TRANSPORTATION-THEORETIC IMAGE COUNTERFORENSICS TO FIRST SIGNIFICANT DIGIT HISTOGRAM FORENSICS

Cecilia Pasquini *, *Pedro Comesaña-Alfaro* †, *Fernando Pérez-González* †,‡, *Giulia Boato* *

* DISI, University of Trento
cecilia.pasquini@unitn.it boato@disi.unitn.it
38123 Trento-ITALY
† EE Telecomunicacion, University of Vigo
‡ GRADIANT (Galician Research and Development Center in Advanced Telecommunications)
36310 Vigo-SPAIN
{pcomesan|fperez}@gts.uvigo.es

ABSTRACT

First-order statistics of First Significant Digits (FSD) have been recently exploited in multimedia forensics as a powerful tool to reveal traces of previous coding operations. As an answer, adversarial approaches aimed at modifying the FSD histogram and fooling such forensic methods have been proposed. However, the existing techniques have limitations in terms of distortion introduced in the multimedia object. In this paper, a transportation-theoretic formulation of the problem is presented which provides a close-to-optimal solution. Such strategy is tested in a well-known image forensic scenario, where FSDs of 8×8-DCT coefficients after single or double quantization are modified in order to restore a certain target histogram and the distortion with respect to the provided compressed image is measured in terms of MSE.

Index Terms— Counterforensics, FSD statistics, multimedia forensics, transportation theory.

1. INTRODUCTION

In the last decades, the issue of authenticating and preserving the integrity of multimedia content has gained more importance, due to the enormous amount of multimedia data created every day and the availability of powerful editing tools, accessible also to non-skilled users. For this reason, the recent research field of passive multimedia forensics started facing the complex problem of detecting the traces of processing previously applied to a given multimedia object, in order to assess the trustability of its content.

As it happened to digital watermarking and steganography, the need of an adversary-aware perspective recently emerged also in multimedia forensics [1]. Indeed, the potential presence of a smart adversary compromises the reliability of forensic methods, as proved by the increasing number of counterforensic techniques proposed in the literature which strongly decrease the performance of forensic detectors. Counterforensic strategies have been introduced, for example, for hiding traces of resampling [2], median filtering [3, 4], lossy compressions [5, 6] and PRNU inconsistencies [7].

However, they usually target specific forensic methods and their optimality under any criterion is not discussed. In this sense, the recent approaches [8] and [9] represent a significant enhancement, since they propose a general procedure to cope with an entire class of image forensic detectors, the ones based on the histogram of the input samples. In particular, in [9] the optimal modification in terms of MSE distortion is derived, by means of a transportation-theoretic formulation that reduces the problem to a single optimization process. However, such optimal solution is obtained when the forensic detector is based on the histogram of a bijective orthonormal transformation of the input signal, for example the block-DCT.

On the other hand, another statistic which is often used in image forensics, but does not fulfill these properties, is the distribution of First Significant Digits (FSD) in the DCT domain, which has been widely investigated in image processing [10] [11]. In JPEG image forensics, the FSD histograms of block-DCT coefficients at certain frequencies are analyzed and exploited in several decision problems. Indeed, forensic methods have been designed for discriminating between uncompressed and compressed images [12], single and double compressed images [13], or even images compressed different numbers of times [14].

Adversarial approaches have also been proposed in [6] and [15], though presenting some limitations in terms of distortion introduced in the image or lack of flexibility with respect to the forensic scenario, as will be discussed in greater detail in Section 3.1.

In this paper, we focus on the problem of FSD histogram modification and propose a method to replicate a set of target histograms starting from the ones of a given image.

With the same meaning as in [8], our attack can be seen as universal to detectors based on FSD first-order histogram, since no specific method is targeted but we consider a generic binary detector taking as input a set of vectors and analyzing their FSD histograms in order to decide between a null hypothesis H_0 and an alternative hypothesis H_1, that can be adapted to suit different forensic problems. In the following, we will always assume hypothesis H_1 is verified for the given image and we want to modify its FSD first-order histogram so that its attacked version belongs to H_0.

Since a generic detector is addressed, we do not express the acceptance region analytically nor rely on specific models for FSD first-order statistics. However, in our framework we assume to have a reference histogram to be replicated for each frequency, obtained by averaging the histograms of a set of images for which H_0 is verified.

Though such assumption might sound restrictive, it proves to be suitable for the forensic scenarios mentioned before. Indeed, it is possible to exploit the fact that the distribution of DCT coefficients at

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the same frequency is similar among images and so are the FSD first-order statistics, even after quantization with the same quality factor. Then, we will assume the decision region to be convex and consider as target the histograms obtained from the averaging operation. This leads to a conservative attack, since it is guarantees that the input signal is moved to the acceptance region regardless of the particular detector (thus preserving the universality of the attack), even if a smaller modification might be sufficient.

The paper is organized as follows: in Section 2, we outline the theoretical framework and present the transportation-theoretic formulation. The novel method proposed in this paper is described in Section 3. In Section 4, we report and comment the results of experiments we performed on a image database. Finally, we conclude the paper and discuss future work in Section 5.

2. PROBLEM FORMULATION

In order to use a compact notation throughout the paper, we formally define the First Significant Digits and their histogram.

Let $\mathbf{y} \in \mathcal{Y}^N \subset \mathbb{R}^N$ be a vector containing the samples of a given discrete signal, where $\mathcal{Y}$ is the set of possible values for every component of $\mathbf{y}$. Then, let $FSD : \mathbb{R} \longrightarrow \mathcal{D}, \mathcal{D} := \{0, \dots, 9\}$, be the function mapping any real value a to its first significant digit, i.e.,

$$FSD(a) = \begin{cases} \left\lfloor \frac{|a|}{10^{\lfloor \log_{10} |a| \rfloor}} \right\rfloor & \text{if } a \neq 0 \\ 0 & \text{otherwise.} \end{cases}$$

With a slight abuse of notation, we will indicate as $FSD(\mathbf{y})$ the vector given by $(FSD(y_1), \dots, FSD(y_N)) \in \mathcal{D}^N$.

Now, we can define the function H mapping any FSD vector $\mathbf{d} \in \mathcal{D}^N$ to a vector $\mathbf{h} \in \{0, \dots, N\}^{10}$, representing its histogram computed by considering bins corresponding to $0, \dots, 9$.

As mentioned before, binary forensic detectors proposed in the literature analyze a number of histograms, corresponding to a set of frequencies. However, if a target histogram for each frequency is provided, the process of replicating such histograms can be performed separately on each frequency by means of the same procedure. Therefore, in the following we will consider a single vector $\bar{\mathbf{y}}$, containing the DCT coefficients at a certain frequency, and a given target histogram $\mathbf{h}^*$. We consider a distance g^y defined over $\mathcal{Y}^N \times \mathcal{Y}^N$ as a measure for comparing $\bar{\mathbf{y}}$ and its modified version.

Then, the problem of modifying $\bar{\mathbf{y}}$ such that its FSD first-order histogram is equal to $\mathbf{h}^*$ minimizing the distortion is equivalent to solving the following optimization problem

$$\mathbf{y}^* = \underset{\{\mathbf{y} | H(FSD(\mathbf{y})) = \mathbf{h}^*\}}{\arg\min} g^y(\bar{\mathbf{y}}, \mathbf{y}). \tag{1}$$

In order to express the problem in (1) in terms of *optimal transportation theory* [16], as it is done in [9], we define $\bar{\mathbf{d}} := FSD(\bar{\mathbf{y}})$ and a similarity measure for FSD vectors with respect to $\bar{\mathbf{d}}$:

$$g^d(\bar{\mathbf{d}}, \mathbf{d}) := \min_{\{\mathbf{y} | FSD(\mathbf{y}) = \mathbf{d}\}} g^y(\bar{\mathbf{y}}, \mathbf{y}).$$

Now, we can state that the solution of (1) is equivalent to following sequence of problems:

$$\mathbf{d}^\sharp = \underset{\{\mathbf{d} | H(\mathbf{d}) = \mathbf{h}^*\}}{\arg\min} g^d(\bar{\mathbf{d}}, \mathbf{d}), \tag{2}$$

$$\mathbf{y}^* = \underset{\{\mathbf{y} | FSD(\mathbf{y}) = \mathbf{d}^\sharp\}}{\arg\min} g^y(\bar{\mathbf{y}}, \mathbf{y}). \tag{3}$$

It is worth noticing that, unlike in [9], we do not need to optimize over a set of histograms, since we consider a single target one.

However, in order to find the optimal solution, $g^d(\bar{\mathbf{d}}, \cdot)$ must be minimized over all the FSD vectors having histogram $\mathbf{h}^*$. If $\mathbf{h}^* = (h_0, \dots, h_9)$, the number of vectors to be considered is given by

$$\binom{N}{h_0}\binom{N - h_0}{h_1} \cdots \binom{N - (h_0 + h_1 + \cdots + h_8)}{h_9}.$$

Such number is generally very high, even for small values of N, thus making the search of the exact $\mathbf{d}^\sharp$ computationally unfeasible.

For this reason, we propose a procedure based on a simple and yet effective strategy, that provides a close-to-optimal solution of (2) and (3) simultaneously.

3. PROPOSED METHOD

As mentioned before, the optimization in (2) requires the evaluation of g^d over every element of $\{\mathbf{d} | H(\mathbf{d}) = \mathbf{h}^*\}$. Regarding (3), it can be solved more easily if we assume that g^y is a component-wise sum

$$g^y(\bar{\mathbf{y}}, \mathbf{y}) = \sum_{j=1}^{N} g(\bar{y}_j, y_j), \tag{4}$$

where g is a symmetric convex function depending on the difference between its input arguments. This is the case of the MSE, the most extensively used distortion measure. Indeed, under these assumptions, minimizing $g^y(\bar{\mathbf{y}}, \cdot)$ is equivalent to minimizing each $g(\bar{y}_j, \cdot)$.

This significantly simplifies solving (3). Indeed, if we define $\mathcal{S}$ to be the subset of $\mathbb{R}$ to which we can move the initial values, then $\mathcal{S}$ is the union of the disjoint sets $\mathcal{S}_d = \{s \in \mathcal{S} | FSD(s) = d\}$, $d \in \mathcal{D}$. For any real value y and any digit d, the elements in $\mathcal{S}_d$ that minimize the absolute difference with respect to y can then be identified.

Then, we define

$$f_{\mathcal{S}}(y, d) := \underset{y' \in \mathcal{S}_d}{\arg\min} |y - y'|,$$

$$Dist_{\mathcal{S}}(y, d) := |y - f_{\mathcal{S}}(y, d)|.$$

If the optimization problem in (3) has more than one minimizer, one of them is arbitrarily chosen, thus guaranteeing that $f_{\mathcal{S}}$ is well-defined. For instance, if $\mathcal{S} = \mathbb{Z} \cdot 10^{-1}$ then $f_{\mathcal{S}}(50, 5) = 50$ ($Dist_{\mathcal{S}}(50, 5) = 0$), $f_{\mathcal{S}}(-50, 7) = -70$ ($Dist_{\mathcal{S}}(-50, 7) = 20$), $f_{\mathcal{S}}(50, 3) = 39.9$ ($Dist_{\mathcal{S}}(50, 3) = 10.1$).

Considering this, in the following we propose a sub-optimal approach to solve (2), while (3) is solved optimally by means of the map $f_{\mathcal{S}}$. The procedure determines a new vector $\mathbf{z}$ starting from a given input vector $\bar{\mathbf{y}}$ and a target histogram $\mathbf{h}^*$. For the sake of simplicity, we assume $\bar{\mathbf{y}}$ is non-negative; otherwise we should just consider the absolute values of its components and recover the signs of the original vector after the transformation. Our approach relies on the heuristic idea that, in order to obtain a low distortion of $\bar{\mathbf{y}}$, the elements with largest values should be modified as less as possible, since they clearly introduce heavier distortion than the smallest ones. To this end, in our method a suitable new digit is selected for every element of $\bar{\mathbf{y}}$ and each new component is chosen by means of $f_{\mathcal{S}}$, as described below.

Precisely, starting from $\mathbf{h}^*$, we define as $\mathcal{D}_0^t$ the unique set of N elements belonging to $\mathcal{D}$ such that its histogram is $\mathbf{h}^*$.

Then, the input vector $\bar{\mathbf{y}}$ is sorted in descending order by means of a permutation[1]

$$\tilde{\mathbf{y}} = \sigma(\bar{\mathbf{y}}),$$

and, starting from $j = 0$ (greatest value) until $j = N - 1$ (smallest value), every component of $\tilde{\mathbf{y}}$ is transformed as follows

$$d_j^+ = \text{argmin}_{d \in \mathcal{D}_j^t} Dist_{\mathcal{S}}(\tilde{y}_j, d),$$

$$z_j^+ = f_{\mathcal{S}}(\tilde{y}_j, d_j^+),$$

$$\mathcal{D}_{j+1}^t = \mathcal{D}_j^t \setminus d_j^+.$$

Finally, the modified vector is given by

$$\mathbf{z} = \sigma^{-1}(\mathbf{z}^+),$$

where σ^{-1} is the inverse permutation of σ, and its FSD histogram is exactly $\mathbf{h}^*$. Therefore, $\mathbf{z}$ and $\mathbf{d} = \sigma^{-1}(\mathbf{d}^+)$ are obtained as approximate solutions instead of the exact ones, $\mathbf{y}^*$ and $\mathbf{d}^\sharp$, respectively.

Because of the sorting operation, for elements with higher values the corresponding new FSD can be chosen among a larger pool of digits; hence, they will be likely kept unaltered or assigned to a digit that leads to a small $Dist_{\mathcal{S}}(\tilde{y}_i, d_t)$. On the other hand, small coefficients might be moved to a new digit that is far from the original one.

Such procedure clearly does not lead to the theoretical optimal solution, since (2) is suboptimally solved. However, the fact that the highest values in $\bar{\mathbf{y}}$ (i.e., the ones that would potentially introduce a higher distortion) are mapped to a new FSD such that $Dist_{\mathcal{S}}$ is low, helps to keep a low distortion between $\bar{\mathbf{y}}$ and $\mathbf{z}$. Specifically, such approach will be particularly effective when the values of $\tilde{y}_j$ decay rapidly as j increases, as it happens for DCT coefficients.

3.1. Observations and comparison to previous art

It is worth noticing that, in the framework of JPEG image forensics described before, the vectors $\bar{\mathbf{y}}$ are a transformation of the signal in the pixel domain. However, as pointed out in [9], if the distortion between the provided image and the modified one is measured in terms of the MSE (or equivalently the PSNR) in the pixel domain, the method proposed in Section 3 can be applied straightforwardly to $\bar{\mathbf{y}}$. Indeed, the orthonormality of the block-DCT transformation allows us to consider the MSE as a distance directly in the DCT domain, thus satisfying the assumptions on g^y required in Section 3.

To the best of our knowledge, all of the forensic detectors based on FSD histograms proposed in the literature only consider non-zero-valued DCT coefficients in their analysis, while null coefficients are discarded and the FSD histogram is computed only for bins $1, \dots, 9$. The formulation in Section 2 copes with the more general case where also the null coefficients can be moved in order to replicate a target histogram defined over the 10 bins corresponding to $0, \dots, 9$. However, our procedure can be easily adapted to the 9 bin case by simply defining $\bar{\mathbf{y}}$ as the vector containing the non-zero coefficients at a DCT frequency and considering $\mathcal{D} = \{1, \dots, 9\}$ when computing the histogram, thus keeping unaltered the null values.

[1] We remark that, since in (4) the function g is the same for every j, any permutation that sorts $\bar{\mathbf{y}}$ in descending order can be used (there might be more than one because of repeated values in $\bar{\mathbf{y}}$).

A significant difference of the proposed method with respect to the approach in [15] is that coefficients are moved in sequence depending on their absolute value, and regardless of their initial distribution. Indeed, in such technique, inspired to waterfilling solutions [17], FSD histogram bins with an exceeding or lacking number of elements with respect to the target histogram are first identified and only transfers from the former to the latter ones are allowed. This generally leads to a quite heavy modification in the DCT coefficients, since it reduces the degrees of freedom in the movement of coefficients.

Unlike [6], the procedure proposed here is able to restore any target histogram and it can then be suitable for a larger number of forensic problems. Indeed, the method in [6] imposes a reasonable upperbound to the distance between every coefficient and its attacked version, but it can be applied only for the case where the attacker wants to restore the statistics of uncompressed images, since it does not allow to produce an arbitrary histogram. On the other hand, as long as a reference target histogram is available, the proposed method can potentially be applied in any hypothesis testing problem where H_0 is "image has been compressed n times" and H_1 is "image has been compressed m times". Furthermore, the procedure in [6] only approximately provides a histogram that verifies Benford's law and, especially for high frequencies (or, in general, frequencies were a strong quantization is performed), such approximation can be not accurate. Indeed, it proves to be effective when the lower frequencies are considered, which is true for most forensic methods proposed in the literature (see [13] and [14]), but might not happen for a generic detector.

4. EXPERIMENTAL RESULTS

In order to evaluate the performance of the proposed method, we considered the forensic scenario described in Section 2, i.e., where a binary forensic detector takes as input a set of FSD histograms corresponding to 8×8 block-DCT coefficients at different frequencies.

The images used in our experiments belong to the UCID database [18].

We considered three different binary hypothesis testing problems, specified in Table 1. In each situation, we are interested in modifying the block-DCT coefficients of images in the decision region corresponding to H_1, in order to be in the decision region of H_0, by introducing a minimal distortion.

	H_0	H_1
A	uncompressed	single compressed
B	uncompressed	double compressed
C	single compressed	double compressed

Table 1.

As introduced in Section 1 reference sets of FSD histograms have been obtained by averaging the histograms of 600 randomly chosen images in UCID for every frequency, from 1 to 64: specifically, we computed a set $(\mathbf{h}_1^{unq}, \dots, \mathbf{h}_{64}^{unq})$ from uncompressed images and the families $(\mathbf{h}_1^{QF_t}, \dots, \mathbf{h}_{64}^{QF_t})$ from single compressed images with quality factors $QF_t = \{50, 60, 70, 80, 90\}$. Then, the averaged histograms have been normalized, so that we have a reference *probability* for each digit, that is transformed into an integer value according to the number of coefficients in each frequency.

A set of images (different of those used for the computation of the target histogram) are applied the processing corresponding to H_1 for the three cases considered in Table 1. For each of them, and each frequency, the FSD histogram is modified in order to yield the target FSD histogram. The set $\mathcal{S}$ has been considered, for each frequency, as a lattice with step equal to the maximum over a row of the 8×8-DCT transformation matrix, in order to encompass in the modification the further distortion due to the quantization in the pixel domain. In a first set of experiments we focus on the nonzero coefficients, i.e., $\mathcal{D} = \{1, \ldots, 9\}$.

In Tables 2 and 3 we report the PSNR corresponding to the average value of the MSE of the modified images with respect to the provided compressed versions (the ones for which H_1 is verified) for each binary decision problem. In order to evaluate the validity of our approach, we implemented the method described in [15] and compared the results obtained when the same target histogram is considered. Indeed, such technique is also designed to replicate a given histogram, thus allowing for a fair comparison with the proposed method. The two methods are denoted in the Tables as **TT** and **WF**, indicating the transportation-theoretic formulation and the waterfilling approach, respectively.

In problem A, images were first single compressed with different quality factors QF_1 and then $(\mathbf{h}_1^{unq}, \ldots, \mathbf{h}_{64}^{unq})$ have been targeted. The same happens in problem B, where images were first compressed with fixed quality factor 75 and then re-compressed with different QF_2. In problem C, images are first compressed with fixed quality factor 75, re-compressed with QF_2 and the histogram sets $(\mathbf{h}_1^{QF_t}, \ldots, \mathbf{h}_{64}^{QF_t})$ were replicated for different QF_t.

As we can see from the tables, the distortion introduced in the image by the proposed method is significantly lower than the one obtained by applying [15]. The difference in terms of PSNR ranges from 3 dB to 12.61 dB.

$\mathbf{QF_1}$	**50**	**60**	**70**	**80**	**90**
TT	41.03	41.56	42.11	43.38	46.22
WF	34.47	34.22	34.58	34.91	34.77

$\mathbf{QF_2}$	**50**	**60**	**70**	**80**	**90**
TT	38.21	38.58	41.33	44.17	42.83
WF	33.14	33.14	34.29	35.86	35.12

Table 2. Case A and B, nonzero coefficients, 738 images.

$\mathbf{QF_t}$	$\mathbf{QF_2}$	**50**	**60**	**70**	**80**	**90**
50	**TT**	42.71	41.37	38.49	39.20	38.61
	WF	38.43	37.32	36.33	35.03	36.05
60	**TT**	42.08	42.95	41.33	39.08	40.70
	WF	33.65	38.20	36.49	36.11	36.55
70	**TT**	39.23	42.25	42.98	42.16	43.57
	WF	32.34	32.77	37.66	37.36	37.45
80	**TT**	36.99	38.93	41.93	44.04	43.82
	WF	32.17	32.40	33.31	39.13	34.57
90	**TT**	36.17	36.73	39.22	44.16	40.85
	WF	32.99	32.46	33.41	35.14	34.35

Table 3. Case C, nonzero coefficients, 738 images.

In a second set of experiments, we included also null coefficients in the modification, i.e., the bin corresponding to 0 is also considered. In this case, the computational complexity significantly increases, since more coefficients need to be moved in every frequency. PSNR results for a subset of images, computed in a similar way to those in Tables 2 and 3, are reported in Tables 4 and 5. They are generally different with respect to the previous case, due to the additional constraint on the null values and the availability of more coefficients to be moved, but still we find similar results as before when considering the difference between the two methods.

$\mathbf{QF_1}$	**50**	**60**	**70**	**80**	**90**
TT	43.67	43.67	43.60	43.54	43.43
WF	37.67	37.79	37.71	37.93	36.98

$\mathbf{QF_2}$	**50**	**60**	**70**	**80**	**90**
TT	43.23	43.19	43.58	43.60	43.63
WF	34.79	34.36	36.88	38.45	37.78

Table 4. Case A and B, zero coefficients included, 300 images.

$\mathbf{QF_t}$	$\mathbf{QF_2}$	**50**	**60**	**70**	**80**	**90**
50	**TT**	34.36	34.26	34.79	36.05	35.28
	WF	33.66	31.63	31.98	32.30	32.28
60	**TT**	35.03	34.89	35.47	36.45	35.96
	WF	31.03	33.67	32.17	32.33	32.34
70	**TT**	35.66	35.76	36.45	37.56	37.04
	WF	30.69	30.81	34.43	32.74	33.05
80	**TT**	36.06	36.49	37.79	39.41	38.74
	WF	30.82	31.13	31.54	34.81	32.27
90	**TT**	37.12	37.14	39.36	42.88	40.71
	WF	31.08	30.98	32.09	33.23	32.87

Table 5. Case C, zero coefficients included, 100 images.

Finally, we also present a comparison with the method in [6], though some observations are in order. Indeed, this procedure leads to a FSD histogram that depends on the input signal and is obtained by means of a random process, thus preventing us to perform a fair comparison. However, in order to compare the quality of the resulting image in a realistic forensic scenario, we applied both approaches to the first 20 DCT frequencies only (for the case A where only non-zero coefficients are considered in both methods), as state-of-the-art forensic detectors limit their analysis to these frequencies. The behavior of the average MSE (whose corresponding PSNR is reported in Table 6) varies together with the quality factors. This is due to the fact that, when a heavier quantization is performed, the two methods restore histograms that are not very close for frequencies 15-20, because [6] exploits a random signal-dependent process while we impose a conservative reconstruction. On the other hand, when quantization is lighter, the histograms almost coincide and our strategy leads to a better quality in the resulting image.

$\mathbf{QF_1}$	**50**	**60**	**70**	**80**	**90**
TT	44.58	46.00	47.71	50.46	54.50
Method in [6]	45.18	46.16	47.80	49.99	53.65

Table 6. Case A, nonzero coefficients, 738 images considered.

5. CONCLUSION

We have presented a method based on heuristic criteria which provides a close-to-optimal solution for the problem of FSD histogram modification with minimal distortion, expressed in a transportation-theoretic fashion as a two-step optimization process. Considering the promising results obtained in our experiments for the histogram reconstruction phase, we plan to test our attack against state-of-the-art forensic detectors facing different forensic problems. In addition, it would be of great interest to extend our approach to distortion measures different from the MSE (i.e., the PSNR), such as the SSIM or the WPSNR.

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