $\tanh \frac{LLR(x)}{2}$, where LLR(x) is the log-likelihood ratio (LLR) of the source-relay transmission, is an optimal strategy in terms of SNR maximization and bit error rate (BER) minimization. This *tanh*-forwarding strategy also bears a close relation to the so-called check-operation¹.

The question we ask here is what happens with multiple relays. Our studies show that the *tanh*-forwarding strategy that is optimal for the single-relay system is no longer optimal for the parallel-relay system. To compute the *tanh* value requires only the knowledge of the source-relay channel, and not that of the relay-destination channel. This can be an advantage for sing-relay systems (i.e. simplicity), but is also an opportunity lost for parallel-relay systems where an effective estimate-forward strategy must consider the collaborative effect of all the parallel source-relay-destination links.

The major contribution of this paper is the proposition of a threshold-based estimate-forwarding strategy, termed "Zforwarding." The *tanh*-forwarding strategy proposed in [3] [4] uses a nonlinear function of LLR(x), which makes it very difficult to derive the theoretical error probabilities or the ML detector at the destination. In comparison, the new Z-forwarding strategy uses a nonlinear but piece-wise linear function of LLR(x). Specifically, taking the thresholds as the the parameter, we are able to derive the end-to-end BER of the parallel system as a function of the thresholds, and formulate the optimal signal relaying strategy as an optimization problem. We show that the optimal thresholds must depend on all the source-relay and relay-destination channels, as well as the targeting BER. We also derive the optimal maximum-likelihood (ML) detector that can best exploit all the parallel relaying signals at the destination. We show that in a

¹The check operation of a three binary random variables a, b, c forming a check of $a \oplus b = c$ is $\tanh \frac{LLR(a)}{2} \tanh \frac{LLR(b)}{2} = \tanh \frac{LLR(c)}{2}$.

single-relay system, Z-forwarding delivers practically the same performance as *tanh*-forwarding, but in a multi-relay system, it delivers noticeably better performance than *tanh*-forwarding.

II. SYSTEM MODEL

The relay system model, shown in Fig. 1, consists of a source S, a destination D, and M parallel relays R_i , i =1, 2...M. Since the technical objective of this paper is not about relay selection, all the relays are assumed to be active relays, participating in every communication session in a collaborative and trustworthy manner. We consider binary phase shift keying (BPSK), and quasi-static fading for all the channels, where the fading coefficients are fixed over the course of each communication session, but changes independently from session to session. (Quasi-static fading is when user cooperation is most helpful.) Following the conventions in the literature [15], we assume that the channel state information (CSI) is known for all the links, and the optimal parameters of the relay mechanisms are computed based on these CSIs in a centralized manner. This can be expensive to achieve in practice, but it sheds useful insight into what optimal strategies are like, as well as provides an error rate lower bound of what can be achieved.

Each communication session consists of two phases. In the first phase, the source S broadcasts the signal x_S to all the relays R_i , with an average power level of E_{SR} (per data bit). The relay R_i receives:

$$y_{SR_i} = \sqrt{E_{SR}} h_{SR_i} x_S + n_{SR_i}, \quad i = 1, 2, \cdots, M,$$
 (1)

where $x_S \in \{+1, -1\}$, h_{SR_i} is the Rayleigh fading coefficient, and n_{SR_i} is a complex additive white Gaussian noise (AWGN): $n_{SR_i} \sim \mathcal{N}(0, \sigma_{SR_i}^2)$. The per-session SNR of the channel S- R_i is given by $\gamma_{SR_i} = \frac{E_{SR}h_{SR_i}^2}{2\sigma_{SR_i}^2}$.

In the second phase, all the relays send their processed signals $\beta_i l_i$ through orthogonal channels (e.g. time division or frequency division), which reach the destination D as:

$$y_{R_iD} = h_{R_iD}\beta_i l_i + n_{R_iD}, \quad i = 1, 2, \cdots, M,$$
 (2)

where $n_{R_iD} \sim \mathcal{N}(0, \sigma_{R_iD}^2)$, and β_i is the power amplification factor at relay R_i . Each relay R_i use the same transmit power E_{R_iD} , and the per-session SNR of the R_i -D channel is given by $\gamma_{R_iD} = \frac{E_{R_iD}h_{R_iD}^2}{2\sigma_{R_iD}^2} = \frac{E(|\beta l_i|^2)h_{R_iD}^2}{2\sigma_{R_iD}^2}$. The destination D collects all the M signals y_{R_iD} to make the best decision x_D .



III. Z-FORWARDING STRATEGY

We now present the proposed Z-forwarding strategy in the context of parallel relays (Fig. 1).

Upon receiving the signals from the source S, each relay R_i first calculates the LLR of the reception, which is a function of the *i*th source-relay channel:

$$LLR_{i} = \frac{2\sqrt{E_{SR}}h_{SR_{i}}}{\sigma_{SR_{i}}^{2}}y_{SR_{i}}, \quad i = 1, 2, \cdots, M.$$
(3)

Inserting (1) into (3), the LLR of the ith source-relay transmission becomes

$$LLR_{i} = \frac{2\sqrt{E_{SR}}h_{SR_{i}}}{\sigma_{SR_{i}}^{2}}(\sqrt{E_{SR}}h_{SR_{i}}x_{S} + n_{SR_{i}})$$
$$= \frac{2E_{SR}h_{SR_{i}}^{2}}{\sigma_{SR_{i}}^{2}}x_{S} + \frac{2\sqrt{E_{SR}}h_{SR_{i}}}{\sigma_{SR_{i}}^{2}}n_{SR_{i}}$$
$$= m_{i}x_{S} + n_{1i}$$
(4)

where $m_i = \frac{2E_{SR}h_{SR_i}^2}{\sigma_{SR_i}^2}$ and $n_{1i} \sim \mathcal{N}(0, \sigma_{1i}^2) = \mathcal{N}(0, \frac{4E_{SR}h_{SR_i}^2}{\sigma_{SR_i}^4})$. In the proposed Z-forwarding, what the relay S_i forwards

In the proposed Z-forwarding, what the relay S_i forwards to the destination is a simple three-segment piece-wise linear function of LLR_i . Specifically, for each source-destination channel S- D_i , we define a threshold $t_i (\geq 0)$, such that the message forwarded to the destination, l_i , is a truncated version of LLR_i with respect to t_i .

$$l_{i} = \begin{cases} t_{i}, & LLR_{i} \ge t_{i}, \\ -t_{i}, & LLR_{i} \le -t_{i}, \\ LLR_{i}, & \text{otherwise}, \end{cases}$$
(5)

Clearly, the determination and optimization of the threshold t_i for $i = 1, 2, \dots, M$ is key to the system design (next subsection). When $t_i = \infty$, the relays will forward the exact LLRs, and the estimate-forward system degenerates to the conventional amplify-forward. When $t_i = 0$, the system degenerates to the conventional decode-forward. While preparing for the manuscript, we learned the work of [16], which presents a special case of Z-forwarding, with a fixed threshold set to $t = 2\sqrt{3}$ for all the relays, irrespective of number of relays and the quality of channels. Such a fixed choice is simple (and slightly better than AF and DF), but far from optimal.

A. BER Performance and Threshold Selection

Before proceeding to threshold optimization, we note that for single relay systems, tanh(LLR/2) is shown to be the optimal (soft) relaying message that will maximize the endto-end SNR or minimize the BER at the destination [3]. However, since $tanh(LLR_i/2)$ is a nonlinear function whose probability density function (pdf) is hard to track, the optimal estimator at the destination becomes rather hard to derive also, especially when there are multiple relays. Further, as one would expect, in a multi-relay system with distinctive sourcerelay-destination channels, the optimal process must account for all the specific channel conditions.

There are a number of ways to define system optimality, including maximum mutual information and minimum bit error rate. The idea here is to formulate the average BER at the final destination as a function of the relay thresholds t_i and

the instantaneous channel condition, and derive the optimal thresholds t_i that will minimize the BER.

For simplicity, we start with the typical diamond network with a source, a destination and two active relay nodes, where the destination integrates the two receptions R_1 and R_2 via maximum ratio combining (MRC).

To compute the BER requires the knowledge of the exact pdf of each signal transmitted through each source-relaydestination channel. Let $f(l_i|x_S = +1)$ and $f(l_i|x_S = -1)$ be the pdf of l_i , the message forwarded by the relay S_i , conditioned on $x_s = +1$ and $x_s = -1$, respectively. From (5), the exact pdf of l_i can be expressed by

$$f(l_i|x_S = +1) = \delta(l_i - t_i)Q(\frac{t_i - m_i}{\sigma_{1i}}) +$$
(6)
$$\frac{1}{\sqrt{2\pi\sigma_{1i}}}e^{\frac{-(l_i - m_i)^2}{2\sigma_{1i}^2}} + \delta(l_i + t_i)Q(\frac{t_i + m_i}{\sigma_{1i}}),$$

$$f(l_i|x_S = -1) = \delta(l_i - t_i)Q(\frac{t_i + m_i}{\sigma_{1i}}) +$$

$$\frac{1}{\sqrt{2\pi\sigma_{1i}}}e^{\frac{-(l_i + m_i)^2}{2\sigma_{1i}^2}} + \delta(l_i + t_i)Q(\frac{t_i - m_i}{\sigma_{1i}}),$$

where $\delta(x)$ is the Dirac delta function of x. Let $f(n_{R_iD})$ be the pdf of the noise n_{R_iD} . We have

$$f(n_{R_iD}) = \frac{1}{\sqrt{2\pi\sigma_{R_iD}}} e^{\frac{-(n_{R_iD})^2}{2\sigma_{R_iD}^2}}.$$
 (7)

The pdf of the received signal from each relay node R_i over a specific relay channel can be expressed by

$$f(y_{R_iD}|x_S=+1) = \frac{1}{\beta_i h_{R_iD}} f(\frac{l_i}{\beta_i h_{R_iD}} |x_S=+1) \otimes f(n_{R_iD}),$$
(8)

$$f(y_{R_iD}|x_S = -1) = \frac{1}{\beta_i h_{R_iD}} f(\frac{l_i}{\beta_i h_{R_iD}} |x_S = -1) \otimes f(n_{R_iD}),$$
(9)

where \otimes indicates the convolutional operation.

The destination uses the MRC method to combine the signals and then proceeds with ML decoding. Let α_i be the combining weight for the signal from relay R_i . To facilitate the computation of α_i , we consider approximating the truncated LLRs with as a Gaussian distribution² with mean μ_i and variance $\tilde{\sigma}_i^2$ [17]. Consequently, the LLRs extracted from the source-relay channel can be approximated by

$$l_i = \mu_i x_S + \tilde{n}_i \tag{10}$$

$$\mu_{i} = t(Q(\frac{t_{i} - m_{i}}{\sigma_{1i}}) + Q(\frac{t_{i} + m_{i}}{\sigma_{1i}})) + \int_{-t}^{t} |l_{i}| \frac{1}{\sqrt{2\pi\sigma_{1i}}} e^{\frac{-(l_{i} - m_{i})^{2}}{2\sigma_{1i}^{3}}} dl_{i}$$
(11)
$$\tilde{\sigma}_{i}^{2} = E\left[|l_{i}|^{2}\right] - \mu_{i}^{2}$$

²The approximation is only used to compute the MRC combining weights, the exact pdfs will be used to compute the BER.

Where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} du$. Thus the received signal at the destination from relay *i* can be written as

$$y_{R_iD} = h_{R_iD}\beta_i(\mu_i x_S + \tilde{n}_i) + n_{R_iD}$$
$$= h_{R_iD}\beta_i\mu_i x_S + h_{R_iD}\beta_i\tilde{n}_i + n_{R_iD}, \qquad (12)$$

Which leads to the weight factors for MRC

$$\alpha_i = \frac{\beta_i \mu_i h_{R_i D}}{\beta_i^2 h_{R_i D}^2 \tilde{\sigma}_i^2 + \sigma_{R_i D}^2}.$$
(13)

Thus, the combined signal at the destination becomes

$$y_D = \alpha_1 y_{R_1 D} + \alpha_2 y_{R_2 D} \tag{14}$$

Consider equal-probable transmission of +1 and -1 at the source. Without loss of generality, we use $x_s = +1$ to derive an analytical expression for the BER at the destination:

$$P_{e}(t_{1}, t_{2}) = P(y_{D} < 0) = P(\alpha_{1}y_{SR_{1}} + \alpha_{2}y_{SR_{2}} < 0)$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{-\frac{\alpha_{1}}{\alpha_{2}}y_{R_{1}D}} f(y_{R_{2}D}|x_{S} = +1)f(y_{R_{1}D}|x_{S} = +1)dy_{R_{2}D}dy_{R_{1}D},$$

where t_1 and t_2 are the threholds that will directly affect the BER value.

The problem of finding the optimal thresholds is therefore formulated as the following optimization problem:

min
$$P_e(t_1, t_2)$$
 (15)
s.t. $\beta_i^2 = \frac{E_{R_i D}}{E[|l_i^2|]}, \ i = 1, 2$
 $t_1, t_2 \ge 0$

In general, the BER $P_e(t_1, t_2)$ is a function of the channel conditions of four channel segments, $S-R_1$, $S-R_2$, R_1-D , and R_2 -D. The threshold is evaluated numerically via an exhaustive grid search method [18]. Here we present the optimal threshold values for the system under additive white Gaussian noise (AWGN) channels with $\sigma_{SR_1}^2 = \sigma_{SR_2}^2$ and $\sigma_{R_1D}^2 = \sigma_{R_2D}^2$. In this scenario, the weight coefficient and the threshold are the same for both signals from the two relays. These results, depicted in Fig. 2, fit both the AWGN channel and the block fading channel (as block Rayleigh fading channel SR_i and $R_i D$ can be taken as an equivalent Gaussian channel with SNR equaling $|h_{SR_i}|^2 E_{SR_i} / 2\sigma_{SR_i}^2$ and $|h_{R_iD}|^2 E_{R_iD} / 2\sigma_{R_iD}^2$). A similar analysis can be performed for systems with more than 2 relays, but the computation is considerably more complex.

IV. ML ESTIMATION

There are two good decoding methods for the destination. The first is based on MRC discussed previously (marked out as "Z-MRC" in the simulation): After the destination combines the received signals via MRC (13), it makes a hard decision on y_D :

$$x_D = \begin{cases} 1, & y_D \ge 0\\ -1, & y_D < 0 \end{cases}$$
(16)

An alternative approach is the maximum-likelihood (ML) detection. Given the accurate pdf expression of y_{R_iD} (as



Fig. 2: The optimal thresholds t_i of Z-forwarding in a tworelay system

a function of the threshold), the ML estimator makes the following decision (marked out as "Z-ML" in the simulation):

$$x_D = \begin{cases} +1, \prod_{i=1}^{2} f(y_{R_iD}|x_S=+1) \ge \prod_{i=1}^{2} f(y_{R_iD}|x_S=+1) \\ -1, \text{ o.w.} \end{cases}$$
(17)

V. NUMERICAL RESULTS

We now evaluate the proposed Z-forwarding strategy via Monte Carlo simulation. BPSK modulation is used. The signals that are transmitted, either at the source or at the relay, always have energy normalized to unit before being put on air.

We first test the scheme with different fixed thresholds over AWGN channels in a diamond network. Suppose all the 4 channel segments have the same SNR. The SNR value varies from 0 to 10. As shown in Fig. 3, the BER performance reveals obvious difference with different thresholds. When SNR_{R_iD} is low, smaller thresholds tend to achieve better performance; as SNR increases, the optimal threshold value also increases. This is consistent with the numerical results in Fig. 2.

The BER performances of five schemes, AF, DF, EF based on *tanh*-forwarding [3], and the proposed Z-MRC and Z-ML schemes, for both the single-relay and the two-relay systems, are compared in Fig. 4 and 5 with different channel qualities. While *tanh*-forwarding is optimal and performs on par with our proposed Z-forwarding in the single-relay case, Z-forwarding is 1 dB better than the *tanh*-forwarding at BER of 10^{-6} (as well as the others).

VI. CONCLUSION

We have proposed a parametric approach to soft signal relaying in parallel-relay networks. We derived the theoretical BER performance for a diamond network. The parameters are selected to minimize the overall BER. The analytical tractability of the the proposed Z-forwarding allows the exploitation of ML estimation, in addition to MRC, at the destination. Extensive simulations show that Z-forwarding with ML estimation is capable of better BER than the conventional strategies, including the previously proposed *tanh*-forwarding that is optimal for the single-relay case.



Fig. 3: BER performance of Z-forwarding with different thresholds over AWGN channels, 2 relay nodes, $SNR_{SR_i} = SNR_{R_iD}$



Fig. 4: BER of different schemes under block Rayleigh fading channels. $SNR_{SR_1} = SNR_{R_1D} = SNR_{SR_2} = SNR_{R_2D}$.

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Fig. 5: BER of different schemes under block Rayleigh fading channe. $SNR_{SR_1} = SNR_{SR_2} = SNR_{R_1D} + 3dB = SNR_{R_2D} + 3dB$.

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