# GEOMETRIC MONITORING FOR CSI REDUCTION IN AMPLIFY-AND-FORWARD RELAY NETWORKS

*Antonios Igglezakis, Antonios Deligiannakis, Aggelos Bletsas*

School of Electronic & Computer Engineering, Technical University of Crete, Greece

## ABSTRACT

This work studies the recently proposed geometric monitoring (GM) method in order to reduce the amount of necessary channel state information (CSI) in network (i.e., distributed) setups. Specifically, a network of amplify-and-forward (AF) relays is studied and the GM method is appropriately adapted. The basic idea is that GM could flag time instances where CSI exchange is unnecessary and, thus, the relay network could abstain from CSI exchange. It is found that, compared to alternative approaches, our GM approach can achieve significant CSI reduction in a variety of network setups.

***Index Terms***— CSI reduction, geometric monitoring, amplify-and-forward relays

## I. INTRODUCTION

Typical relay networks are inherently distributed systems where channel state information (CSI) regarding specific links cannot be assumed a priori known at distant parts of the network. The research community has addressed the issue of CSI requirements with a number of ways, including: a) distributed algorithms where only a part of the network is selected and participates in information forwarding and thus, necessary CSI is reduced (e.g., literature in relay selection falls within this category, as in [1]); b) estimation algorithms that offer CSI regarding links two-hop away [2]; c) performance with imperfect or partial CSI (e.g., [3], [4] and references therein). The issue of CSI acquisition and impact on overall performance is even more critical in amplify-and-forward relay networks, due to their *inherently simple* physical layer processing.

**Relation to Prior Art:** In sharp contrast to prior art, this work limits CSI locally, i.e. each terminal is assumed to know only its own channel gain links towards its immediate neighbors and avoids any other type of CSI estimation. The GM method [5], [6], [7], [8], [9], originally proposed in the distributed databases community to predict changes of a global function from local measurements, is adopted. This work puts forth GM for amplify-and-forward, two-hop relaying and studies algorithms that assist the network terminals to abstain from CSI exchange, without degrading overall performance. To the best of our knowledge, this is the first attempt to adapt GM-based methodologies in AF relaying.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

Half-duplex dual-hop relaying is considered [10], where direct communication between the source $S$ and destination $D$ is impractical and reliable communication is possible only through the relays. During the first hop, the source, without exploiting any channel state information (CSI), transmits $N/2$ symbols and the relays listen, while during the second hop, the relays forward an amplified version of the received signal (amplify-and-forward) using the same number of symbols. The channel is assumed to remain constant during the two hops (at least $N$-symbol coherence time) with Rayleigh fading. The received signal in a link $(A \to B)$ between two nodes $A$, $B$ is given by:

$$y_B = \alpha_{AB} x_A + n_B \quad (1)$$

where $x_A$ is the signal transmitted from node $A$, $n_B \sim \mathcal{CN}(0, N_0)$ is the additive white complex Gaussian noise (AWGN) at node $B$ and $\alpha_{AB} \sim \mathcal{CN}(0, \Omega_{AB})$ is the channel gain for the link $A \to B$. If the node $A$ is the source, then $\mathbb{E}\left\{|x_A|^2\right\} = P_{source}$. Similarly, if the node $A$ is the $k$-th relay, then $\mathbb{E}\left\{|x_A|^2\right\} = P_k$. For simplicity in this work, $P_{source}$ and $P_k$ are binary-valued in $\{0, P\}$. For each relay $k \in S_{relay}$, a link from the source to the $k$-th relay is designated by $S \to k$ and a link from the $k$-th relay to the destination by $k \to D$.

Each amplify-and-forward relay $k$ out of the relay set $S_{relay}$ normalizes the received signal $y_k$ from the first phase of the protocol and transmits:

$$x_k = \sqrt{P_k}\frac{y_k}{\sqrt{\mathbb{E}\left\{|y_k|^2\right\}}} \quad (2)$$

during the second phase of the protocol. The mutual information at the final destination for the aforementioned amplify-and-forward strategy is given by [10]:

$$I_{\text{MR-AaF}} = \frac{1}{2}\log_2\left\{1 + \frac{\left|\sum_{k=1}^{K}\sqrt{\frac{P_k}{\Omega_{Sk}P_{source}+N_0}}\alpha_{Sk}\alpha_{kD}\right|^2}{\left(1+\sum_{k=1}^{K}\frac{P_k|\alpha_{kD}|^2}{\Omega_{Sk}P_{source}+N_0}\right)}\frac{P_{source}}{N_0}\right\}$$

The work of A. Igglezakis and A. Bletsas (Sections 2, 4 and 5) was supported by the European Union (European Social Fund - ESF) and Greek national funds through the Operational Program Education and Lifelong Learning of the National Strategic Reference Framework through the Research Funding Program Thales - Investing in knowledge society through the European Social Fund.

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In such setup, the outage probability $\Pr(I_{\text{MR-AaF}} < r)$ i.e. the probability that the system cannot support a target data rate $r$, can be used as a generalized error probability in quasi-static fading for any modulation and error correction scheme. Performance is clearly a function of CSI between source and all active relays, as well as all active relays and destination. This work studies geometric monitoring (GM) approaches that attempt to reduce the amount of necessary exchange of CSI information among the network terminals (active relays, source and destination) with amplify-and-forward relays, without performance degradation. The basic idea is that GM could flag time instances where CSI exchange is unnecessary and, thus, the relay network could abstain from CSI exchange.

## III. REVIEW OF GEOMETRIC MONITORING (GM)

Geometric monitoring [5], [7], [9], [8] enables a set of $n$ distributed nodes to monitor when an arbitrary non-linear function $f$, expressed over the *average* $v$ of data vectors maintained at the individual nodes, falls below a threshold. A coordinator node assists in the whole process. Let $v_k$ denote the data vector maintained by the $k$-th node. The geometric monitoring technique consists of 3 operations: the initialization phase, the monitoring phase and the synchronization phase.

**Initialization Phase.** In the initialization phase (assume that it occurs at time $t_{last\_sent}$), the coordinator communicates with the $n$ nodes and requests their data vectors $v_k$ $(1 \leq k \leq n)$. It then computes the *estimate vector* $e = \frac{\sum_{k=1}^{n} v_k(t_{last\_sent})}{n}$ and transmits $e$ to the nodes. In a nutshell, $e$ constitutes at each time the last known average data vector. At subsequent epochs, unless the nodes decide to communicate, the true global average data vector $v = \frac{\sum_{k=1}^{n} v_k(t_{now})}{n}$ can be different from $e$ and is unknown to the nodes and the coordinator.

**Monitoring Phase.** The monitoring of non-linear functions by the geometric monitoring is made possible by looking at where the average vector $v$ may reside in the domain of the function, rather than looking at the *range* of the function. Let us now describe how this is accomplished. Let $DV_k = v_k(t_{now}) - v_k(t_{last\_sent})$ denote the *delta vector* representing the difference between the current data vector of node $k$ from its last transmitted data vector. Let $u_k = e + DV_k$ denote the *drift vector* of node $k$. [5] proved that $v = e + \frac{\sum_{k=1}^{n} DV_i}{n} = \frac{\sum_{k=1}^{n} u_k}{n}$.

Thus, $v$ lies within the convex hull of the drift vectors $u_k$. The part of the domain where $f$ remains above or equal to the threshold is called the *admissible region*. Assume that we have somehow selected a *convex* subset $CA$ of the admissible region and then each node $k$ simply checks whether its drift vector $u_k$ lies within $CA$. If this is the case for all nodes, then $v$, which is a linear combination of the drift vectors, must also lie within $CA$, meaning that the function has not fallen below the threshold. We demonstrate in Section IV how to determine $CA$ for our problem.

What is *guaranteed* by geometric monitoring is that the function will never fall below the threshold (an event termed as a *global violation*) without *at least one* drift vector exiting $CA$ (a *local violation*). Nodes transmit their data vector to the coordinator only in the event of a local violation. This means that the coordinator will be notified by at least one node in the event of a global violation and will always detect it after launching the synchronization phase. Thus, if all nodes remain silent, then the value of the function remains above (or equal to) the threshold. On the other hand, a local violation does not mean that a global violation has indeed occurred.

**Synchronization Phase.** Upon a local violation, the coordinator requests the local data vectors from the nodes and computes the true value of the monitoring function. It then computes a new estimate vector that it transmits to the nodes. The nodes, in turn, set $DV = 0$, since they have just transmitted their latest data vector, and update the value of $t_{last\_sent}$.

## IV. RELAY ALGORITHMS FOR CSI WITH GM

Given a set $S_{relay}$ of available candidate relays, our algorithm seeks to continuously maintain a subset $S_{active} \subseteq S_{relay}$ of active relays that maintain their mutual information above a specified threshold.

### IV-A. Expressing Mutual Information Function using Geometric Monitoring

The mutual information for the AaF strategy with K candidate relays, but only $|S_{active}|$ active relays, is given by:

$$I_{\text{MR-AaF}} = \frac{1}{2}\log_2\left\{1 + \frac{|\sum_{k\in S_{active}}(v_{k,1} + \mathrm{i}v_{k,2})|^2}{1+\sum_{k\in S_{active}} v_{k,3}} \frac{P_{source}}{N_0}\right\} \quad (3)$$

$$v_{k,1} = Re\left\{\sqrt{\frac{P_k}{\Omega_{Sk}P_{source}+N_0}}\alpha_{Sk}\alpha_{kD}\right\} \quad (4)$$

$$v_{k,2} = Im\left\{\sqrt{\frac{P_k}{\Omega_{Sk}P_{source}+N_0}}\alpha_{Sk}\alpha_{kD}\right\} \quad (5)$$

$$v_{k,3} = \frac{P_k|\alpha_{kD}|^2}{\Omega_{Sk}P_{source}+N_0} \quad (6)$$

The local data vector that each relay maintains is the vector $v_k = [v_{k,1} \quad v_{k,2} \quad v_{k,3}]$. Notice that each relay can compute all components of its local data vector without the knowledge of the channels of other relays. The true global average vector $v = [v_1 \quad v_2 \quad v_3]$ can thus be computed as $v = \frac{1}{|S_{active}|}\sum_{k\in S_{active}} v_k$.

Thus, the mutual information can be expressed, as required by the geometric monitoring framework, as a function of the global average vector $v$:

$$I_{\text{MR-AaF}} = \frac{1}{2}\log_2\left\{1 + \frac{|S_{active}|(v_1^2+v_2^2)}{\frac{1}{|S_{active}|}+v_3}\frac{P_{source}}{N_0}\right\} \quad (7)$$

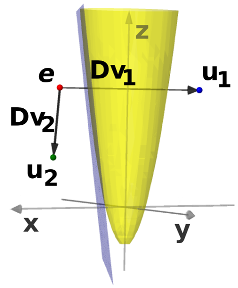


**Fig. 1**: Depicting the estimate vector $e$, 2 drift vectors, the monitoring area of the relays and the inadmissible region (area inside the parabola). Relay 1 exhibits a local violation.

Our algorithm seeks to first select an optimal subset of relays to be active and to then maintain the same subset of active relays as long as their mutual information exceeds a threshold $r$. The above constraint can be expressed as follows:

$$I_{\text{MR-AaF}} = \frac{1}{2}\log_2\left\{1+\frac{|S_{active}|(v_1^2+v_2^2)}{\frac{1}{|S_{active}|}+v_3}\frac{P_{source}}{N_0}\right\} \geq r \Leftrightarrow$$
$$1+\frac{|S_{active}|(v_1^2+v_2^2)}{1/|S_{active}|+v_3}\frac{P_{source}}{N_0} \geq 2^{2r} \Leftrightarrow$$
$$(v_1^2+v_2^2) \geq (2^{2r}-1)(\frac{1}{|S_{active}|}+v_3)\frac{N_0}{|S_{active}|P_{source}}$$

Defining $\lambda = \frac{N_0}{|S_{active}|P_{source}}(2^{2r}-1)$ and $\beta = \frac{\lambda}{|S_{active}|}$, our constraint can now be equivalently expressed as:

$$f(v) = f(v_1,v_2,v_3) = (v_1^2+v_2^2) - \lambda v_3 \geq \beta \quad (8)$$

Figure 1 depicts the monitored function. The inadmissible region (the area of the domain where $f(v) < \frac{\lambda}{|S_{active}|}$) is the area inside a region that resembles a parabola. The algorithm then picks a convex subset $CA$ of the admissible region in order to perform the tests for local violations. To achieve this, the active relays select a point $bp$ ($bp$ should preferably be close to $e$) of the parabola and then draw the tangent hyperplane at the parabola, crossing $bp$ (the value of $bp$ is presented later in this section). An active relay refrains from communication as long as its drift vector remains within the half-space (including the hyperplane itself) determined by the hyperplane and $e$.

### IV-B. Actions of the Destination Node

**Initialization Phase.** The destination node acts as the coordinator in our case and asks all relays for their local data vectors. It then selects a set of active relays, computes the new estimate vector solely from the local data vectors of the relays that were selected to be active and transmits this estimate vector, along with a list of the nodes that were selected to be active (note that the latter information could be transmitted with a simple bitmap containing $k$ bits).

**Reaction to Local Violations.** Upon a local violation, the destination node requests the local data vectors from all relays and performs the actions described in the Initialization phase.

### IV-C. Actions of the Active Relays

Each active relay estimates its channel gains $\alpha_{Sk}$ and $\alpha_{kD}$ and calculates its $v_k$ vector. After each such estimation, it updates its delta $DV_k$ and drift $u_k$ vectors. In order to check for a local violation, it first considers the point $bp$ with coordinates $[c\times e_1,\ c\times e_2,\ e_3]$, where $e_i$ denotes the $i$-th component of the estimate vector $e$ and $c = \sqrt{\frac{\lambda e_3+\beta}{e_1^2+e_2^2}}$. The perpendicular vector $pv$ to the tangent plane at bp is $pv = [2ce_1,\ 2ce_2,\ -\lambda]$. No local violation occurs if both $e$ and $u_k$ lie in the same half-space of the tangent at $bp$, meaning that:

$$((u_k - bp)\cdot pv)((e-bp)\cdot pv) \geq 0 \quad (9)$$

The symbol '$\cdot$' denotes the dot product of two vectors. At each transmission of its data vector (either due to a local violation, or because the coordinator requested it) the value of $t_{last\_sent}$ is updated.

## V. EXPERIMENTAL EVALUATION

We performed an extensive experimental evaluation in order to test the performance of our proposed approach, termed as *GM* hereafter. Table I mentions the parameters that we modified in our simulations, with the default values of these parameters being underlined. We compared the performance of the following three algorithms:

- *Full communication*, in which both active and inactive relays transmit their measured parameters after each channel estimation. The destination node then decides the optimal set $S_{active}$ of active relays such that $I_{\text{MR-AaF}} \geq r$.
- *Subset communication*, in which only active relays send info message on each channel estimation. The destination node changes the decision on the active relays when the target data rate is not achievable.
- *Geometric monitoring (GM)*, which utilizes the techniques presented in Section IV. The data vectors and the estimate vector contain 3 components (each assumed to consume 4 bytes), while the decision on the set of active nodes is sent through a bitmap.

In our simulations, the channel estimations were generated according to the following model:

| | |
|---|---|
| $SNR = \frac{P_{source}}{N_0}$ | $E_{MSE} = \frac{P_{source}}{1+SNR}$ |
| $\hat{\alpha}_{SK} = \alpha_{SK} + Ee_{SK}$, $\Omega_{SK} = 1$, where $\alpha_{SK} \sim \mathcal{CN}(0,\Omega_{SK})$ , $Ee_{SK} \sim \mathcal{CN}(0,E_{MSE})$ | |

The above relations are valid for $\hat{\alpha}_{KD}$ as well. All the algorithms have been executed with the same data set as input. We repeated the channel estimations generation and

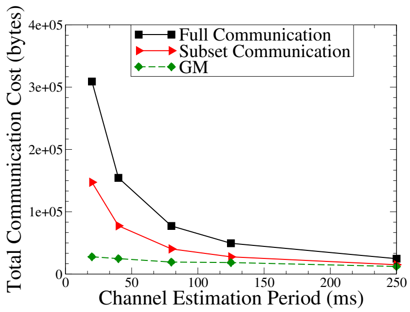


**Fig. 2**: Total communication cost varying the channel estimation period.

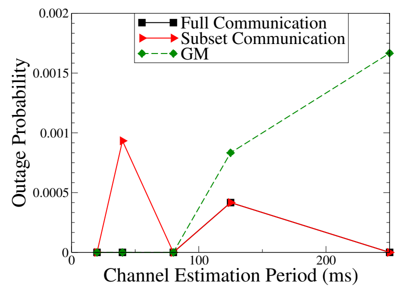


**Fig. 3**: Outage probability varying the channel estimation period.

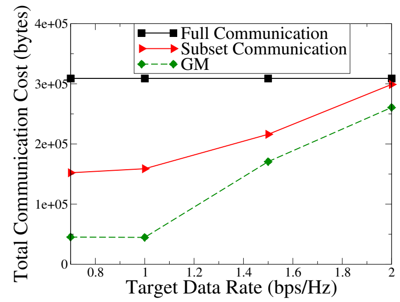


**Fig. 4**: Total communication cost with variable target data rate.

**Table I**: Simulation parameters. Default values are underlined.

| Parameters | Values |
|---|---|
| Simulated Time (sec) | 30 |
| Candidate Relays $K$ | 3 5 7 <u>10</u> |
| Signal-Noise Ratio $SNR$ (dB) | 0 4 6 <u>9</u> 14 19 |
| Signal Power (dBm) | -20 |
| Channel Estimation Period $T_0$ (ms) | <u>20</u> 40 80 125 250 500 |
| Channel Coherence Time $T_c$ (ms) | $Tc \sim N(500, [\underline{20}, 40, 80, 100])$ |
| Target Data Rate $r$ (bps/Hz) | 0.7 <u>1</u> 1.5 2 |

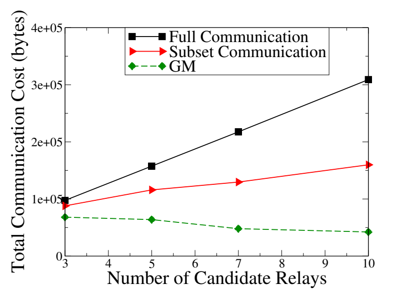


**Fig. 5**: Total communication cost varying the number of candidate relays.

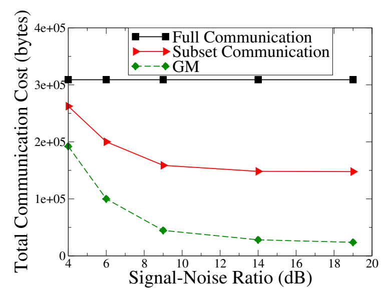


**Fig. 6**: Total communication cost varying the SNR ratio.

the execution of the algorithm for 10 times and present in this section the average values of those metrics.

In Figs. 2, 3 we plot the total communication cost (expressed in bytes) and the outage probability for all three tested algorithms as we vary the channel estimation period for SNR=14db. The reduction in communication cost for our GM algorithm is significant (up to 91.1% reduction compared to Full Communication and 81.1% reduction compared to Subset Communication), with the benefits being larger for shorter channel estimation periods. The shorter the channel estimation period, the larger the number of times that the relays in our GM algorithm refrain from communication with the destination node. On the other hand, the outage probabilities by all techniques (Fig. 3) remain comparable and below 0.17% in all cases.

In Fig. 4 we plot the total communication cost as we vary the desired data target rate, with the remaining parameters obtaining their default values. Our GM algorithm exhibits benefits in all cases, with the benefits being larger for smaller target data rates. As the target data rate increases, it becomes harder for the selected subset of active relays to sustain it. Geometrically, this translates to a wider parabola (parameters $\lambda$, $\beta$ and, therefore, $c$ all increase with larger data rates), bringing the estimate vector closer to the inadmissible region and allowing a smaller area where the drift vectors may reside without a local violation. Even though not shown due to space constraints, the outage probabilities corresponding to Figs. 4-6 were comparable for all algorithms.

In Fig. 5 we depict the total communication cost as we vary the number of candidate relays. As expected, the cost of Full Communication increases linearly with the number of candidate relays. For Subset Communication, the corresponding increase in the total communication cost is smaller and is due to the increase of the number of relays chosen to be active. While GM exhibits a similar increase in the number of relays chosen to be active (results omitted due to space constraints), GM actually manages to reduce the number of transmitted messages as with more candidate relays it can select a subset of active relays with larger distance (compared to having few candidate relays to choose from) of the estimate vector from the inadmissible region. Finally, Fig. 6 demonstates that the benefits of our GM algorithm are significant for a wide range of SNR values, but increase with the value of SNR. This is to be expected, as $E_{MSE}$ decreases with the increase of SNR.

## VI. CONCLUSIONS

We presented an algorithm that utilizes the recently proposed geometric monitoring method to reduce the amount of necessary channel state information (CSI) in a network of amplify-and-forward (AF) relays. Using our techniques, relays can frequently refrain from exchanging CSI information. The net result is a significant reduction in CSI compared to other approaches, especially with a larger number of candidate relays, smaller channel estimation periods and larger SNR.

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