MULTI-GROUP MULTI-WAY RELAYING WITH REDUCED NUMBER OF RELAY ANTENNAS

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ABSTRACT

In this paper, multi-group multi-way relaying is considered. There are L groups with K nodes in each group. Each node wants to share d data streams with all the other nodes in its group. A single MIMO relay assists the communications. The relay does not have enough antennas to spatially separate the data streams. However, the relay assists in performing interference alignment at the receivers. In order to find the interference alignment solution, we generalize the concept of signal and channel alignment developed for the MIMO Y channel and the two-way relay channel to group signal alignment and group channel alignment. In comparison to conventional multi-group multi-way relaying schemes [1, 2], where at least $R \geq LKd - d$ antennas are required, in our proposed scheme, exploiting the multiple antennas at the nodes, only $R \geq LKd - Ld$ antennas are needed. The number of antennas required at the nodes to achieve this is also derived. It is shown that the proposed interference alignment based scheme achieves more degrees of freedom than the reference schemes without interference alignment.

Index Terms— Multi-way relaying, group communication, interference alignment.

1. INTRODUCTION

In applications like video conferencing, K nodes form a group and each node within the group wants to share its information with all the other nodes in the group. In the absence of direct links between these nodes, a single relay with R antennas can assist their communication. In the first phase called multiple access (MAC) phase, all the nodes transmit their signals to the relay. Then, the relay broadcasts multiple linearly processed versions of the signals received in the MAC phase to all the nodes in multiple broadcast (BC) phases. This is called multi-way relaying [3]. In this paper, we focus on half-duplex amplify-and-forward multi-way relaying.

Single group multi-way relaying has been considered in several works, see [4, 5, 6, 7, 8] and the references therein. In [4, 5, 6, 7, 8], K single antenna nodes form a group and each node shares its information with all the other nodes in the group through a relay with R antennas. It is assumed that the relay has enough antennas so that all K data streams from the nodes can be spatially separated at the relay, i.e., $R \geq K$ [4, 5, 6, 7, 8].

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Multi-group multi-way relaying is investigated in [1] and [2]. Here, the case of L groups with each group consisting of K nodes is considered. In [1], the nodes have $N = 1$ antenna. The K data streams from each of the L groups are spatially separated at the relay and hence, $R \geq LK$ antennas are required at the relay. In [2], multi-antenna nodes are considered. Equipped with N antennas, each node transmits $d = N$ data streams simultaneously. Using the fact that the self-interference can be cancelled at the nodes, the required number of relay antennas is shown to be $R \geq LKd - d$.

In all references [4, 5, 6, 7, 8, 1, 2], the data streams are spatially separated at the relay and hence, the relay needs a large number of antennas. In this paper, the multi-group case is considered. For $N \geq d$, using the concept of interference alignment (IA), we show that only $R \geq LKd - Ld$ antennas at the relay are sufficient for an interference-free communication. The basic idea is as follows: as the relay is not interested in the messages shared by the nodes, it is not necessary to spatially separate all the LKd data streams at the relay. The relay only needs to make sure that after performing IA at the nodes, the useful signals are separable from the interference signals and the dimension of the useful signal space is sufficiently large to separate all the useful data streams. It is assumed that the nodes can perfectly cancel the self interference and hence, signals from each group should be in a subspace of dimension at least $Kd - d$. In addition to this, the signals from different groups should be spatially separable at the relay. Hence, $R \geq LKd - Ld$ is necessary. In this paper, we focus on the case $R = LKd - Ld$, which is $(L-1)d$ antennas less than the other schemes available in the literature.

IA is a tri-linear problem. In this paper, we propose one possible approach to solve this problem as follows: Using the fact that the relay has only $R = LKd - Ld$ antennas, we decouple IA into three linear problems namely, group signal alignment (GSA), group channel alignment (GCA), and transceive zero forcing. GSA and GCA are generalizations of the concepts of signal and channel alignment, respectively, proposed in [9] and [10] for the multi-pair two-way relay channel and for the MIMO Y channel, respectively. A multi-way relay network is a generalization of multi-pair two-way relay network, where the number of nodes in a group is limited to two. MIMO Y channel consist of a single group of K nodes and each node has $K - 1$ independent data streams with each data stream to be transmitted to one of the $K - 1$ nodes. Signal alignment is also considered in [11] for a MIMO multi-group multi-way relay channel. [11] is a generalization of MIMO Y channel to the case of multiple groups. In [11] Signal and channel alignment are performed to achieve interference free communication. In comparison to [11], in the current paper, each node shares d data streams with all the other nodes in its group. All the signal alignment based algorithm utilize the fact that the self-interference can be cancelled at the receiver and hence, the useful and the self-interference are designed to align with each other at the relay. In this paper, each node is interested in all the data streams transmitted by all the nodes in its group and hence, the useful signal subspace is of larger dimension than the subspace spanned by the self-interference signal. Hence, there is more possibility in performing signal alignment in comparison to [9], [10], [11]. At the same time, group signal alignment should take care that all the members in the group are able to decode all the data streams after performing self-interference cancelation. As each node is interested in the data streams transmitted by all the nodes in its group, similar to [1, 2], in the current paper, $K-1$ BC phases are necessary. In this paper, IA is performed during each of the $K-1$ phases. The relay filters corresponding to each of the BC phases are designed such that after the joint processing of the useful signal received in all the BC phases, the receiver will be able to determine all its desired data streams. We propose a closed form solution to jointly process the useful signals received in $K-1$ BC phases.

The organization of the paper is as follows. The system model is introduced in Section 2. In Section 3, the proposed IA algorithm is described. Section 4 evaluates the performance of the proposed schemes in terms of the sum rate of the system. Section 5 concludes the paper.

We use lower case letters for scalars and lower case bold letters and upper case bold letters to denote column vectors and matrices, respectively. $(.)^*$, $(.)^{\mathrm{T}}$ and $(.)^{\mathrm{H}}$ denote the complex conjugate, transpose and complex conjugate transpose of the element within the brackets, respectively.

2. SYSTEM MODEL

In this paper, multi-group multi-way relaying is considered [3]. There are L groups, each with K nodes. The k^{th} node in the l^{th} group is denoted by (l,k). Node (l,k) has N_{lk} antennas for $l=1,\ldots,L$ and $k=1,\ldots,K$. Figure 1 shows an example scenario with 3 groups and 3 nodes in each group. Each of the LK nodes wants to share $d \leq N_{lk}$ data streams with all the other nodes in its group. There is no direct link between the nodes and a single relay with $R = LKd - Kd$ antennas assists their communication. In the first phase called MAC phase, all the LK nodes transmit their signals to the relay. Then, the relay broadcasts different linearly processed versions of the signals received in the MAC phase to all the LK nodes in $K-1$ BC phases. The channels are assumed to be constant over the $K-1$ BC phases. Global channel state information is assumed to be available at the nodes and at the relay. Let $\mathbf{d}_{lk}$ and $\mathbf{V}_{lk}$ denote the data symbols and the transmit filter matrix, respectively, of node (l,k). Each node has a maximum transmit power P_{node}. Let $\mathbf{H}^{\text{m}}_{lk}$ denote the MIMO channel matrix between node (l,k) and the relay in the MAC phase. The signal received at the relay is given by

$$\mathbf{r} = \sum_{l=1}^{L}\sum_{k=1}^{K} \mathbf{H}^{\text{m}}_{lk}\mathbf{V}_{lk}\mathbf{d}_{lk} + \mathbf{n}_{\text{r}}, \tag{1}$$

where $\mathbf{n}_{\text{r}}$ denotes the noise vector at the relay. The components of the noise vector at the relay are i.i.d. complex Gaussian random variables which follow $\mathcal{CN}(0,\sigma_{\text{r}}^2)$. Let $\mathbf{G}^p$ denote the matrix representing the linear signal processing performed at the relay for the p^{th} BC phase with $p=1,\ldots,K-1$. The relay has a transmit power P_{relay} available for each transmission phase. Consider the receiving node (l',k'). Let $\mathbf{H}^{\text{b}}_{l'k'}$ denote the MIMO channel matrix between node (l',k') and the relay during the BC phases. The received signal $\mathbf{y}^p_{l'k'}$ at node (l',k') in the p^{th} BC phase is given by

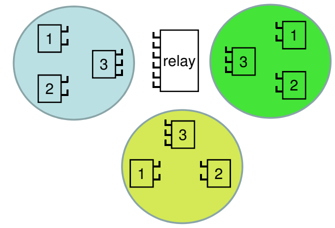

Fig. 1. Multi-group multi-way relay network for $L=3$, $K=3$ and $R=6$.

$$\begin{aligned}\mathbf{y}^p_{l'k'} &= \mathbf{H}^{\text{b}}_{l'k'}\mathbf{G}^p\left(\textstyle\sum_{\substack{k=1\\k\neq k'}}^{K}\mathbf{H}^{\text{m}}_{l'k}\mathbf{V}_{l'k}\mathbf{d}_{l'k} + \mathbf{H}^{\text{m}}_{l'k'}\mathbf{V}_{l'k'}\mathbf{d}_{l'k'}\right)\\ &\quad + \mathbf{H}^{\text{b}}_{l'k'}\mathbf{G}^p\textstyle\sum_{\substack{l=1\\l\neq l'}}^{L}\sum_{k=1}^{K}\mathbf{H}^{\text{m}}_{lk}\mathbf{V}_{lk}\mathbf{d}_{lk} + \widetilde{\mathbf{n}}_{l'k'}\end{aligned} \tag{2}$$

where $\widetilde{\mathbf{n}}_{l'k'} = \mathbf{H}^{\text{b}}_{l'k'}\mathbf{G}^p\mathbf{n}_{\text{r}} + \mathbf{n}_{l'k'}$ is the effective noise at receiver (l',k') with $\mathbf{n}_{l'k'}$ denoting the noise at node (l',k'). The components of the noise vector $\mathbf{n}_{l',k'}$ are i.i.d. complex Gaussian random variables which follow $\mathcal{CN}(0,\sigma^2_{l'k'})$. In (2), the first term corresponds to the useful signals. The second and the third terms correspond to the self interference and unknown inter-group interferences, respectively. It is assumed that the self interference can be perfectly cancelled. In this paper, the transmit and relay filters are designed to align all the interference signals during each of the $K-1$ BC phases within a $N_{l',k'}-d$ dimensional interference subspace at receiver (l',k') for $l'=1,\ldots,L$ and $k'=1,\ldots,K$ and the useful signals within a d-dimensional useful subspace disjoint from the interference subspace. The receive filter is designed in two stages. The first stage receive filter $\mathbf{U}^{\text{H}}_{l'k'}$ is applied to the received signal during each of the BC phases to nullify the interference signals. The output of the first stage receive filter at node (l',k') is given by

$$\begin{aligned}\mathbf{s}^p_{l'k'} &= \mathbf{U}^{\text{H}}_{l'k'}\mathbf{H}^{\text{b}}_{l'k'}\mathbf{G}^p\textstyle\sum_{\substack{k=1\\k\neq k'}}^{K}\mathbf{H}^{\text{m}}_{l'k}\mathbf{V}_{l'k}\mathbf{d}_{l'k} + \\ &\mathbf{U}^{\text{H}}_{l'k'}\mathbf{H}^{\text{b}}_{l'k'}\mathbf{G}^p\textstyle\sum_{\substack{l=1\\l\neq l'}}^{L}\sum_{k=1}^{K}\mathbf{H}^{\text{m}}_{lk}\mathbf{V}_{lk}\mathbf{d}_{lk} + \mathbf{U}^{\text{H}}_{l'k'}\widetilde{\mathbf{n}}_{l'k'}.\end{aligned} \tag{3}$$

Note that the first stage receive filter nullifies the interferences but not the useful signal in the d-dimensional useful subspace. These two conditions are represented as

$$\mathbf{U}^{\text{H}}_{l'k'}\mathbf{H}^{\text{b}}_{l'k'}\mathbf{G}^p\mathbf{H}^{\text{m}}_{lk}\mathbf{V}_{lk} = \mathbf{0} \ \text{ for all } l \neq l' \tag{4}$$

$$\text{rank}\left(\mathbf{U}^{\text{H}}_{l'k'}\mathbf{H}^{\text{b}}_{l'k'}\mathbf{G}^p\mathbf{H}^{\text{m}}_{l'k}\mathbf{V}_{l'k}\right) = d \tag{5}$$

for $k,k'=1,\ldots,K$, and $l,l'=1,\ldots,L$. In the d-dimensional useful subspace, there are $(K-1)d$ useful signals. These $(K-1)d$ useful signals cannot be spatially separated in a single BC phase. However, during each of the $K-1$ BC phases, the relay transmits d linearly independent linear combinations of the $(K-1)d$ useful signals and hence, joint processing over all BC phases can be utilized to spatially separate all the useful signals. In order to achieve this, the output signal of the first stage receive filter is concatenated over all the $K-1$ phases and the second stage receive filter $\mathbf{Q}^{\text{H}}_{l'k'}$ is applied to the concatenated signal to spatially separate all the useful signals coming from different nodes within the same group. Let $\hat{\mathbf{d}}_{lk}$ contain all the $(K-1)d$ estimated symbols. Then $\hat{\mathbf{d}}_{lk}$ is given by

$$\hat{\mathbf{d}}_{lk} = \mathbf{Q}^{\text{H}}_{l'k'}\left[\mathbf{s}^{1\text{T}}_{l'k'}\ \ldots\ \mathbf{s}^{(K-1)\text{T}}_{l'k'}\right]^{\text{T}}. \tag{6}$$

3. INTERFERENCE ALIGNMENT ALGORITHM

In this section, the proposed IA algorithm is described. The IA conditions given in (4) are a set of trilinear equations. In the following, we propose an approach to decouple them into three linear problems, namely, group signal alignment (GSA), group channel alignment (GCA) and transceive zero forcing. After achieving IA in each of the BC phases, the useful signals received during all the $K-1$ BC phases are jointly processed to separate the useful signals. In the following subsections, GSA, GCA, transceive zero forcing and the joint processing to separate the useful signals are described in detail.

3.1. Group Signal Alignment

In this section, the concept of GSA is described. In the MAC phase, each of the KL nodes transmits d data streams to the relay. However, the relay has only $L(K-1)d$ antennas and, therefore, the relay space is of dimension $L(K-1)d$. In the $L(K-1)d$-dimensional relay space, LKd data streams cannot be spatially separated. However, if the Kd data streams from the K nodes of group l are designed to be within a $(K-1)d$-dimensional relay subspace, then at the receiver, $(K-1)d$ useful signals can be obtained by subtracting the self interference signal. GSA is given by the following linear dependency equation:

$$\underbrace{\begin{bmatrix} \mathbf{H}_{l1}^{\mathrm{m}} & \mathbf{H}_{l2}^{\mathrm{m}} & \dots & \mathbf{H}_{lK}^{\mathrm{m}} \end{bmatrix}}_{\mathbf{H}_l^{\mathrm{m}}} \begin{bmatrix} \mathbf{V}_{l1} \\ \mathbf{V}_{l2} \\ \vdots \\ \mathbf{V}_{lK} \end{bmatrix} = \mathbf{0} \qquad (7)$$

for $l = 1, \dots, L$. Note that in addition to (7), $\mathbf{V}_{lk}$ needs to be of full rank d so that the d data streams transmitted by node (l, j) span a d-dimensional subspace at the relay. (7) has a non-trivial solution with each $\mathbf{V}_{lk}$ of rank d, if and only if the dimension of the null space of $\mathbf{H}_l^{\mathrm{m}}$ is at least d. The channels from different nodes to the relay are uncorrelated and, hence, $\mathbf{H}_l^{\mathrm{m}}$ is almost surely full rank. The size of the matrix $\mathbf{H}_l^{\mathrm{m}}$ is $R \times \sum_{k=1}^{K} N_{lk}$ with $R = L(K-1)d$. Hence, GSA is feasible if and only if the following condition holds:

$$\sum_{k=1}^{K} N_{lk} \geq R + d \ \text{ for } l = 1, \dots, L. \qquad (8)$$

Note that the matrix $\mathbf{V}_{lk}$ gives the subspace along which the d data streams need to be satisfied so that GSA is achieved. However, the d data streams can be transmitted in any d linearly independent directions in this subspace. Hence, for any arbitrary matrix $\mathbf{C}_{lk}$ of size $d \times d$ and rank d, $\mathbf{V}_{lk}\mathbf{C}_{lk}$ is also a solution for GSA.

3.2. Group Channel Alignment

In this section, the concept of group channel alignment is introduced. After GSA, there are $L(K-1)d$ effective data streams. In each of the $K-1$ BC phases, we want to achieve IA. This is achieved through GCA followed by transceive zero forcing at the relay. Similar to GSA, in GCA all the nodes choose their first stage receive filter such that the effective channel, $\mathbf{U}_{l'k'}^{\mathrm{H}}\mathbf{H}_{l'k'}^{\mathrm{b}}$, of all the nodes in a group l span only a $(K-1)d$-dimensional subspace at the relay. The relay with $L(K-1)d$ antennas can spatially separate the subspaces of the channels corresponding to different groups. Similar to GSA, GCA can be expressed as

$$\begin{bmatrix} \mathbf{U}_{l'1}^{\mathrm{H}} & \mathbf{U}_{l'2}^{\mathrm{H}} & \dots & \mathbf{U}_{l'K}^{\mathrm{H}} \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{H}_{l'1}^{\mathrm{b}} \\ \mathbf{H}_{l'2}^{\mathrm{b}} \\ \vdots \\ \mathbf{H}_{l'K}^{\mathrm{b}} \end{bmatrix}}_{\mathbf{H}_l^{\mathrm{b}}} = \mathbf{0} \qquad (9)$$

for $l' = 1, \dots, L$. From (7) and (9), it can be seen that GSA and GCA are dual problems. The solution for GCA can be obtained similar to GSA and the feasibility condition for GCA is the same as that of GSA.

Remark: Note that in this paper, it is assumed that the channel is constant during the $K-1$ BC phases. However, an extension to the case where the channel is differente in each of the $K-1$ BC phases is easily possible. A new first stage receive filter satisfying the GCA condition has to be determined during each of the $K-1$ BC phases.

3.3. Transceive Zero Forcing

In this section, the relay filters are designed for the $K-1$ BC phases. After GSA and GCA, there are $L(K-1)d$ effective data streams and $L(K-1)d$ effective channels. The relay can perform transceive zero forcing to transmit these $L(K-1)d$ effective data streams to all the nodes. Let $\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}}$ and $\mathbf{G}_{\mathrm{tx}}$ denote the receive and transmit zero forcing matrices, respectively. Let $\mathbf{P}_p$ be a block diagonal precoding matrix that makes sure that the receiver gets linearly independent linear combinations of the effective data streams during each of the BC phases. Then, the relay filter $\mathbf{G}_p$ for $p = 1, \dots, K-1$ in the p^{th} BC phase can be expressed as

$$\mathbf{G}_p = \mathbf{G}_{\mathrm{tx}}\mathbf{P}_p\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}}. \qquad (10)$$

In the following, we determine $\mathbf{G}_{\mathrm{rx}}$, $\mathbf{G}_{\mathrm{tx}}$ and $\mathbf{P}_p$. Let the columns of matrix $\widetilde{\mathbf{H}}_l^{\mathrm{m}}$ denote the basis of the $(K-1)d$-dimensional subspace spanned by the signals from the K nodes of group l given by $\widetilde{\mathbf{H}}_l^{\mathrm{m}} = \begin{bmatrix} \mathbf{H}_{l1}^{\mathrm{m}}\mathbf{V}_{l1} & \mathbf{H}_{l2}^{\mathrm{m}}\mathbf{V}_{l2} & \dots & \mathbf{H}_{l(K-1)}^{\mathrm{m}}\mathbf{V}_{l(K-1)} \end{bmatrix}$. Then the receive zero forcing matrix that spatially separates signals from all the L groups of nodes is given by

$$\mathbf{G}_{\mathrm{rx}}^{\mathrm{H}} = \begin{bmatrix} \widetilde{\mathbf{H}}_1^{\mathrm{m}} & \widetilde{\mathbf{H}}_2^{\mathrm{m}} & \dots & \widetilde{\mathbf{H}}_L^{\mathrm{m}} \end{bmatrix}^{-1}. \qquad (11)$$

Note that the matrix on the right hand side of (11) is a square matrix and for arbitrarily generated channel matrices, it is full rank with probability one. Let the rows of the matrix $\widetilde{\mathbf{H}}_{l'}^{\mathrm{b}}$ denote the basis of the $(K-1)d$ dimensional subspace spanned by the effective channels corresponding to the K nodes of group l'. Then the transmit zero forcing matrix that spatially separates the channels corresponding to all the L groups of nodes is

$$\mathbf{G}_{\mathrm{tx}} = \begin{bmatrix} \left(\widetilde{\mathbf{H}}_1^{\mathrm{b}}\right)^{\mathrm{T}} & \left(\widetilde{\mathbf{H}}_2^{\mathrm{b}}\right)^{\mathrm{T}} & \dots & \left(\widetilde{\mathbf{H}}_L^{\mathrm{b}}\right)^{\mathrm{T}} \end{bmatrix}^{(-1)\mathrm{T}}. \qquad (12)$$

Note that the transmit zero forcing filter $\mathbf{G}_{\mathrm{tx}}$ is based on the effective channel including the first stage receive filter $\mathbf{U}_{lj}^{\mathrm{H}}$. Hence, at each of the LK nodes, the interferences will be zero after the first stage receive filter. If we consider the received signals, then the interference will be in an $N_{lk} - d$ dimensional interference subspace orthogonal to the d-dimensional subspace spanned by the columns of the matrix $\mathbf{U}_{lj}^{\mathrm{H}}$. However, the useful signals are not nullified by $\mathbf{G}_{\mathrm{tx}}$ and hence, will be non-zero after the first stage receive filter. This means before the first stage receive filters, the useful signals are in a subspace disjoint from the interference subspace. Hence, IA is achieved through GCA and transceive zero forcing.

After the first stage receive filter, the useful signals are in a d-dimensional subspace. Assuming the self-interference has been perfectly cancelled, the d-dimensional useful subspace contains d-linearly independent linear combinations of the $(K-1)d$ effective signals which are again linear combinations of the $(K-1)d$ data streams transmitted by all the nodes within the group. The linear combinations depend on the block diagonal matrix $\mathbf{P}_p$ and the zero forcing matrices $\mathbf{G}_{\text{rx}}^{\text{H}}$ and $\mathbf{G}_{\text{tx}}$ at the relay and the channel matrices $\mathbf{H}_{l'k'}^{\text{b}}$. The matrices $\mathbf{G}_{\text{rx}}^{\text{H}}$, $\mathbf{G}_{\text{tx}}$ and $\mathbf{H}_{l'k'}^{\text{b}}$ are constant for all the p phases. The matrix $\mathbf{P}_p$ has to be chosen such that in $K-1$ BC phases, $(K-1)d$ linearly independent linear combinations are received. Any p arbitrary choices of block diagonal matrices will almost surely be a valid solution. However, one can choose $\mathbf{P}_p$ for $p = 1, \ldots, K-1$ such that some utility function, e.g. signal to noise ratio, is maximized in the system. In this paper, we arbitrarily choose $\mathbf{P}_p$ and leave the optimization of $\mathbf{P}_p$ for future work.

Remark: If the channel coefficients are different in each of the $K-1$ BC phases and are independent of each other, then linearly independent linear combinations of the effective data streams will be almost surely guaranteed with $\mathbf{P}_p = \mathbf{I}$.

3.4. Group Signal Separation

In this section, the second stage receive filter is designed to separate the useful signals received from the nodes within the group. Let $\mathbf{H}_{l'k'k}^{(p)\text{eff}} = \mathbf{U}_{l'k'}^{\text{H}} \mathbf{H}_{l'k'}^{\text{b}} \mathbf{G}_p \mathbf{H}_{l'k}^{\text{m}} \mathbf{V}_{l'k}$ denote the effective channel between nodes k and k' of group l' in the BC phase p. Then, the effective channel from all the $K-1$ nodes in group l' to node k' is given by

$$\mathbf{H}_{l'k'}^{(p)\text{eff}} = \left[\begin{array}{ccccc} \mathbf{H}_{l'k'1}^{(p)\text{eff}} & \ldots & \mathbf{H}_{l'k'j}^{(p)\text{eff}} & \ldots & \mathbf{H}_{l'k'K}^{(p)\text{eff}} \end{array} \right]_{j \neq k'}. \quad (13)$$

The second stage receive filter is applied to the concatenation of the signals received in the $K-1$ BC phases. The effective channel from all the $K-1$ nodes to node j in the group l of all the $K-1$ BC phases is given by

$$\mathbf{H}_{l'k'}^{\text{eff}} = \left[\begin{array}{cccc} \left(\mathbf{H}_{l'k'}^{(1)\text{eff}}\right)^{\text{T}} & \left(\mathbf{H}_{l'k'}^{(2)\text{eff}}\right)^{\text{T}} & \ldots & \left(\mathbf{H}_{l'k'}^{(K-1)\text{eff}}\right)^{\text{T}} \end{array} \right]^{\text{T}}. \quad (14)$$

The second stage receive filter is designed as the zero forcing filter to spatially separate the $(K-1)d$ data streams and is given by

$$\mathbf{Q}_{l'k'}^{\text{H}} = \left(\mathbf{H}_{l'k'}^{\text{eff}}\right)^{-1}. \quad (15)$$

4. PERFORMANCE ANALYSIS

In this section, the degrees of freedom and the sum rate performance of the proposed multi-group multi-way relaying IA scheme is investigated. In this paper it is assumed that $R = LKd - Ld$. In section 3, the feasibility condition for GSA and GCA is derived as $\sum_{k=1}^{K} N_{lk} \geq L(K-1)d + d$. We define the degrees of freedom (DoF) as the total number of data streams received by all the nodes in one time slot. Each node receives d data streams from all the other $K-1$ nodes of its group. Hence, each node receives $(K-1)d$ data streams. There are LK nodes in the system. This results in $LK(K-1)d$ data streams in the system. The communication takes place in K time slot. Hence, $L(K-1)d = LKd - d$ degrees of freedom are achieved. In [1] and [2], the relay needs $R \geq LKd$ and $R \geq LKd - d$ antennas, respectively. The proposed IA based scheme requires $(L-1)d$ less antennas than [1] and [2], i.e., the proposed scheme IA achieve more DoF for a given number of antennas at the relay. The gain in the DoF is due to the fact that the proposed IA based scheme utilizes the multiple antennas at the nodes and jointly design the transmit, relay and receive filters. In the following we consider, 3 different scenarios shown in Table 1 to compare the DoF of the proposed scheme with [1] and [2].

Scenario	L	K	R	d	N_{l1}	N_{l2}	N_{l3}	N_{l4}
A	3	3	6	1	2	2	3	-
B	3	4	9	1	2	2	3	3
C	4	4	12	1	2	3	4	4

Table 1. Parameters of the investigated scenarios

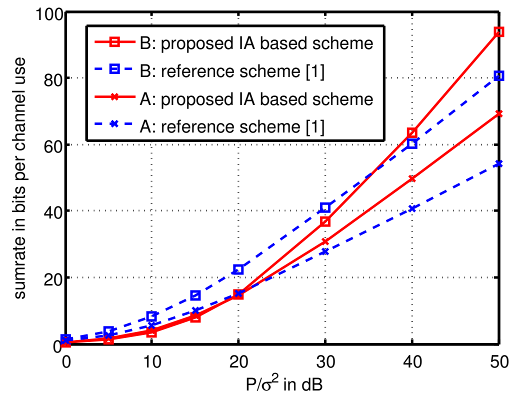

Fig. 2. Sum rate performance

In scenario A, the proposed IA based scheme achieves 6 DoF. [1] and [2] needs atleast 8 antennas to serve all the nodes simulataneously. With 6 antennas [1] and [2] can serve only 2 groups simulataneously and hence, only a DoF of 4 is achieved. Similarly, in scenarios B and C, the proposed IA based scheme achieves 9 and 12 DoF, respectively. However, both [1] and [2] achieves only 6 and 9 in scenarios B and C, respectively.

As described above, the proposed IA based scheme achieves 2, 3, and 3 additional DoF than both [1] and [2] in scenario A, B, and C, respectively. In the following, we consider the sum rate achieved by the proposed IA based scheme and zero-forcing criterion based multicasting strategy in [1] for the scenarios A and B. Figure 2 shows the sum rate performance as a function of P/σ^2. P is the transmit power available at each node. The relay has a transmit power of $9P$ and $12P$ in scenario A and B, respectively. σ^2 is the noise power per antenna at the relay and at each of the nodes. The channel matrices are generated randomly using the i.i.d. frequency flat Rayleigh MIMO channel model. From Figure 2, it can be seen that that the proposed IA based scheme performs better than the reference scheme at medium and high SNR.

5. CONCLUSION

In this paper, IA in multi-group multi-way relay networks is considered. The relay has only $LKd - Kd$ antennas and, hence, cannot spatially separate all the nodes' signals. The multiple antennas at the nodes and at the relays are utilized to perform IA to make the communication possible. IA is achieved through group signal alignment, group channel alignment and transceive zero forcing. The feasibility condition is derived as $\sum_{k=1}^{K} N_{lk} \geq L(K-1)d + d$ for $l = 1, \ldots, L$. It is shown that the proposed scheme achieves more degrees of freedom than the reference schemes and has a better sum rate performance at medium to high SNR.

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