AN AMPLIFY-AND-FORWARD SCHEME FOR COGNITIVE RADIOS

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ABSTRACT

In this paper, we propose an opportunistic amplify-and-forward relaying scheme for a cognitive radio network, which is aimed at allowing a secondary user (SU) to transmit over the same time-frequency slot of a primary user (PU). In our scheme, the SU amplifies and transmits the PU signal it receives, by using as relaying gain the information symbols that the SU wishes to transmit towards its own secondary receiver. The information theoretic limits of the proposed protocol are investigated by showing that, in some operative conditions of practical interest, the SU can embed its information symbols in the PU signal, without violating the cognitive radio principle of protecting the PU transmission and, at the same time, by attaining low transmission rates.

1. INTRODUCTION

Recently, measurement studies have confirmed [1] that the licensed radio spectrum is relatively unused across many time and/or frequency slots. Taking into account the severe under-utilization of the licensed spectrum, the Federal Communications Commission (FCC) has recommended [2] new policies, which enable to significantly improve spectrum efficiency by allowing secondary users (SUs) to temporarily utilize a licensed band, under the constraint that the SUs generate minimal interference for the licensed (primary) users. Regulatory, standardization, and deployment activities on these new *cognitive radio schemes* are also carried out in Europe [3]. Cognitive radio technology enables [4–6] SUs to perform spectrum sensing (i.e., to detect spectrum white spaces), spectrum management (i.e., to select the best available channel), spectrum sharing (i.e., to coordinate access to the channel with other users) and spectrum mobility (i.e., to vacate the channel when a PU is detected). Most of the literature on spectrum underlay for cognitive radios [7]– [11] relies on the idea of minimizing the interference caused by the SU, by carefully assessing the risk that the SU signal is received by a primary receiver, while a primary transmitter is actively communicating with it. Herein, we turn this concept "head over heels", by allowing the SU transmission to be completely concurrent and superimposed to the PU one, albeit in a "symbiotic" form, in order to possibly improve the primary link quality rather than degrading it. Specifically, our simple idea is to combine the strategy of cooperative [12]– [22] amplify-and-forward (AF) relaying with the insertion of information-bearing symbols from the SU in the relaying gain.

The fundamental theoretic limits of the cognitive radio approach can be studied by modeling the cognitive radio channel as the classical interference channel [23–26]. Within this framework, it has been shown in [24] that, if the PU and SU are allowed to cooperate and to jointly design their encoder/decoder pairs, then the capacity achieving strategy for the SU is to perform superposition coding of its codeword (generated by performing Costa precoding [27]) as well as of the codeword of the PU and transmit over the same time-frequency slot used by the PU. However, similarly to Costa's dirty-paper coding, such a strategy requires the SU to have *noncausal* knowledge of the PU codeword. Even though the noncausal constraint can be relaxed if the PU and SU are in close proximity of each other, strict coordination between PU and SU, at both the physical and medium access control layers, is required. In contrast, in our proposed scheme, the PU and SU simultaneously transmit over the same time or frequency slot, without requiring any noncausal side information. Such an idea is similar in spirit to the concept of hiding information onto another signal without significantly distorting it [28, 29]. However, the proposed approach differs from information hiding mainly because superimposition of the SU symbols on the PU signal not only preserves the information of the PU, but also improves the performance of the primary link.

In the proposed cognitive radio scheme, the SU transmits one symbol in each slot assigned to the PU in a specific frame. We show that the channel as seen by the PU is a block inter-symbol interference (ISI) system, whereas the channel as seen by the SU is a fast flat-fading link. The achievable rates for both the PU and SU are evaluated in terms of ergodic channel capacity. The main results are that, if the primary system is properly designed to equalize the ISI channel, the concurrent transmission of the SU improves the capacity of the PU link, provided that the primary transmitter is closer to the secondary transmitter than to the primary receiver; moreover, the SU earns an unlicensed channel, albeit with low transmission rates.

2. THE PROPOSED COOPERATIVE PROTOCOL

The considered wireless network encompasses a primary user (PU), employing the primary transmitter/receiver pair represented in Fig. 1 by node 1 (PTx) and node 3 (PRx), respectively, and a secondary user (SU), employing the secondary transmitter/receiver pair represented by node 2 (STx) and node 4 (SRx), respectively. The PTx transmits a sequence $x_{\mathrm{PU}}(\cdot)$ of independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex symbols with variance $\sigma^2_{\mathrm{PU}} = \mathsf{P}_{\mathrm{PU}}$, arranged in frames of M symbols, whose duration is comparable with the coherence time of the wireless channel; the STx exploits the PU transmission to opportunistically deliver to the SRx a sequence $x_{\mathrm{SU}}(\cdot) \in \mathbb{C}$ of i.i.d. zero-mean circularly symmetric complex symbols with variance σ^2_{SU}, statistically independent of $x_{\mathrm{PU}}(\cdot)$, whose rate is one symbol per PU frame.

The channel corresponding to the $i \to \ell$ link is assumed to be frequency non-selective and quasi-stationary, i.e., it is characterized by the single fading coefficient $g_{i\ell}$, which remains constant within one frame but is allowed to vary independently from frame to frame;

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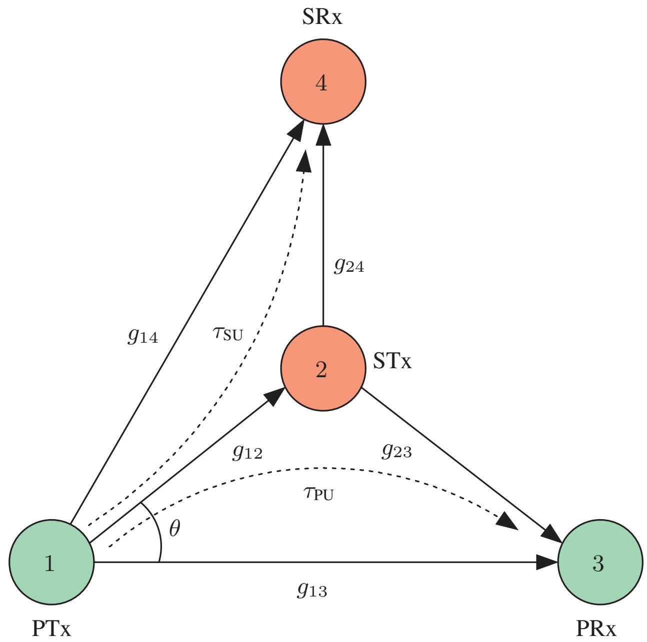

Fig. 1. The wireless network model: in green, the PU trasmitting/receiving nodes, in red the SU transmitting/receiving nodes.

moreover, the fading coefficients of different links are statistically independent, with $g_{i\ell} \sim \mathcal{CN}(0, \sigma^2_{i\ell})$ and $\sigma^2_{i\ell} \triangleq d_{i\ell}^{-\eta}$, where $d_{i\ell}$ is the distance between node i and node ℓ, with η denoting the path-loss exponent. Furthermore, all the considered channel parameters are statistically independent of the symbol sequences $x_{\rm PU}(\cdot)$ and $x_{\rm SU}(\cdot)$. Each link is characterized by a constant time delay that accounts for the processing time at the node and the propagation delay of the path. More specifically, with reference to Fig. 1, since only relative delays are important, the delays of the PTx $\to$ PRx and PTx $\to$ SRx links are conventionally set to zero; $\tau_{\rm PU} > 0$ denotes the integer cumulative delay for the PTx $\to$ STx link, processing at the STx, and STx $\to$ PRx link; whereas $\tau_{\rm SU} > 0$ is the integer cumulative delay for the PTx $\to$ STx link, processing at the STx, and STx $\to$ SRx link. We assume hereinafter that the frame length M is chosen such as both the delays $\tau_{\rm PU}$ and $\tau_{\rm SU}$ are much smaller than M.

The transmission takes place according to the following protocol: the signal transmitted by the PTx towards the PRx is also overheard by the STx; specifically, the discrete-time baseband equivalent received signal at the STx during the mth symbol period is

$$r_2(m) = g_{12}\, x_{\rm PU}(m) + v_2(m)\,, \qquad m \in \{0, 1, \ldots, M-1\} \quad (1)$$

where the sequence $v_2(\cdot)$ denotes additive white Gaussian noise (AWGN) at the SRx, modeled as a sequence of i.i.d. $\mathcal{CN}(0, N_0)$ random variables (RVs), statistically independent of g_{12} and $x_{\rm PU}(\cdot)$. Let $x_{\rm SU}$ denote[1] the SU symbol to be transmitted during the considered PU frame, the SU adopts an opportunistic A&F relaying scheme by transmitting over the same time-frequency slot of the PU, wherein the received samples (1) are amplified at the STx by the scaling factor $x_{\rm SU}$ and forwarded to the SRx. The variance of $x_{\rm SU}$ is adjusted at the STx according to the average power constraint

[1]Since we are focusing on a single PU frame, and the SU transmits one symbol per frame, the time index of the SU symbol can be safely dropped.

$\mathbb{E}\left[|x_{\rm SU}\, r_2(m)|^2\right] = \mathsf{P}_{\rm SU}$, i.e.,

$$\sigma^2_{\rm SU} = \frac{\mathsf{P}_{\rm SU}}{\sigma^2_{12}\, \mathsf{P}_{\rm PU} + N_0} = \frac{\mathsf{SNR}_{\rm SU}}{1 + \sigma^2_{12}\, \mathsf{SNR}_{\rm PU}} \quad (2)$$

with $\mathsf{SNR}_{\rm PU} \triangleq \mathsf{P}_{\rm PU}/N_0$ and $\mathsf{SNR}_{\rm SU} \triangleq \mathsf{P}_{\rm SU}/N_0$ representing the signal-to-noise ratio (SNR) of the PU and SU transmissions, respectively. In the following two subsections, we separately derive and discuss the received signal models at the PRx and SRx.

The discrete-time baseband equivalent received signal at the PRx can be written as

$$y_3(m) = g_{13}\, x_{\rm PU}(m) + g_{23}\, x_{\rm SU}\, r_2(m - \tau_{\rm PU}) + v_3(m) \quad (3)$$

for $m \in \{0, 1, \ldots, M-1\}$, where the sequence $v_3(\cdot)$ denotes AWGN at the PRx, modeled as a sequence of i.i.d. $\mathcal{CN}(0, N_0)$ RVs, statistically independent of g_{13}, $x_{\rm PU}(\cdot)$, g_{23}, $x_{\rm SU}$, and $r_2(\cdot)$. Setting $y_3(m) \equiv y_{\rm PU}(m)$, eq. (3) can be equivalently rewritten as

$$y_{\rm PU}(m) = g_{\rm PU}(m) * x_{\rm PU}(m) + v_{\rm PU}(m) \quad (4)$$

where

$$g_{\rm PU}(m) \triangleq g_{13}\, \delta(m) + g_{12}\, g_{23}\, x_{\rm SU}\, \delta(m - \tau_{\rm PU}) \quad (5)$$

$$v_{\rm PU}(m) \triangleq g_{23}\, x_{\rm SU}\, v_2(m - \tau_{\rm PU}) + v_3(m) \quad (6)$$

represent the impulse response of the overall PU relay channel towards the PRx and the *noise-plus-interference* term at the PRx, respectively. It can be seen from (4) and (5) that the PU experiments ISI through a *two-ray frequency-selective channel* $g_{\rm PU}(m)$, where in particular the second channel tap gain (at delay $m = \tau_{\rm PU}$) incorporates the contribution of the SU transmitted symbol $x_{\rm SU}$.

Similarly to (3), the signal received at the SRx is given by

$$y_4(m) = g_{14}\, x_{\rm PU}(m) + g_{24}\, x_{\rm SU}\, r_2(m - \tau_{\rm SU}) + v_4(m) \quad (7)$$

for $m \in \{0, 1, \ldots, M-1\}$, where the sequence $v_4(\cdot)$ denotes AWGN at the SRx, modeled as a sequence of i.i.d. $\mathcal{CN}(0, N_0)$ RVs, statistically independent of g_{14}, $x_{\rm PU}(\cdot)$, g_{24}, $x_{\rm SU}$, and $r_2(\cdot)$. Setting $y_4(m) \equiv y_{\rm SU}(m)$, eq. (7) can be equivalently rewritten as

$$y_{\rm SU}(m) = g_{\rm SU}(m)\, x_{\rm SU} + v_{\rm SU}(m) \quad (8)$$

where

$$\begin{aligned} g_{\rm SU}(m) &\triangleq g_{24}\, r_2(m - \tau_{\rm SU}) \\ &= g_{24}\, g_{12}\, x_{\rm PU}(m - \tau_{\rm SU}) + g_{24}\, v_2(m - \tau_{\rm SU}) \quad (9) \\ v_{\rm SU}(m) &\triangleq g_{14}\, x_{\rm PU}(m) + v_4(m) \quad (10) \end{aligned}$$

represent the time-varying fading gain towards the SRx and the *noise-plus-interference* term at the SRx, respectively. It results from (8) and (9) that the SU sees a *fast flat-fading channel* characterized by the channel gain $g_{\rm SU}(m)$, whose dynamic in particular depends on the PU transmitted symbol $x_{\rm PU}(m)$.

3. CAPACITY ANALYSIS OF THE PRIMARY USER

As a benchmark, we first study the performance limit of the PU link in the case of the direct PTx $\to$ PRx transmission, i.e., when the SU is silent. In this case, the model for the received signal at the PRx can be simply obtained by setting $x_{\rm SU} = 0$ in (3)–(6), thus yielding $y_{\rm PU}(m) = g_{13}\, x_{\rm PU}(m) + v_3(m)$, which shows that the direct PU transmission sees a *block flat-fading* channel with AWGN. It is assumed in this network scenario that the training session for the PU link is long enough to estimate the realization of g_{13} with negligible error at the receiver, and both the PTx and PRx know the statistical distribution of g_{13}. Furthermore, if we assume that the PU transmission involves a large number of frames, an appropriate performance measure for the direct PU link is the *ergodic channel capacity* [30] with receiver CSI, which is given by (see [31])

$$\mathsf{C}_{\text{PU,direct}} = f(\mathsf{ASNR}_{\text{PU,direct}}) \tag{11}$$

where

$$f(A) \triangleq -\exp\left(\frac{1}{A}\right) \text{Ei}\left(-\frac{1}{A}\right) \tag{12}$$

where $\text{Ei}(x) \triangleq \int_{-\infty}^{x} \frac{\exp(u)}{u} \, \mathrm{d}u$ denotes the exponential integral function, and $\mathsf{ASNR}_{\text{PU,direct}} \triangleq \sigma_{13}^2 \, \mathsf{SNR}_{\text{PU}}$ is the average (over the channel) SNR (ASNR) of the PU at the PRx (node 3) when $x_{\text{SU}} = 0$.

Let us now consider the case in which the STx is active, i.e., $x_{\text{SU}} \neq 0$; in this case, as discussed in Section 2, PU transmission experiments frequency-selective fading [see eq. (5)]. Evaluation of the ergodic capacity of a single-user frequency-selective channel with CSI at the receiver can be carried out [30] by decomposing the channel into an equivalent number of independent frequency-flat (i.e., memoryless) subchannels. Let $\mathbf{x}_{\text{PU}} \triangleq [x_{\text{PU}}(0), x_{\text{PU}}(1), \ldots, x_{\text{PU}}(M-1)]^{\text{T}} \in \mathbb{C}^M$ and $\tilde{\mathbf{y}}_{\text{PU}} \in \mathbb{C}^M$ represent a PU frame of M symbols and the corresponding received block, respectively. In the sequel, we assume that the training sequence for the PU transmission is long enough to acquire CSI at the PRx with negligible error: specifically, the PRx is assumed to have perfect knowledge of the realization $\boldsymbol{\alpha}_{\text{PU}} \in \mathbb{C}^2$ of the channel vector $\mathbf{g}_{\text{PU}} \triangleq [g_{13}, g_{12} \, g_{23} \, x_{\text{SU}}]^{\text{T}} \in \mathbb{C}^2$, which collects the two taps of the channel (5); from an information-theoretic viewpoint, it is noteworthy that this is equivalent to assume that the channel output consists of the pair $(\tilde{\mathbf{y}}_{\text{PU}}, \mathbf{g}_{\text{PU}})$. Let $\mathsf{I}(\mathbf{x}_{\text{PU}}; \tilde{\mathbf{y}}_{\text{PU}} \,|\, \mathbf{g}_{\text{PU}})$ be the conditional mutual information [32] between $\mathbf{x}_{\text{PU}}$ and $\tilde{\mathbf{y}}_{\text{PU}}$, given $\mathbf{g}_{\text{PU}}$, in the presence of the SU, the ergodic channel capacity of the PU with receiver CSI can be expressed [30] as

$$\mathsf{C}_{\text{PU}} \triangleq \lim_{M \to +\infty} \mathsf{C}_{\text{PU}}(M) \tag{13}$$

where $\mathsf{C}_{\text{PU}}(M)$ (in nats/symbol) is obtained by taking the supremum of $\mathsf{I}(\mathbf{x}_{\text{PU}}; \tilde{\mathbf{y}}_{\text{PU}} \,|\, \mathbf{g}_{\text{PU}})/M$ over all possible distributions of the vector $\mathbf{x}_{\text{PU}}$ that satisfy the power constraint $\mathbb{E}[\|\mathbf{x}_{\text{PU}}\|^2] = M \, \mathsf{P}_{\text{PU}}$.

An upper bound on C_{PU} can be obtained by assuming that the PRx additionally has perfect knowledge of the realization $\alpha_{12} \in \mathbb{C}$ of the fading coefficient g_{12} characterizing the PTx $\to$ STx link.[2] In this case, it can be shown (details are omitted) that

$$\mathsf{C}_{\text{PU}} \leq \mathsf{C}_{\text{PU,upper}} \triangleq \mathbb{E}\left\{ f[\Gamma_{3,\text{upper}}(|g_{23}|^2 \, |x_{\text{SU}}|^2)] \right\} \tag{14}$$

with

$$\Gamma_{3,\text{upper}}(|g_{23}|^2 \, |x_{\text{SU}}|^2) \triangleq \mathsf{ASNR}_{\text{PU,direct}} \, \frac{1 + |g_{23}|^2 \, |x_{\text{SU}}|^2 \, \frac{\sigma_{12}^2}{\sigma_{13}^2}}{1 + |g_{23}|^2 \, |x_{\text{SU}}|^2} \, . \tag{15}$$

To find a lower bound on C_{PU}, we observe that the Gaussian distribution might not be the one maximizing $\mathsf{I}(\tilde{x}_{\text{PU}}(\ell); \tilde{y}_{\text{PU}}(\ell) \,|\, \mathbf{g}_{\text{PU}})$ and, thus, we choose $\tilde{x}_{\text{PU}}(\ell) \equiv \tilde{x}_{\text{PU,G}}(\ell) \sim \mathcal{CN}(0, \mathsf{P}_{\text{PU}})$. Moreover, let $v_{\text{PU,G}}(\cdot)$ be a sequence of i.i.d. circularly symmetric complex Gaussian RVs having the same mean and variance as $v_{\text{PU}}(\cdot)$ in (6), i.e., $v_{\text{PU,G}}(\cdot) \sim \mathcal{CN}\left[0, N_0 \, (1 + \sigma_{23}^2 \, \sigma_{\text{SU}}^2)\right]$, independent of $x_{\text{PU,G}}(\cdot)$, by replacing $x_{\text{PU}}(\ell)$ and $v_{\text{PU}}(\ell)$ in (4) with $x_{\text{PU,G}}(\ell)$ and $v_{\text{PU,G}}(\ell)$, using the results of [33], it can be shown (details are omitted) that

$$\mathsf{C}_{\text{PU}} \geq \mathsf{C}_{\text{PU,lower}} \triangleq \mathbb{E}\left\{ f[\Gamma_{3,\text{lower}}(|g_{23}|^2 \, |x_{\text{SU}}|^2)] \right\} \tag{16}$$

with

$$\Gamma_{3,\text{lower}}(|g_{23}|^2 \, |x_{\text{SU}}|^2) \triangleq \mathsf{ASNR}_{\text{PU,direct}} \, \frac{1 + |g_{23}|^2 \, |x_{\text{SU}}|^2 \, \frac{\sigma_{12}^2}{\sigma_{13}^2}}{1 + \sigma_{23}^2 \, \sigma_{\text{SU}}^2} \, . \tag{17}$$

Let us first consider the case where $d_{12} > d_{13}$, that is, the PTx is closer to the PRx than to the STx, which yields $\sigma_{12}^2 < \sigma_{13}^2$: in this case, one has from (15) that $\Gamma_{3,\text{upper}}(|g_{23}|^2 \, |x_{\text{SU}}|^2) < \mathsf{ASNR}_{\text{PU,direct}}$ for each realization of $g_{23} \, x_{\text{SU}}$. Hence, since $f(A)$ is a monotonically increasing function of $A \geq 0$, by comparing (11) and (14) one readily obtains that $\mathsf{C}_{\text{PU,upper}} < \mathsf{C}_{\text{PU,direct}}$, which shows that *the capacity of the PU is worsened by SU transmission when $d_{12} > d_{13}$*, since the SU prevalently forwards noise in this case. In other words, when the SU is active, the cognitive radio principle of protecting the PU is violated if the PTx is farther from the STx than from the PRx. Since the SU can determine whether $d_{12} \leq d_{13}$ or, equivalently, $\sigma_{12}^2 \geq \sigma_{13}^2$, i.e., it belongs to the the *symbiotic region*, in the sequel, we restrict our attention to this case. Remembering (2), we get

$$\sigma_{23}^2 \, \sigma_{\text{SU}}^2 = \frac{\sigma_{23}^2 \, \mathsf{SNR}_{\text{SU}}}{1 + \sigma_{12}^2 \, \mathsf{SNR}_{\text{PU}}} < \frac{\sigma_{23}^2 \, \mathsf{SNR}_{\text{SU}}}{\sigma_{12}^2 \, \mathsf{SNR}_{\text{PU}}} = \frac{\mathsf{AINR}_3}{\mathsf{ASNR}_2} \tag{18}$$

where $\mathsf{ASNR}_2 \triangleq \sigma_{12}^2 \, \mathsf{SNR}_{\text{PU}}$ is the ASNR at the STx [see eq. (1)], whereas $\mathsf{AINR}_3 \triangleq \sigma_{23}^2 \, \mathsf{SNR}_{\text{SU}}$ is the average (over the channel) interference-to-noise ratio (AINR) at the PRx [see eq. (3)]. It can be inferred from (18) that, in the *low-interference regime*, i.e., when $\mathsf{AINR}_3 \ll \mathsf{ASNR}_2$, the beneficial increase in frequency diversity prevails against the adverse noise propagation effect since $\sigma_{23}^2 \, \sigma_{\text{SU}}^2 \ll \epsilon$, with $\epsilon > 0$ being a sufficiently small real number. In such a case, by invoking the Chebychev's inequality [34], one additionally has $\mathsf{P}\left(|g_{23}|^2 \, |x_{\text{SU}}|^2 \geq \epsilon\right) \leq (\sigma_{23}^2 \, \sigma_{\text{SU}}^2)/\epsilon \ll 1$ and, consequently, the RV $|g_{23}|^2 \, |x_{\text{SU}}|^2$ takes on values significantly smaller than one, with high probability. Henceforth, in the low-interference regime, $\Gamma_{3,\text{upper}}(|g_{23}|^2 \, |x_{\text{SU}}|^2)$ in (15) and $\Gamma_{3,\text{lower}}(|g_{23}|^2 \, |x_{\text{SU}}|^2)$ in (17) can be approximated as

$$\begin{aligned} \Gamma_{3,\text{upper}}(|g_{23}|^2 \, |x_{\text{SU}}|^2) \approx \Gamma_{3,\text{lower}}(|g_{23}|^2 \, |x_{\text{SU}}|^2) \approx \Gamma_3(|g_{23}|^2 \, |x_{\text{SU}}|^2) \\ \triangleq \mathsf{ASNR}_{\text{PU,direct}} \left(1 + |g_{23}|^2 \, |x_{\text{SU}}|^2 \, \frac{\sigma_{12}^2}{\sigma_{13}^2}\right) \end{aligned} \tag{19}$$

which leads to

$$\mathsf{C}_{\text{PU}} \approx \mathsf{C}_{\text{PU,lower}} \approx \mathsf{C}_{\text{PU,upper}} \approx \mathbb{E}\left\{ f[\Gamma_3(|g_{23}|^2 \, |x_{\text{SU}}|^2)] \right\} \, . \tag{20}$$

Since $\Gamma_3(|g_{23}|^2 \, |x_{\text{SU}}|^2) > \mathsf{ASNR}_{\text{PU,direct}}$ for each realization of $g_{23} \, x_{\text{SU}}$, recalling again that $f(A)$ is a monotonically increasing function of $A \geq 0$, and comparing (11) with (20), one readily obtains that $\mathsf{C}_{\text{PU}} > \mathsf{C}_{\text{PU,direct}}$, which shows that *the capacity of the PU improves as a result of the SU transmission in the low-interference regime*, no matter what the distributions of g_{23} and x_{SU} are. In summary, we can conclude that, when the STx is sufficiently closer to the PTx than to the PRx, which represents the most interesting situation in a spectrum underlay cognitive radio network, the cognitive radio principle of protecting the PU is not only violated but even a performance improvement can be gained by the PU in terms of ergodic channel capacity (see Fig. 2 for a numerical example).

[2] To acquire g_{12} in practice, the PRx would require additional help from the STx in the form of channel-state feedback.

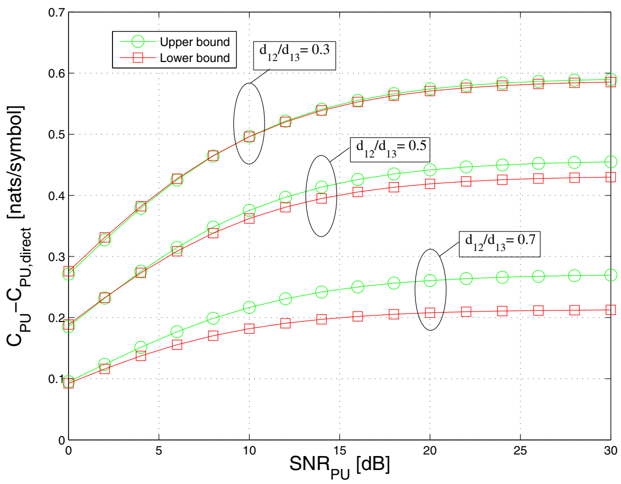

Fig. 2. Capacity gain $\mathsf{C}_{\text{PU}} - \mathsf{C}_{\text{PU,direct}}$ of the PU versus SNR_{PU} ($\mathsf{P}_{\text{PU}} = \mathsf{P}_{\text{SU}} = 1$, $\eta = 2$, $\theta = \pi/3$, QPSK SU symbols).

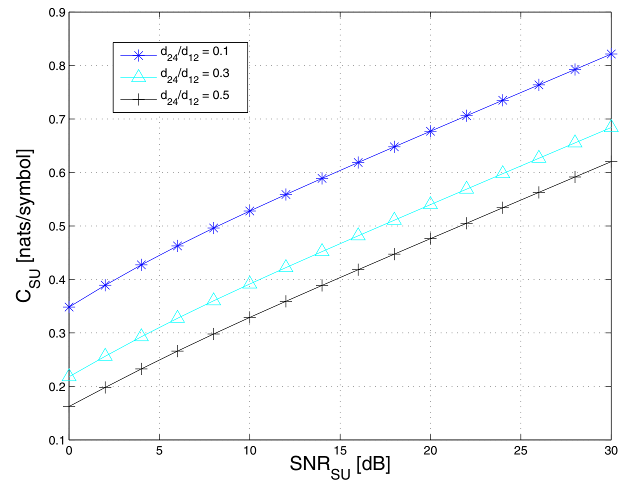

Fig. 3. Capacity C_{SU} of the SU versus SNR_{SU} ($\mathsf{P}_{\text{PU}} = \mathsf{P}_{\text{SU}} = 1$, $\eta = 2$, $M = 16$, QPSK PU symbols).

4. CAPACITY ANALYSIS OF THE SECONDARY USER

In this section, we evaluate the achievable throughput of the SU communication link. As discussed in Section 2, the SU sees a fast flat-fading channel [see eq. (8)]. We assume that also the STx is active over several PU frames and, thus, it amplifies and forwards many PU symbols: hence, similarly to the primary transmission, an appropriate limit for the SU throughput is given by the ergodic channel capacity, whose definition however depends on the channel information assumption. Moreover, according to the results of the capacity analysis of the PU, we assume hereinafter that the ASNR at the STx is sufficiently large, i.e., $\mathsf{ASNR}_2 = \sigma_{12}^2 \, \mathsf{SNR}_{\text{PU}} \gg 1$, which is a reasonable assumption for SNR values of practical interest when the STx is not too far from the PTx; in this case, the noise term in (1) can be neglected, allowing one to simplify (9) as follows

$$g_{\text{SU}}(m) \approx g_{24} \, g_{12} \, x_{\text{PU}}(m - \tau_{\text{SU}}) \, . \tag{21}$$

Let $\mathbf{y}_{\text{SU}} \triangleq [y_{\text{SU}}(0), y_{\text{SU}}(1), \ldots, y_{\text{SU}}(M-1)]^{\text{T}} \in \mathbb{C}^M$ be the vector of samples given by (8) that the SU observes over a frame, accounting for (8), (10), and (21), the data block $\mathbf{y}_{\text{SU}}$ can be expressed as

$$\mathbf{y}_{\text{SU}} = (g_{24} \, g_{12} \, \mathbf{J} \, \check{\mathbf{x}}_{\text{PU}}) \, x_{\text{SU}} + g_{14} \, \mathbf{x}_{\text{PU}} + \mathbf{v}_4 \tag{22}$$

where $\mathbf{J} \in \mathbb{C}^{M \times (M+\tau_{\text{SU}})}$ is obtained from $\mathbf{I}_{M+\tau_{\text{SU}}}$ by picking its first M rows, $\check{\mathbf{x}}_{\text{PU}} \triangleq [x_{\text{PU}}(-\tau_{\text{SU}}), \ldots, x_{\text{PU}}(-1), \mathbf{x}_{\text{PU}}^T]^{\text{T}} \in \mathbb{C}^{M+\tau_{\text{SU}}}$, $\mathbf{v}_4 \triangleq [v_4(0), v_4(1), \ldots, v_4(M-1)]^{\text{T}} \sim \mathcal{CN}(\mathbf{0}_M, N_0 \, \mathbf{I}_M)$. It is important to observe that $\mathbf{y}_{\text{SU}}$ also depends on the block $\check{\mathbf{x}}_{\text{PU}}$ of PU symbols transmitted by the PTx. The capacity of the SU link depends on what is known about the channel vector $\mathbf{g}_{\text{SU}} \triangleq [g_{24} \, g_{12}, g_{14}]^{\text{T}} \in \mathbb{C}^2$ and $\check{\mathbf{x}}_{\text{PU}}$ at the STx and SRx.

Herein, we assume that the SRx has perfect knowledge of both the realization of the channel vector $\mathbf{g}_{\text{SU}}$ and the PU symbol block $\check{\mathbf{x}}_{\text{PU}}$, and both the STx and SRx know the statistical distribution of $(\mathbf{g}_{\text{SU}}, \check{\mathbf{x}}_{\text{PU}})$. Such an assumption is reasonable when, besides having knowledge of the training symbol transmitted per frame by the STx, the SRx additionally knows the training signal sent by the PTx and, moreover, it is sufficiently close to both the PTx and STx such that to reliably estimate the channel vector $\mathbf{g}_{\text{SU}}$. In this case, the channel output consists of the triplet $(\mathbf{y}_{\text{SU}}, \mathbf{g}_{\text{PU}}, \check{\mathbf{x}}_{\text{PU}})$ and, thus, the mutual information between channel input and output (in nats/frame) is represented by $\mathsf{I}(x_{\text{SU}}; \mathbf{y}_{\text{SU}}, \mathbf{g}_{\text{SU}}, \check{\mathbf{x}}_{\text{PU}}) = \mathsf{I}(x_{\text{SU}}; \mathbf{y}_{\text{SU}} \,|\, \mathbf{g}_{\text{SU}}, \check{\mathbf{x}}_{\text{PU}})$. The capacity C_{SU} is the supremum of $\mathsf{I}(x_{\text{SU}}; \mathbf{y}_{\text{SU}} \,|\, \mathbf{g}_{\text{SU}}, \check{\mathbf{x}}_{\text{PU}})/M$ over all distributions on x_{SU} satisfying (2). It can be shown (details are omitted for the lack of space) that

$$\mathsf{C}_{\text{SU}} = \frac{1}{M} \, \mathbb{E}_{\mathbf{g}_{\text{SU}}, \check{\mathbf{x}}_{\text{PU}}} \left\{ \ln \left[1 + \frac{\sigma_{\text{SU}}^2 \, |g_{24}|^2 \, |g_{12}|^2}{N_0} \cdot \sum_{m=0}^{M-1} |x_{\text{PU}}(m - \tau_{\text{SU}})|^2 \right] \right\} . \tag{23}$$

The capacity in (23) (see Fig. 3 for a numerical example) can be achieved if the SU is capable to decode the PU symbols with arbitrarily small error probability: in order for this to be true, the information rate R_{PU} (in nats/symbol) of the PU must be smaller than the ergodic channel capacity $\mathsf{C}_{\text{PU}\to\text{SU}}$ (in nats/symbol) of the overall link between the PTx and the SRx, i.e., $\mathsf{R}_{\text{PU}} < \mathsf{C}_{\text{PU}\to\text{SU}}$. Due to the complete symmetry between the PRx and SRx with respect to the PTx, we can obtain an upper bound on $\mathsf{C}_{\text{PU}\to\text{SU}}$ exactly as we have done to get $\mathsf{C}_{\text{PU,upper}}$, hence obtaining, similarly to (14), the inequality $\mathsf{C}_{\text{PU}\to\text{SU}} \le \mathbb{E}\{f[\Gamma_{4,\text{upper}}(g_{24} \, x_{\text{SU}})]\}$, with $\Gamma_{4,\text{upper}}(|g_{24}|^2 \, |x_{\text{SU}}|^2) \triangleq \mathsf{ASNR}_{4,x_{\text{SU}}=0}(1+|g_{24}|^2 \, |x_{\text{SU}}|^2 \, \sigma_{12}^2/\sigma_{14}^2)/(1+|g_{24}|^2 \, |x_{\text{SU}}|^2)$, where $\mathsf{ASNR}_{4,x_{\text{SU}}=0} \triangleq \sigma_{14}^2 \, \mathsf{SNR}_{\text{PU}}$ is the ASNR at the SRx when $x_{\text{SU}} = 0$.

5. CONCLUSIONS

We proposed an amplify-and-forward scheme which allows a SU to concurrently transmit in the same frequency band of a PU, without requiring any noncausal knowledge of the PU information symbols. When the STx is closer to the PTx than to the PRx, the cognitive radio principle of protecting the PU is not only violated, but even a performance improvement can be gained by the PU. When the SRx is sufficiently close to the STx, the secondary link can support more than one bit per symbol for moderate-to-high SNR values if the SRx is able to decode the PU data. It is noteworthy that, while the transmission of a single SU symbol per frame gives low information rates, typically several PUs multiplex the frame resources in time or in frequency, and the SU might be active in parallel over all the channels allocated for the PUs, therefore potentially attaining larger transmission rates, without adding interference.

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