ROBUST BEAMFORMING IN TWO-WAY RELAY NETWORKS: QUARTICALLY PERTURBED CHANCE CONSTRAINED FORMULATION AND TRACTABLE APPROXIMATION

Shuai Ma†*, Anthony Man-Cho So*‡*, and Kehu Yang*†

†ISN Lab, Xidian University, Xi'an 710071, China
‡ Dept. of Sys. Eng. & Eng. Mgmt., The Chinese University of Hong Kong, Shatin, N. T., Hong Kong
Email: {mashuai@stu., yang001@}xidian.edu.cn†, manchoso@se.cuhk.edu.hk‡

ABSTRACT

In this paper, we consider an outage-based robust beamforming problem in two-way relay networks under the imperfect channel state information (CSI) scenario. Specficially, our goal is to minimize the relay transmit power while keeping the probability of each user's signal-to-interference-plus-noise ratio (SINR) outage as caused by the imperfect CSI below a given threshold. Assuming that the CSI errors follow a complex Gaussian distribution, the probabilistic SINR constraints involve *quartic* polynomials of complex Gaussian random variables, which, to the best of our knowledge, have not been treated from a computational perspective before. Using moment inequalities for Gaussian polynomials and the semidefinite relaxation technique, we propose a new tractable approximation approach for tackling such constraints. Simulation results show that the proposed method outperforms the existing robust approaches when the CSI errors are large.

Index Terms— Two-way relaying, robust beamforming, chance constrained optimization, semidefinite relaxation

1. INTRODUCTION

In recent years, two-way relaying—*i.e.*, the use of relay nodes to establish a communication link between two users—has attracted significant interest, as it can greatly improve the spectral efficiency and extend the coverage of wireless networks [1–4]. To facilitate information exchange, the most commonly adopted transmit strategy is beamforming, which requires channel state information (CSI). However, in practice, the available CSI is typically inaccurate due to errors in channel estimation and limited feedback. Such imperfect CSI will lead to residual self-interference and degrade the performance of two-way relaying systems. This motivates the study of relay beamforming schemes that are robust against CSI errors.

Currently, there are two main approaches to modelling CSI errors, which, in the context of relay beamforming, lead to two different classes of robust design problems. The first is to assume that the CSI errors lie within a given bounded set, and the beamformer is designed so that it is robust against the worst-case quality-of-service (QoS) under the given CSI error model [5, 6]. The second is to assume that the CSI errors follow a probabilistic model (typically Gaussian), and the design goal is to provide a certain level of QoS with high probability; cf. [7]. In this paper, we consider the latter approach and assume that the CSI errors arising in the two-way relay system follow a Gaussian distribution. Our goal is to minimize the average transmit power at the relay nodes while satisfying signal-to-interference-and-noise (SINR) constraints of the users with high probability. The main technical challenge of such a formulation lies in the probabilistic SINR constraints, which involve *quartic* polynomials of complex Gaussian random variables and are intractable in general. Prior works (see, e.g., [5–7]) tackle those constraints simply by ignoring the higher-order (*i.e.*, quadratic or above) error terms and applying standard techniques from robust optimization [8] or chance-constrained optimization [9] to the simplified constraints. However, such an approach is not satisfactory, as it is not clear whether the higher-order error terms really have a negligible effect on system performance.

In this paper, we develop a new approach for handling chance constraints that involve a general quartic Gaussian polynomial. By combining this approach with the semidefinite relaxation (SDR) technique [10], we show that the aforementioned probabilistic SINR constraints can be approximated by a set of linear matrix inequalities, which are efficiently computable. Lastly, we demonstrate the efficacy of our approach via simulations.

Notations: We use $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$, $\mathrm{Tr}(\cdot)$, $\|\cdot\|$, $\odot$, and $\mathbb{C}^N$ to denote conjugate, transpose, conjugate transpose, trace, Frobenius norm, Hadamard product, and the set of N-dimensional complex vectors, respectively. We write $\boldsymbol{x} \sim \mathcal{CN}(\boldsymbol{m}, \boldsymbol{V})$ to mean that $\boldsymbol{x} - \boldsymbol{m}$ is a circularly symmetric complex Gaussian random vector with covariance $\boldsymbol{V}$.

This work was supported in part by the Major Project of National Natural Science Foundation of China (NSFC) Grant 10990012, in part by the 111 Project No. B08038, and in part by the Hong Kong Research Grants Council (RGC) General Research Fund (GRF) Project CUHK 416012.

978-1-4799-2893-4/14/$31.00 ©2014 IEEE

2. PROBLEM FORMULATION

2.1. System Model

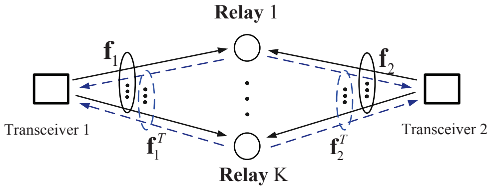

Fig. 1. A two-way relay network

Consider a two-way relay network, in which a pair of users communicate with each other through K relay nodes; see Fig. 1. Each node in the network is equipped with a single antenna and is half-duplex. The communication is performed in two phases. In the first, or the multiple access (MAC) phase, both users transmit their data simultaneously to the relay nodes. In the second, or the broadcast (BC) phase, the relay nodes broadcast the weighted versions of their received signals to the two users over the same channels. We assume that all channels are flat fading and constant during the two phases. Moreover, we assume that the channels in the MAC phase are reciprocal to those in the BC phase. Now, let $\boldsymbol{f}_i \in \mathbb{C}^K$ denote the channel vector from the ith user to the relay nodes, where $i = 1, 2$. Then, in the MAC phase, the received signal vector at the relay nodes can be modelled as

$$\boldsymbol{y}_R = \boldsymbol{f}_1 s_1 + \boldsymbol{f}_2 s_2 + \boldsymbol{n}_R,$$

where s_i is the signal from the ith user with $\mathbb{E}\left[|s_i|^2\right] = P_i$, and $\boldsymbol{n}_R \sim \mathcal{CN}\left(0, \sigma_R^2 \boldsymbol{I}\right)$ is the additive noise at the relay nodes. In the BC phase, the relay nodes broadcast the weighted signal

$$\boldsymbol{x}_R = \boldsymbol{w} \odot \boldsymbol{y}_R = (\boldsymbol{w} \odot \boldsymbol{f}_1)\, s_1 + (\boldsymbol{w} \odot \boldsymbol{f}_2)\, s_2 + \boldsymbol{w} \odot \boldsymbol{n}_R, \quad (1)$$

where $\boldsymbol{w} \in \mathbb{C}^K$ denotes the relay beamformer. The received signal at the ith user is then modelled as

$$\begin{aligned} y_i &= \boldsymbol{f}_i^T \boldsymbol{x}_R + n_i \\ &= \boldsymbol{f}_i^T \left[(\boldsymbol{w} \odot \boldsymbol{f}_i)\, s_i + (\boldsymbol{w} \odot \boldsymbol{f}_j)\, s_j + (\boldsymbol{w} \odot \boldsymbol{n}_R)\right] + n_i, \end{aligned} \quad (2)$$

where $i, j \in \{1, 2\}$ and $i \neq j$, and $n_i \sim \mathcal{CN}\left(0, \sigma_i^2\right)$ is the additive noise at the ith user.

2.2. Imperfect Channel State Information

To capture the effect of CSI errors, we model the actual channels between the users and the relay nodes as

$$\boldsymbol{f}_i = \widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i, \quad i = 1, 2, \quad (3)$$

where $\widehat{\boldsymbol{f}}_i \in \mathbb{C}^K$ is the estimated channel between the ith user and the relay nodes, and $\boldsymbol{e}_i \in \mathbb{C}^K$ is the corresponding channel error vector. Note that in the process of acquiring the CSI, there are two main sources of error, namely, estimation error and quantization error. We consider the scenario where the estimation is not very accurate, but the amount of bits available for feeding back the CSI—which determines the size of the quantization codebook—is sufficient. In this case, the estimation error will be the dominant error in the acquired CSI. It is known that when estimating channels using the minimum mean square error method, the CSI errors tend to follow a Gaussian distribution [11]. Hence, we adopt a Gaussian channel error model; *i.e.*, $\boldsymbol{e}_i \sim \mathcal{CN}\left(\boldsymbol{0}, \boldsymbol{E}_i\right)$ with $\boldsymbol{E}_i \succeq \boldsymbol{0}$. For simplicity, we shall assume that $\boldsymbol{E}_i = \eta_i^2 \boldsymbol{I}$ for some $\eta_i > 0$ in the sequel.

2.3. Outage-Based Robust Beamforming at Relay Nodes

Using (2) and (3), we can express the received signal at the ith user (where $i = 1, 2$) as

$$\begin{aligned} y_i = &\underbrace{(\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)^T \left(\boldsymbol{w} \odot (\widehat{\boldsymbol{f}}_j + \boldsymbol{e}_j)\right) s_j}_{\text{desired signal}} + \underbrace{\widehat{\boldsymbol{f}}_i^T (\boldsymbol{w} \odot \widehat{\boldsymbol{f}}_i) s_i}_{\text{self-interference \#1}} \\ &+ \underbrace{\left\{\boldsymbol{e}_i^T \left(\boldsymbol{w} \odot (\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)\right) + \widehat{\boldsymbol{f}}_i^T (\boldsymbol{w} \odot \boldsymbol{e}_i)\right\} s_i}_{\text{self-interference \#2}} \\ &+ \underbrace{(\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)^T (\boldsymbol{w} \odot \boldsymbol{n}_R) + n_i}_{\text{noise}}. \end{aligned}$$

Note that the part labeled "self-interference #1" can be cancelled using known channel estimates and beamformer coefficients. Hence, the SINR of the ith user can be written as

$$\mathrm{SINR}_i\left(\boldsymbol{w}, \boldsymbol{e}\right) = \frac{P_j \left\|(\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)^T \left(\boldsymbol{w} \odot (\widehat{\boldsymbol{f}}_j + \boldsymbol{e}_j)\right)\right\|^2}{\Phi_i + \sigma_i^2}, \quad (4)$$

where $\Phi_i = P_i \left\|\boldsymbol{e}_i^T \left(\boldsymbol{w} \odot (\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)\right) + \widehat{\boldsymbol{f}}_i^T \left(\boldsymbol{w} \odot \boldsymbol{e}_i\right)\right\|^2 + \sigma_R^2 \cdot \mathrm{Tr}\left(\left(\boldsymbol{w}\boldsymbol{w}^H\right) \odot \left((\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)^* (\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)^T\right)\right)$, and $\boldsymbol{e} = (\boldsymbol{e}_1, \boldsymbol{e}_2)$. Furthermore, using (1), we can express the average transmit power of the relay nodes as

$$\begin{aligned} P_R &= \mathbb{E}\left[\sum_{i=1}^{2} \left\|\boldsymbol{w} \odot (\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i) s_i\right\|^2 + \left\|\boldsymbol{w} \odot \boldsymbol{n}_R\right\|^2\right] \\ &= \mathrm{Tr}\left[\left(\boldsymbol{w}\boldsymbol{w}^H\right) \odot \left(\sum_{i=1}^{2} P_i (\widehat{\boldsymbol{f}}_i \widehat{\boldsymbol{f}}_i^H + \boldsymbol{E}_i) + \sigma_R^2 \boldsymbol{I}\right)\right]. \end{aligned}$$

Now, we are interested in designing the beamformer $\boldsymbol{w}$ so that the average transmit power P_R is minimized while the users' SINR outage constraints are satisfied. Specifically, consider the following problem:

$$\min_{\boldsymbol{w}} \quad P_R \quad (5a)$$

$$\text{s.t.} \quad \Pr\left\{\mathrm{SINR}_i\left(\boldsymbol{w}, \boldsymbol{e}\right) \leq \gamma_i\right\} \leq p_i, \quad i = 1, 2, \quad (5b)$$

where γ_i is the SINR threshold for the i-th user, and $p_i \in (0, 1]$ is its maximum SINR outage probability.

It is easy to verify that the chance constraint (5b) involves a quartic polynomial of complex Gaussian random variables. As such, it is generally non-convex and does not admit a closed-form expression. Currently, a popular approach for tackling chance constraints is to replace them with tractable safe approximations.[1] However, existing safe approximations (see, e.g., [9, 12, 13]) apply only to chance constraints that are linear or quadratic in the random variables. In the sequel, we develop a new approach for constructing tractable safe approximations of chance constraints that involve a general quartic Gaussian polynomial. Then, we show how this approach can be combined with the SDR technique [10] to obtain a tractable approximation of the outage-constrained power minimization problem (5).

3. PROPOSED METHOD

To begin, observe that by (4), the chance constraint (5b) is equivalent to

$$\Pr(Q_i(\boldsymbol{w}, \boldsymbol{e}) \geq 0) \leq p_i, \quad i = 1, 2, \tag{6}$$

where

$$Q_i\left(\boldsymbol{w}, \boldsymbol{e}\right) \triangleq \Phi_i + \sigma_i^2 - \frac{P_j}{\gamma_i}\left\|(\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)^T \left(\boldsymbol{w} \odot (\widehat{\boldsymbol{f}}_j + \boldsymbol{e}_j)\right)\right\|^2$$

and $j \neq i$. A straightforward but tedious calculation yields

$$Q_i\left(\boldsymbol{w}, \boldsymbol{e}\right) = \text{vec}(\boldsymbol{W})^H \text{vec}\left(\sigma_R^2 \boldsymbol{F}_i + P_i \boldsymbol{F}_{ii} - \frac{P_j}{\gamma_i}\boldsymbol{F}_{ij}\right) + \sigma_i^2,$$

where $\boldsymbol{W} = \boldsymbol{w}\boldsymbol{w}^H$, $\boldsymbol{F}_i = \left((\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)^*(\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)^T\right) \odot \boldsymbol{I}$,

$$\begin{aligned}\boldsymbol{F}_{ii} &= 4(\widehat{\boldsymbol{f}}_i \odot \boldsymbol{e}_i)^*(\widehat{\boldsymbol{f}}_i \odot \boldsymbol{e}_i)^T + (\boldsymbol{e}_i \odot \boldsymbol{e}_i)^* (\boldsymbol{e}_i \odot \boldsymbol{e}_i)^T \\ &\quad + 2\left(\boldsymbol{e}_i \odot \boldsymbol{e}_i\right)^* (\boldsymbol{e}_i \odot \widehat{\boldsymbol{f}}_i)^T + 2(\boldsymbol{e}_i \odot \widehat{\boldsymbol{f}}_i)^*(\boldsymbol{e}_i \odot \boldsymbol{e}_i)^T, \\ \boldsymbol{F}_{ij} &= \left((\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)^* \odot (\widehat{\boldsymbol{f}}_j + \boldsymbol{e}_j)^*\right)\left((\widehat{\boldsymbol{f}}_i + \boldsymbol{e}_i)^T \odot (\widehat{\boldsymbol{f}}_j + \boldsymbol{e}_j)^T\right).\end{aligned}$$

The following theorem shows that a certain set of second-order cone constraints can serve as a tractable safe approximation of the chance constraints (6). Its proof can be found in the Appendix.

Theorem 1 *Let $\xi_1, \ldots, \xi_m$ be independent standard (*i.e.*, zero mean and unit variance) real Gaussian random variables. Consider the function Q: $\mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ defined via*

$$\begin{aligned}Q\left(\boldsymbol{x}, \boldsymbol{\xi}\right) &= -a_0\left(\boldsymbol{x}\right) + \sum_{i=1}^{m} \xi_i a_i\left(\boldsymbol{x}\right) + \sum_{i,j=1}^{m} \xi_i\xi_j a_{i,j}\left(\boldsymbol{x}\right) \\ &\quad + \sum_{i,j,k=1}^{m} \xi_i\xi_j\xi_k a_{i,j,k}\left(\boldsymbol{x}\right) + \sum_{i,j,k,l=1}^{m} \xi_i\xi_j\xi_k\xi_l a_{i,j,k,l}\left(\boldsymbol{x}\right),\end{aligned}$$

where $a_0(\cdot)$ is affine and $a_i(\cdot)$, $a_{i,j}(\cdot)$, $a_{i,j,k}(\cdot)$, $a_{i,j,k,l}(\cdot)$ are linear in its argument. Consider the chance constraint

$$\Pr\left(Q\left(\boldsymbol{x}, \boldsymbol{\xi}\right) \geq 0\right) \leq \epsilon, \tag{7}$$

where $\epsilon > 0$ is given. Set

$$\bar{q}(\epsilon) \triangleq \begin{cases} \dfrac{-\ln\epsilon + \sqrt{(\ln\epsilon)^2 - 8\ln\epsilon}}{4} & \text{if } \epsilon \in (0, \exp(-8)], \\ 2 & \text{otherwise} \end{cases}$$

and $\bar{Q}(\boldsymbol{x}, \boldsymbol{\xi}) \triangleq Q(\boldsymbol{x}, \boldsymbol{\xi}) + a_0(\boldsymbol{x})$. Then, the following hold:

(a) For each $\boldsymbol{x} \in \mathbb{R}^n$ and $\boldsymbol{\xi} \in \mathbb{R}^m$, we have $\bar{Q}(\boldsymbol{x}, \boldsymbol{\xi})^2 = \boldsymbol{x}^T \boldsymbol{U}(\boldsymbol{\xi})\boldsymbol{x}$ for some $\boldsymbol{U}(\boldsymbol{\xi}) \succeq \boldsymbol{0}$.

(b) Let $\boldsymbol{U} \triangleq \mathbb{E}[\boldsymbol{U}(\boldsymbol{\xi})] \succeq \boldsymbol{0}$ and

$$c(\epsilon) \triangleq \begin{cases} (\bar{q}(\epsilon) - 1)^2 \exp\left(\dfrac{2\bar{q}(\epsilon)}{\bar{q}(\epsilon) - 1}\right) & \text{if } \bar{q}(\epsilon) > 2, \\ 1/\sqrt{\epsilon} & \text{if } \bar{q}(\epsilon) = 2. \end{cases} \tag{8}$$

The second-order cone constraint

$$a_0(\boldsymbol{x}) \geq c(\epsilon)\|\boldsymbol{U}^{1/2}\boldsymbol{x}\| \tag{9}$$

serves as a tractable safe approximation of the chance constraint (7).

Remark. Theorem 1 also applies to the case where $\xi_1, \ldots, \xi_m$ are independent mean-zero Gaussian random variables, as their variances can be absorbed into the functions $a_i(\cdot)$, $a_{i,j}(\cdot)$, $a_{i,j,k}(\cdot)$, $a_{i,j,k,l}(\cdot)$.

To apply Theorem 1 to the chance constraint (6), define

$$a_0^i\left(\boldsymbol{W}\right) \triangleq \text{vec}(\boldsymbol{W})^H \text{vec}\left(\frac{P_j}{\gamma_i}\widehat{\boldsymbol{F}}_{ij} - \sigma_R^2\widehat{\boldsymbol{F}}_i\right) - \sigma_i^2, \quad i = 1, 2,$$

where $\widehat{\boldsymbol{F}}_{ij} = (\widehat{\boldsymbol{f}}_i^* \odot \widehat{\boldsymbol{f}}_j^*)(\widehat{\boldsymbol{f}}_i \odot \widehat{\boldsymbol{f}}_j)^T$, $\widehat{\boldsymbol{F}}_i = (\widehat{\boldsymbol{f}}_i^* \widehat{\boldsymbol{f}}_i^T) \odot \boldsymbol{I}$, and $j \neq i$. Since $Q_i(\cdot, \cdot)$ depends on $\boldsymbol{w}$ only through the term $\boldsymbol{W} = \boldsymbol{w}\boldsymbol{w}^H$, by a slight abuse of notation, we shall write $Q_i(\boldsymbol{W}, \boldsymbol{e})$ for $Q_i(\boldsymbol{w}, \boldsymbol{e})$. Then, we have

$$\bar{Q}_i\left(\boldsymbol{W}, \boldsymbol{e}\right) \triangleq Q_i\left(\boldsymbol{W}, \boldsymbol{e}\right) + a_0^i\left(\boldsymbol{W}\right) = \text{vec}(\boldsymbol{W})^H \text{vec}\left(\bar{\boldsymbol{F}}_i\right)$$

and

$$\mathbb{E}\left[\left|\bar{Q}_i\left(\boldsymbol{W}, \boldsymbol{e}\right)\right|^2\right] = \text{vec}(\boldsymbol{W})^H \boldsymbol{U}_i \text{vec}\left(\boldsymbol{W}\right),$$

where

$$\begin{aligned}\bar{\boldsymbol{F}}_i &= \sigma_R^2(\boldsymbol{F}_i - \widehat{\boldsymbol{F}}_i) + P_i\boldsymbol{F}_{ii} - \frac{P_j}{\gamma_i}(\boldsymbol{F}_{ij} - \widehat{\boldsymbol{F}}_{ij}), \\ \boldsymbol{U}_i &= \mathbb{E}\left[\text{vec}\left(\bar{\boldsymbol{F}}_i\right)\text{vec}\left(\bar{\boldsymbol{F}}_i\right)^H\right].\end{aligned}$$

By Theorem 1, the following set of second-order cone constraints serves as a safe approximation of the chance constraint (6):

$$a_0^i(\boldsymbol{W}) \geq c(p_i)\|\boldsymbol{U}_i^{1/2}\text{vec}(\boldsymbol{W})\|, \quad i = 1, 2,$$

[1]That is, a set of efficiently computable constraints whose feasible solutions are also feasible for the original chance constraint.

where $c(\cdot)$ is given by (8). Hence, we obtain the following safe approximation of problem (5):

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & P_R \\ \text{s.t.} \quad & a_0^i(\boldsymbol{W}) \geq c(p_i)\|\boldsymbol{U}_i^{1/2}\text{vec}(\boldsymbol{W})\|, \ \ i=1,2, \\ & \boldsymbol{W} = \boldsymbol{w}\boldsymbol{w}^H. \end{aligned} \tag{10}$$

Note that problem (10) is still intractable due to the non-convex rank-one constraint $\boldsymbol{W} = \boldsymbol{w}\boldsymbol{w}^H$. To obtain a tractable approximation, we employ the SDR technique [10] and relax the rank-one constraint to the positive semidefinite constraint $\boldsymbol{W} \succeq \boldsymbol{0}$. The resulting problem is then a semidefinite program (SDP), which can be readily solved by off-the-shelf solvers [14]. In the case where the SDP solution $\boldsymbol{W}^*$ has rank greater than one, a Gaussian randomization procedure [10] can be applied to extract a rank-one solution from $\boldsymbol{W}^*$.

4. NUMERICAL SIMULATION AND DISCUSSION

To demonstrate the performance of the proposed method, numerical simulations are performed. The simulation settings are as follows. There are $K = 6$ relay nodes. All the relay channels are assumed to be Rayleigh flat fading; *i.e.*, $\widehat{\boldsymbol{f}}_i \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$ for $i = 1, 2$. The transmit power of the two users is set to the same value, with $P_1 = P_2 = 10$dB. The power of the noise at all the nodes is assumed to take same value, with $\sigma_1 = \sigma_2 = \sigma_R = 1$. The covariance matrices of the CSI errors are assumed to take the form $\boldsymbol{E}_1 = \boldsymbol{E}_2 = \eta^2\boldsymbol{I}$, where $\eta > 0$ is a power parameter. The SINR outage threshold of the two users are assumed to be the same; *i.e.*, $\gamma = \gamma_1 = \gamma_2$. The SINR outage probabilities of the two users are also assumed to be the same, with $p_1 = p_2 = 10\%$. For comparison, we consider the non-robust method [2] and the Bernstein-type inequality method [9] under the same settings. We remark that since the latter method is designed only for chance constraints involving a quadratic Gaussian polynomial, we drop the cubic and quartic CSI error terms in $Q_i(\boldsymbol{W}, \boldsymbol{e})$ (where $i = 1, 2$) when applying this method.

Fig. 2 compares the empirical cumulative distribution functions (CDFs) of the SINR experienced by the two users. Each curve represents the SINR values achieved over 2000 random channel realizations with γ set to 3dB or 7dB, and $\eta = 0.1732$. From Fig. 2, we see that the non-robust method [2] cannot always guarantee an SINR outage probability of 10%: About 60% and 70% of the achieved SINR values are lower than the SINR threshold of $\gamma = 3$dB and $\gamma = 7$dB, respectively. On the other hand, the SINR outage probabilities of the Bernstein-type inequality method [9] and the proposed method are both lower than 10%, which satisfy the requirements. In addition, we see from Table 1 that the relay transmit power P_R required by the proposed method is much lower than that by the method in [9], except when the SINR threshold is very small. This suggests that the proposed method is less conservative than that of [9].

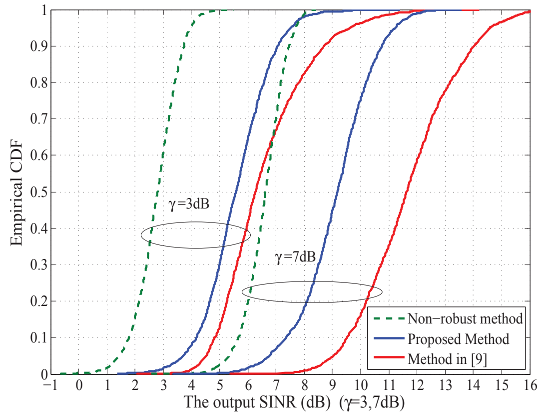

Fig. 2. The empirical CDF of the output SINR ($\gamma = 3, 7$dB)

Table 1. Transmit power P_R (dB)

γ(dB)	1	3	5	7	9
Method in [9]	4.4	11.1	18.9	48.1	87.6
Proposed Method	4.9	7.0	11.5	18.2	31.2

5. APPENDIX: PROOF OF THEOREM 1

By assumption, for each $\boldsymbol{\xi} \in \mathbb{R}^m$, the function $\boldsymbol{x} \mapsto \bar{Q}(\boldsymbol{x}, \boldsymbol{\xi})$ is linear in $\boldsymbol{x} \in \mathbb{R}^n$. This implies that $\boldsymbol{x} \mapsto \bar{Q}(\boldsymbol{x}, \boldsymbol{\xi})^2$ is a non-negative homogeneous quadratic polynomial in $\boldsymbol{x} \in \mathbb{R}^n$. This establishes (a).

To prove (b), we invoke [15, Theorem 5.10], which states that for any $q \geq 2$,

$$\mathbb{E}\left[\left|\bar{Q}(\boldsymbol{x}, \boldsymbol{\xi})\right|^q\right]^{1/q} \leq (q-1)^2\mathbb{E}\left[\left|\bar{Q}(\boldsymbol{x}, \boldsymbol{\xi})\right|^2\right]^{1/2}.$$

This, together with Markov's inequality and the result in (a), implies that for any $q \geq 2$,

$$\begin{aligned} \Pr\left(\left|\bar{Q}(\boldsymbol{x}, \boldsymbol{\xi})\right| \geq t\right) &\leq t^{-q}\mathbb{E}\left[\left|\bar{Q}(\boldsymbol{x}, \boldsymbol{\xi})\right|^q\right] \\ &= \left[t^{-1}(q-1)^2\|\boldsymbol{U}^{1/2}\boldsymbol{x}\|\right]^q. \end{aligned}$$

By setting $q = \bar{q}(\epsilon)$, we have $q \geq 2$. Moreover, whenever $t \geq c(\epsilon)\|\boldsymbol{U}^{1/2}\boldsymbol{x}\|$, we have $\Pr\left(\left|\bar{Q}(\boldsymbol{x}, \boldsymbol{\xi})\right| \geq t\right) \leq \epsilon$. It follows that

$$\begin{aligned} \Pr\left(Q(\boldsymbol{x}, \boldsymbol{\xi}) \geq 0\right) &= \Pr\left(\bar{Q}(\boldsymbol{x}, \boldsymbol{\xi}) \geq a_0(\boldsymbol{x})\right) \\ &\leq \Pr\left(\left|\bar{Q}(\boldsymbol{x}, \boldsymbol{\xi})\right| \geq a_0(\boldsymbol{x})\right) \\ &\leq \epsilon \end{aligned}$$

whenever (9) holds. In particular, the second-order cone constraint (9) is a tractable safe approximation of (7), as desired.

6. REFERENCES

[1] R. Zhang, Y.-C. Liang, C. C. Chai, and S. Cui, "Optimal beamforming for two–way multi–antenna relay channel with analogue network coding," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, pp. 699–712, 2009.

[2] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Optimal distributed beamforming for two–way relay networks," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1238–1250, 2010.

[3] M. Zeng, R. Zhang, and S. Cui, "On design of collaborative beamforming for two–way relay neworks," *IEEE Trans. Signal Process.*, vol. 59, no. 5, pp. 2284–2295, 2011.

[4] S. ShahbazPanahi and M. Dong, "Achievable rate region under joint distributed beamforming and power allocation for two–way relay networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 11, pp. 4026–4037, 2012.

[5] A. Aziz, M. Zeng, J. Zhou, C. N. Georghiades, and S. Cui, "Robust beamforming with channel uncertainty for two–way relay networks," in *Proc. IEEE ICC 2012*, 2012, pp. 3632–3636.

[6] M. Tao and R. Wang, "Robust relay beamforming for two–way relay networks," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1052–1055, 2012.

[7] D. Ponukumati, F. Gao, and C. Xing, "Robust peer–to–peer relay beamforming: A probabilistic approach," *IEEE Commun. Lett.*, vol. 17, no. 2, pp. 305–308, 2013.

[8] A. Ben-Tal, L. El Ghaoui, and A. Nemirovski, *Robust Optimization*, ser. Princeton Series in Applied Mathematics. Princeton, NJ: Princeton University Press, 2009.

[9] K.-Y. Wang, T.-H. Chang, W.-K. Ma, A. M.-C. So, and C.-Y. Chi, "Probabilistic SINR constrained robust transmit beamforming: A Bernstein–type inequality based conservative approach," in *Proc. IEEE ICASSP 2011*, 2011, pp. 3080–3083.

[10] Z.-Q. Luo, W.-K. Ma, A. M.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, 2010.

[11] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, ser. Prentice–Hall Signal Processing Series. New Jersey: PTR Prentice–Hall, Inc., 1993.

[12] S. A. Vorobyov, A. B. Gershman, and Y. Rong, "On the relationship between the worst–case optimization–based and probability–constrained approaches to robust adaptive beamforming," in *Proc. IEEE ICASSP 2007*, 2007, pp. II–977–II–980.

[13] S.-S. Cheung, A. M.-C. So, and K. Wang, "Linear matrix inequalities with stochastically dependent perturbations and applications to chance–constrained semidefinite optimization," *SIAM J. Optim.*, vol. 22, no. 4, pp. 1394–1430, 2012.

[14] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming," http://cvxr.com/cvx, 2011.

[15] S. Janson, *Gaussian Hilbert Spaces*. Cambridge: Cambridge University Press, 1997.