# PARTIAL CSI FEEDBACK DESIGN FOR INTERFERENCE ALIGNMENT IN MIMO CELLULAR NETWORKS

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**ABSTRACT**

Interference alignment (IA) can achieve the optimal capacity scaling with respect to SNR but most existing IA designs require full channel state information (CSI) at the transmitters. In this paper, we consider IA processing with partial CSI feedback in MIMO cellular networks and we use the *feedback dimension* to quantify the first order CSI feedback cost. Conventional IA cannot be used because only *partial CSI knowledge* can be used to design the IA precoders. Therefore, we establish a new set of feasibility conditions for IA under the proposed partial CSI feedback scheme. Based on these results, we formulate the problem of CSI feedback dimension minimization subject to the constraints of IA feasibility. We further propose an asymptotically optimal solution and derive closed-form trade-off results between the CSI feedback cost and IA performance in MIMO cellular networks.

***Index Terms*—** MIMO cellular networks, interference alignment (IA), partial CSI feedback

## 1. INTRODUCTION

Inter-cell interference is one of the most important performance bottlenecks in wireless networks. There are many works on interference mitigation techniques and most conventional approaches rely on channel orthogonalization [1, 2] to avoid the interference. However, these schemes are far from optimal [2]. Recently, interference alignment (IA) was proposed as an effective means to mitigate interference in $K$-user interference channels [3,4]. By aligning the interference from different Base stations (BS) into a lower dimensional subspace at each mobile station (MS), IA can achieve the optimal capacity scaling with respect to (w.r.t.) SNR. As such, there is a surge in the research interest of IA and it has been extended to other topologies such as MIMO cellular networks in [5, 6].

Despite the fact the IA can achieve substantial throughput gain, conventional IA designs [3–6] require full channel state information at the BS side (CSIT). Such full CSIT requirement is quite difficult to achieve in practice due to limited CSI feedback capacity in the reverse link. As such, naive IA design will be very sensitive to CSIT errors [7, 8] and it is important to take into account the CSI feedback constraint in the IA design. There are, in general, two ways to reduce the CSI feedback overhead, namely *CSI quantization* and *CSI filtering*. While CSI quantization is well-studied [7–10], the CSI filtering techniques to reduce feedback overhead are relatively less explored. In [11], a CSI filtering scheme by feeding back CSI *submatrices* is proposed to reduce the CSI feedback in MIMO interference network. In [12], a CSI filtering scheme with zero-forcing IA is proposed to eliminate the intercell CSI feedback in MIMO cellular networks. However, a more systematic understanding is still needed to determine how much CSI feedback is required for IA processing. In this paper, we propose a novel framework of CSI filtering in MIMO cellular networks, and we analyze the associated tradeoff between CSI feedback cost and the IA degrees of freedom (DoF) performance. To achieve these goals, we shall address the following challenges.

- **IA Feasibility Conditions under Partial CSI Feedback:** It is well known that the IA scheme is not always feasible and the feasibility conditions are topology specific. The IA feasibility condition is studied for MIMO interference channels in [13–16], and for MIMO cellular networks in [17]. However, these works have assumed full CSIT and hence the results cannot be used in our scenario in which only partial CSI is available.
- **CSI Feedback Minimization:** Further, it remains a question what is the CSI filtering scheme with the *least* amount of CSI feedback overhead to support IA. Such a question involves minimization of the CSI feedback cost subject to IA constraints. However, this problem is highly non-trivial because of the combinatorial nature of CSI filtering scheme design.

*Notation*s: Uppercase and lowercase boldface letters denote matrices and vectors respectively. The operators $(\cdot)^T$, $(\cdot)^\dagger$, rank$(\cdot)$, $|\cdot|$, $\lfloor\cdot\rfloor$ and $\lceil\cdot\rceil$ are the transpose, conjugate transpose, rank, cardinality, integer floor and integer ceiling respectively; $\mathbb{N}^r(\mathbf{A}) = \{\mathbf{u} \mid \mathbf{u}^\dagger\mathbf{A} = \mathbf{0}\}$ is the left null space of $\mathbf{A}$; $\mathbb{U}(A, B) = \left\{\mathbf{U} \in \mathbb{C}^{A\times B} : \mathbf{U}^\dagger\mathbf{U} = \mathbf{I}\right\}$ is the set of $A \times B$ ($A \geq B$) semi-unitary matrices; $\mathbb{P}(\mathbf{A}) = \{a\mathbf{A} : a \in \mathbb{C}\}$ and $d \mid M$ denotes that integer $M$ is divisible by integer $d$.

## 2. SYSTEM MODEL

### 2.1. MIMO Cellular Networks

Consider a MIMO cellular network with $G$ BSs and each BS serves $K$ MSs as illustrated in Figure 1. Each BS and MS are equipped with $N$ and $M$ antennas respectively, and $d$ data streams are transmitted to each MS from its serving BS. We focus on the case when $M \leq (G-1)Kd + d$ because otherwise, i.e., $M > (G-1)Kd + d$, the number of antennas at the MS is over-sufficient to cancel all the inter-cell interference using pure zero forcing. Denote the $k$-th MS of BS $j$ as the $(j, k)$-th MS, the channel matrix from the $i$-th BS to the $(j, k)$-th MS as $\mathbf{H}_{jk,i} \in \mathbb{C}^{M\times N}$.

**Assumption 1** (Channel Matrices). *Assume the elements of $\mathbf{H}_{jk,i}$ are i.i.d. complex Gaussian random variables with zero mean and unit variance. The CSIs are observable at the MSs and the CSI feedback from the $(j, k)$-th MS will be received error-free by BS $j$. Fur-*

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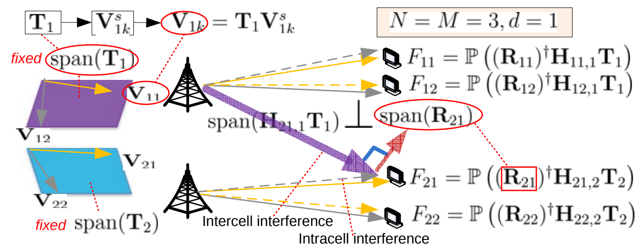


**Fig. 1**. Toy Example of *fixed* outer precoder at the BSs to reduce the CSI feedback dimension for IA.

*thermore, we assume the BSs $\{1, \cdots, G\}$ have backhaul connections such that the feedback CSI can be shared among them.*

### 2.2. Interference Alignment under Partial CSI Feedback

Denote the precoder and decorrelator for the $(j,k)$-th MS as $\mathbf{V}_{jk} \in \mathbb{C}^{N\times d}$, $\mathbf{U}_{jk} \in \mathbb{C}^{M\times d}$ respectively. To simplify the interference alignment structure in MIMO cellular networks, we consider using a *two-stage precoding* structure for $\{\mathbf{V}_{jk}\}$.

**Definition 1** (Two Stage Precoding at the BS)**.** Two stage precoding *is applied at each of the BSs $\{1, \cdots, G\}$, i.e., the precoder $\mathbf{V}_{jk} \in \mathbb{C}^{N\times d}$ is given by $\mathbf{V}_{jk} = \mathbf{T}_j\mathbf{V}^s_{jk}$, where the semi-unitary matrix $\mathbf{T}_j \in \mathbb{U}(N, Kd)$, $N \geq Kd$, is the* outer precoder *for intercell interference nulling and $\mathbf{V}^s_{jk} \in \mathbb{U}(Kd, d)$ is the* inner precoder *for intracell interference nulling between the MSs.*

With two stage precoding, IA is to find out the outer precoders $\{\mathbf{T}_i \in \mathbb{U}(N, Kd) : \forall i\}$, inner precoders $\{\mathbf{V}^s_{jk} \in \mathbb{C}^{Kd\times d} : \forall j, k\}$ and decorrelators $\{\mathbf{U}_{jk} : \forall j, k\}$ such that:

$$\mathrm{rank}(\mathbf{U}^{\dagger}_{jk}\mathbf{H}_{jk,j}\mathbf{T}_j\mathbf{V}^s_{jk}) = d, \forall j, k; \tag{1}$$

$$\mathbf{U}^{\dagger}_{jk}\mathbf{H}_{jk,j}\mathbf{T}_j\mathbf{V}^s_{jp} = \mathbf{0},\ \forall j, k \neq p; \qquad \text{(intracell IA)} \tag{2}$$

$$\mathbf{U}^{\dagger}_{jk}\mathbf{H}_{jk,i}\mathbf{T}_i = \mathbf{0}, \forall j, k, i \neq j. \qquad \text{(intercell IA)} \tag{3}$$

Denote the partial CSI feedback at the $(j,k)$-th MS as $F_{jk}$. Let $\mathbb{G}(A, B)$ be the Grassmann manifold of $A$ dimensional subspaces in $\mathbb{C}^{B\times 1}$. In the literature, there are some works [7–9] that feedback the full channel direction information (CDI)[1], i.e., $F_{jk} = \big(\ \cdots,\ \ \mathbb{P}(\mathbf{H}_{jk,i}),\ \ \cdots\ \big)_{\forall i}, \forall j, k$, which correspond to a CSI *feedback dimension*[2] of $G^2K(MN-1)$. However, we show below that it is possible to substantially reduce the CSI feedback cost for IA.

**Example 1** (Fixed Out Precoders)**.** *Consider a MIMO cellular network as illustrated in Figure 1. Suppose BS 1, 2 use* fixed *outer precoder $\mathbf{T}_1, \mathbf{T}_2 \in \mathbb{U}(3, 2)$. The intercell interference space at the (2,1)-th MS (i.e., span$(\mathbf{H}_{21,1}\mathbf{T}_1)$) can be canceled by choosing its decorrelatorar: $\mathbf{U}_{21} = \mathbf{R}_{21} \in \mathbb{U}(3, 1)$, where $(\mathbf{R}_{21})^{\dagger}\mathbf{H}_{21,1}\mathbf{T}_1 = \mathbf{0}$. Hence, BS 2 only needs to know $F_{21} = \mathbb{P}\left((\mathbf{R}_{21})^{\dagger}\mathbf{H}_{21,2}\mathbf{T}_2\right)$, $F_{22} = \mathbb{P}\left((\mathbf{R}_{22})^{\dagger}\mathbf{H}_{22,2}\mathbf{T}_2\right)$, where $(\mathbf{R}_{22})^{\dagger}\mathbf{H}_{22,2}\mathbf{T}_2\mathbf{V}^s_{21} = \mathbf{0}$, $(\mathbf{R}_{21})^{\dagger}\mathbf{H}_{21,2}\mathbf{T}_2\mathbf{V}^s_{22} = \mathbf{0}$, to compute the inner precoders (similarly for BS 1). Hence, IA (1)-(3) can be achieved with a feedback dimension of $4\times(2\times 1-1) = 4$ instead of $4\times 2\times(3\times 2-1) = 40$ in full CDI feedback.*

Note that the strategy described in Example 1 can be generalized to MIMO cellular networks with a *subset* of BSs to have *fixed* out precoders. Another CSI feedback reduction strategy is to directly reduce the *size* of the CSI matrices, i.e., feeding back CSI *submatrices* only as in [11]. By embracing both strategies of CSI feedback reduction, we give the structure form of the partial CSI feedback $F_{jk}$ in a step-by-step manner (Def. 2-4).

**Definition 2** (Partitioning of BSs)**.** *The BSs {1,...,G} are partitioned into the type-I BSs, $\mathbb{B}^I_g = \{1, \cdots, g\}$ and the type-II BSs, $\mathbb{B}^{II}_g = \{g+1, \cdots, G\}$. Type-II BSs have* fixed *outer precoder $\mathbf{T}^{II}_i \in \mathbb{U}(N, Kd)$, $i \in \mathbb{B}^{II}_g$.*

**Definition 3** (CSI Submatrix Feedback)**.** *Denote the CSI submatrices $\{\mathbf{H}^s_{jk,i}\}$ as*

$$\mathbf{H}^s_{jk,i} = \begin{cases} \begin{bmatrix} \mathbf{I}_{m_{jk}} & \mathbf{0} \end{bmatrix} \mathbf{H}_{jk,i} \begin{bmatrix} \mathbf{I}_{n_i} & \mathbf{0} \end{bmatrix}^T, & \forall j, k, i \in \mathbb{B}^I_g \\ \begin{bmatrix} \mathbf{I}_{m_{jk}} & \mathbf{0} \end{bmatrix} \mathbf{H}_{jk,i}, & \forall j, k, i \in \mathbb{B}^{II}_g \end{cases}. \tag{4}$$

Note that $g = |\mathbb{B}^I_g|$ in Def. 2 is the number of the type-I BS and $m_{jk}$ and $n_i$ in Def. 3 control the size of the CSI submatrices to feedback. Based on this, we formally have the following forms of partial CSI feedback.

**Definition 4** (Partial CSI feedback $F_{jk}$)**.** *The partial CSI feedback $F_{jk}$ at the $(j,k)$-th MS is a $l_{jk}$ tuple and is given by*

$$\begin{aligned} F_{jk} &= \left(\cdots, \mathbb{P}\left(\mathbf{H}^e_{jk,i}\right), \cdots\right)_{i\in\mathbb{B}^I_g \bigcup\{j\}} \\ &\in \mathbb{G}\left(1, B_{jk,1}\right) \times \cdots \mathbb{G}\left(1, B_{jk,l_{jk}}\right), \forall j, k; \end{aligned} \tag{5}$$

*where $l_{jk} = |\mathbb{B}^I_g \bigcup\{j\}|$ is the number of projective spaces in $F_{jk}$, $\mathbb{G}\left(1, B_{jk,i}\right)$, $i \leq l_{jk}$ is the $i$-th Grassmann manifold that contains the $i$-th projective space of $F_{jk}$, $\mathbf{H}^e_{jk,i}$ denotes the effective CSI and is given by*

$$\mathbf{H}^e_{jk,i} = \begin{cases} (\mathbf{R}_{jk})^{\dagger}\mathbf{H}^s_{jk,i} \in \mathbb{C}^{A_{jk}\times n_i}, & \forall j, k, i \in \mathbb{B}^I_g \\ (\mathbf{R}_{jk})^{\dagger}\mathbf{H}^s_{jk,j}\mathbf{T}^{II}_j \in \mathbb{C}^{A_{jk}\times Kd}, & \forall k, i = j \in \mathbb{B}^{II}_g \end{cases} \tag{6}$$

*$\mathbf{R}_{jk} \in \mathbb{U}(m_{jk}, A_{jk})$ defines*[3] *the left null space of the intercell interference from all type-II BSs at the $(j,k)$-th MS:*

$$\mathit{span}(\mathbf{R}_{jk}) = \mathbb{N}^r\left(\begin{bmatrix} \cdots & \mathbf{H}^s_{jk,i}\mathbf{T}^{II}_i & \cdots \end{bmatrix}_{i\in\mathbb{B}^{II}_g\setminus\{j\}}\right), \forall j, k; \tag{7}$$

*and $A_{jk} = m_{jk} - \left|\mathbb{B}^{II}_g\setminus\{j\}\right| Kd$, $\forall j, k$, $B_{jk,i} = KdA_{jk}$ when $i = l_{jk}$, $j \in \mathbb{B}^{II}_g$, $B_{jk,i} = n_iA_{jk}$ otherwise.*

Note that there is no need to feedback the intercell CSIs $\left\{\mathbf{H}^s_{jk,i} : \forall j, k, i \in \mathbb{B}^{II}_g\setminus\{j\}\right\}$ (as in (5)) because the intercell interference from type-II BSs can be canceled by designing the decorrelator $\mathbf{U}_{jk}$ in the subspace spanned by $\mathbf{R}_{jk}$. From Def. 4, the partial CSI feedback $\{F_{jk}\}$ is parameterized by $\mathcal{L} = \left\{\{m_{jk} : \forall j, k\}, g, \{n_i : \forall i \in \mathbb{B}^I_g\}\right\}$. The associated CSI feedback cost for a given feedback profile $\mathcal{L}$ is defined in the following.

[1] For example, in IA designs, if $\mathbf{U}^{\dagger}\mathbf{H}\mathbf{V} = \mathbf{0}$ then $\mathbf{U}^{\dagger}(a\mathbf{H})\mathbf{V} = 0$, $\forall a \in \mathbb{C}$. Hence, it is sufficient to feeding back the CDI for IA, i.e., $\mathbb{P}(\mathbf{H}) = \{a\mathbf{H} : a \in \mathbb{C}\}$, which is contained in $\mathbb{G}(1, MN)$ [18].

[2] The feedback dimension equals the sum dimension of the feedback projective spaces. We formally define this notion in Def. 5.

[3] We define $\mathbf{R}_{jk} = \mathbf{I}$ when $\mathbb{B}^{II}_g\setminus\{j\} = \emptyset$.

**Definition 5** (CSI Feedback Dimension)**.** *Define the feedback dimension $D$ as the sum of the dimension of the Grassmann manifolds [18] $\{\mathbb{G}(1, B_{jk,i}) : \forall j, k, i\}$, i.e.,*

$$D(\mathcal{L}) = \sum_{j=1}^{G}\sum_{k=1}^{K}\sum_{i=1}^{l_{jk}}(B_{jk,i} - 1). \tag{8}$$

Note that the feedback dimension in Def. 5 is a first order measure of CSI feedback cost because it is directly proportional to the total number of CSI feedback bits. For instance, suppose we have $B$ bits to feedback a CSI contained in a Grassmann manifold with dimension $D$. If we want to keep a constant CSI distortion $\Delta$, the CSI feedback bits $B$ should scale linearly with $D$ as $B = \mathcal{O}(D \log \frac{1}{\Delta})$ [18, 19]. Next, we discuss the IA constraints under the proposed partial CSI feedback scheme.

**Constraints 1** (IA under $\mathcal{L}$)**.** *Given the CSI feedback profile $\mathcal{L}$ and the outer precoders $\{\mathbf{T}_i^{II} \in \mathbb{U}(N, Kd) : i \in \mathbb{B}_g^{II}\}$ for the type-II BSs, find the outer precoders $\{\mathbf{T}_i^{I} \in \mathbb{U}(N, Kd) : i \in \mathbb{B}_g^{I}\}$ for type-I BSs, the inner precoders $\{\mathbf{V}_{jk}^{s} \in \mathbb{U}(Kd, d) : \forall j, k\}$ for all BSs and decorrelators $\{\mathbf{U}_{jk}\}$ for all MSs, to satisfy the following conditions:*

$$\textit{rank}(\mathbf{U}_{jk}^{\dagger}\mathbf{H}_{jk,j}\mathbf{T}_j\mathbf{V}_{jk}^{s}) = d, \forall j, k; \tag{9}$$

$$\mathbf{U}_{jk}^{\dagger}\mathbf{H}_{jk,j}\mathbf{T}_j\mathbf{V}_{jp}^{s} = \mathbf{0}, \forall j, k \neq p; \quad \textit{(intracell IA)} \tag{10}$$

$$\mathbf{U}_{jk}^{\dagger}\mathbf{H}_{jk,i}\mathbf{T}_i = \mathbf{0}, \forall j, k, i \neq j; \quad \textit{(intercell IA)} \tag{11}$$

$$\left\{\mathbf{T}_j^{I} : i \in \mathbb{B}_g^{I}\right\}, \{\mathbf{V}_{jk}^{s} : \forall j, k\} \textit{ can only be adaptive to } \{F_{jk} : \forall j, k\} \textit{ according to } \mathcal{L}. \tag{12}$$

*where $\mathbf{T}_j = \mathbf{T}_j^{I}$, $j \in \mathbb{B}_g^{I}$ and $\mathbf{T}_j = \mathbf{T}_j^{II}$, $j \in \mathbb{B}_g^{II}$.*

Note that (9)-(11) refers to the *IA constraints* and (12) refers to the *CSI knowledge constraint*. Compared with conventional IA with full CSIT (1)-(3), there is one unique challenge, namely the *CSI knowledge* associated with Constraints 1 under partial CSIT knowledge. Adjusting the feedback profile $\mathcal{L}$ may reduce the CSI feedback dimension $D(\mathcal{L})$ in (8) but the IA constraints may no longer be feasible. To reduce the CSI feedback, we formulate below the problem of CSI feedback dimension minimization subject to the IA constraint in MIMO cellular networks.

**Problem 1** (Feedback Dimension Minimization)**.**

$$\min_{\mathcal{L}} \quad D(\mathcal{L}) \tag{13}$$

$$\textit{s.t.} \quad 0 \leq g \leq G; \tag{14}$$

$$n_i \leq N, \forall i \in \mathbb{B}_g^{I}; m_{jk} \leq M, \forall j, k; \tag{15}$$

$$\textit{Constraints 1 is feasible under } \mathcal{L}. \tag{16}$$

However, Problem 1 is very difficult due to the *implicit* constraint (16) on $\mathcal{L}$ and the combinatorial nature of the optimization variable ($\mathcal{L}$).

## 3. IA FEASIBILITY CONDITIONS UNDER $\mathcal{L}$

In this section, we study constraint (16) and specify its necessary and sufficient conditions in Thm. 1, 2 respectively. We first introduce an equivalent IA constraint transformation to explicitly handle the *CSI knowledge constraint* in (12).

**Constraints 2** (IA Constraint Transformation under $\mathcal{L}$)**.** *Find $\tilde{\mathbf{T}}_i^{I} \in \mathbb{U}(n_i, Kd)$, $n_i \geq Kd$, $i \in \mathbb{B}_g^{I}$, and $\tilde{\mathbf{U}}_{jk} \in \mathbb{U}(A_{jk}, d)$, $A_{jk} \geq d$, $\forall j, k$, to satisfy the following equations:*

$$(\tilde{\mathbf{U}}_{jk})^{\dagger}\mathbf{H}_{jk,i}^{e}\tilde{\mathbf{T}}_i^{I} = \mathbf{0}, \forall j, k, i \in \mathbb{B}_g^{I}\backslash\{j\}. \tag{17}$$

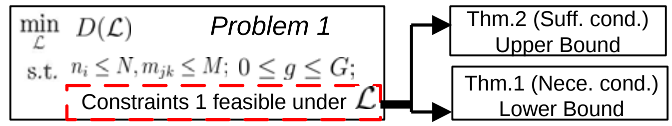


**Fig. 2**. Relationship between Problem 1 and Thm. 1, 2.

The equivalent relationship between Constraints 1 and Constraints 2 is established in the lemma below.

**Lemma 1** (Equivalence of Constraints 1 and 2)**.** *Given the CSI feedback profile $\mathcal{L}$ and the outer precoders $\{\mathbf{T}_i^{II} \in \mathbb{U}(N, Kd) : i \in \mathbb{B}_g^{II}\}$ for type-II BSs, Constraints 2 is feasible iff Constraints 1 is feasible.*

*Proof.* Please refer to [20]. □

Note that IA Constraints 2 contain the intercell IA constraints from type-I BSs only as in (17). Consequently, the aforementioned *CSI knowledge constraint* (12) is automatically satisfied by using Constraint 2 and Lemma 1. Based on Lemma 1 and Constraints 2, we obtain the following necessary feasibility conditions for Constraint 1.

**Theorem 1** (Necessary Conditions for IA Feasible under $\mathcal{L}$)**.** *Under a given feedback profile $\mathcal{L}$, Constraints 1 is feasible only if: 1) $m_{jk} - \sum_{i \in \mathbb{B}_g^{II}\backslash\{j\}} Kd - d \geq 0, \forall j, k$, 2) $N \geq Kd$, $n_i \geq Kd$, $i \in \mathbb{B}_g^{I}$, 3) $\forall \mathcal{J}_{sub}^{[r]} \subseteq \{(j,k) : \forall j, k\}, \mathcal{J}_{sub}^{[t]} \subseteq \mathbb{B}_g^{I}$,*

$$\sum_{(j,k)\in\mathcal{J}_{sub}^{[r]}} \left(m_{jk} - \left|\mathbb{B}_g^{II}\backslash\{j\}\right| Kd - d\right) + \sum_{i\in\mathcal{J}_{sub}^{[t]}} K \times (n_i - Kd) \geq \sum_{j\in\mathcal{J}_{sub}^{[r]}} \sum_{i\in\mathcal{J}_{sub}^{[t]}\backslash\{j\}} Kd. \tag{18}$$

*Proof.* Please refer to [20]. □

For instance, if we have 0 type-I BS ($g = 0$) and $m_{jk} = m$, $\forall j, k$, in $\mathcal{L}$, then Theorem 1 requires that $N \geq Kd$, $m \geq (G-1)Kd + d$ must be satisfied for $\mathcal{L}$ to be IA feasible (see Example 1); if we have 0 type-II BS ($g = G$) and $m_{jk} = m, \forall j, k$, $n_i = n, \forall i$, in $\mathcal{L}$, then Theorem 1 requires that $m \geq d$, $n \geq Kd$ and $m + n \geq (GK+1)d$ must be satisfied for $\mathcal{L}$ to be IA feasible (see Example 2). We also have that the conditions in Theorem 1 are sufficient in the divisible cases.

**Theorem 2** (Necessary Conditions for IA Feasible)**.** *When $d \mid n_i$, $\forall i \in \mathbb{B}_g^{I}$, or $Kd \mid (m_{jk} - d)$, $\forall j, k$, the conditions in Thm. 1 are also sufficient for Constraints 1 to be feasible.*

*Proof.* Please refer to [20]. □

**Remark 1** (Backward Compatibility with Previous Results)**.** *When $g = G$, $K = 1$, $m_{jk} = M$, $n_i = N$, Thm. 2 agrees with the previous results of Corollary 3.4 in [16]. When $g = G$, $m_{jk} = M$, $n_i = N$ and both $d \mid n_i$, $d \mid m_{jk}$, $\forall j, k, i$, then Thm. 2 agrees with the result of Thm. 2 in [17].*

**Algorithm 1** Antenna Reduction in $\mathcal{L}_0$.

- **Step 1**: Construct the max flow graph $\mathcal{N} = (\mathcal{V}, \mathcal{E})$ [21]:

1. The vertices are given by $\mathcal{V} = \{a, b, u_{jk}, v_i, c_{ji,k}\}, \forall j, k, i \in \mathbb{B}_{g_0}^{I}$, where $a$, $b$ are the source, destination node respectively and $u_{jk}, v_i, c_{ji,k}$ are the intermediate nodes in $\mathcal{N}$.
2. The edges are given by $\mathcal{E} = \{(a, u_{jk}), (a, v_i), (u_{jk}, c_{jk,i}), (v_i, c_{jk,i}), (c_{jk,i}, b) : \forall j, k, i \in \mathbb{B}_{g_0}^{I}\}$, where $(u, v)$ denotes the edge from node $u$ to node $v$.
3. The edge capacities are given by $c(a, u_{jk}) = c(u_{jk}, c_{jk,i}) = (m_{jk} - \sum_{i \in \mathbb{B}_{g_0}^{II} \setminus \{j\}} Kd - d)$, $c(a, v_i) = c(v_i, c_{jk,i}) = K(n_i - Kd)$, $c(c_{jk,i}, t) = Kd, \forall j, k, i \in \mathbb{B}_{g_0}^{I}$, where $c(u, v)$ denotes the edge capacity on the edge $(u, v)$.

- **Step 2**: Find the max flow solutions $\{f(a, b) : (a, b) \in \mathcal{E}\}$ [21] for $\mathcal{N}$ and perform antenna reduction as

$$n_i = N_0 - \left\lfloor \frac{c(a, v_i) - f(a, v_i)}{Kd} \right\rfloor d, \; i \in \mathbb{B}_{g_0}^{I};$$

$$m_{jk} = M - \lfloor c(a, u_{jk}) - f(a, u_{jk}) \rfloor, \forall j, k.$$

## 4. ASYMPTOTICALLY OPTIMAL SOLUTION

In this section, we solve Problem 1 by replacing the constraint (16) with the feasibility conditions specified in Thm. 1 and 2. Figure 2 summarizes the relationship between Problem 1 and Thm. 1, 2.

We first design an achievable feasible feedback profile solution $\mathcal{L}_0$ that satisfies the sufficient conditions in Thm. 2. While the solution is a suboptimal upper bound of the minimum feedback dimension $D(\mathcal{L}^*)$, we later show later that it is asymptotically optimal as $G \to \infty$. Denote $N_0 = \min\left(GKd, \left\lfloor \frac{N}{d} \right\rfloor d\right)$. The designed solution $\mathcal{L}_0$ is obtained by first aggressively selecting the *largest* number of *type-II* BSs, and then further performing antenna reduction. Specifically, $\mathcal{L}_0 = \left\{\{m_{jk} : \forall j, k\}, g_0, \{n_i : i \in \mathbb{B}_{g_0}^{I}\}\right\}$ where $g_0 = \left\lceil \frac{G((G-1)Kd - M + d)}{N_0 - Kd} \right\rceil$, and $m_{jk}$, $n_i$ are obtained from Alg. 1.

By using the necessary conditions in Thm. 1, 2, we have the following characterizations on $D(\mathcal{L}_0)$. Denote $N_1 = \min(GKd, N)$, $g_1 = \left\lfloor \frac{G((G-1)Kd - M + d)}{N_1 - Kd} \right\rfloor$.

**Theorem 3** (Characterizations on $D(\mathcal{L}_0)$). *1) The feedback profile $\mathcal{L}_0$ is a feasible solution of Problem 1; 2) $D(\mathcal{L}_0)$ and the optimal feedback dimension $D(\mathcal{L}^*)$ are bounded by*

$$D(\mathcal{L}_0) \geq D(\mathcal{L}^*) \geq D_{low} \triangleq KGN_1 g_1 (M - (G - g_1)Kd) - KG^2; \tag{19}$$

*3) If $N = aGKd$, $M = bGKd$, $0 < a, b < 1$, $1 < a + b$, then*

$$\lim_{G \to \infty} \frac{D(\mathcal{L}^*)}{G^4 K^3 d^2} = \lim_{G \to \infty} \frac{D(\mathcal{L}_0)}{G^4 K^3 d^2} = \frac{(1 - a)(1 - b)^2}{a}. \tag{20}$$

*Proof.* Please refer to [20]. □

From (20), the proposed solution $\mathcal{L}_0$ is an asymptotically optimal solution of Problem 1.

## 5. NUMERICAL RESULTS

In this section, we verify the performance of the proposed feedback scheme in a $G = 3$, $K = 2$, $N = M = 4$, $d = 1$ MIMO cellular networks. Two baselines are considered, namely baseline 1 of full CDI feedback as in [7–9] and baseline 2 of CSI submatrix feedback as in [11].

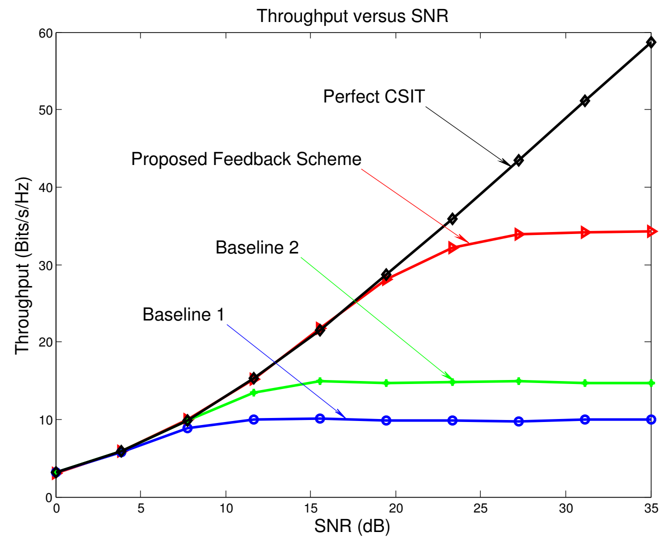


**Fig. 3**. Throughput versus transmit SNR under $B_{tot} = 800$ in a $G = 3$, $K = 2$, $N = M = 4$, $d = 1$ network.

We obtain $\mathcal{L} = \{\{m_{1k} = m_{2k} = 4, m_{3k} = 3 : k = 1, 2\}, g = 2, \{n_1 = n_2 = 4\}\}$ for the proposed scheme and the sum feedback dimension for the proposed scheme, baseline 1 and baseline 2 are 110, 198, and 270 respectively. Figure 3 illustrates the network throughput versus the transmit SNR $P$ under a sum feedback bits of $B_{tot} = 800$. The proposed scheme achieves substantial throughput gain over the baselines. This is because the proposed scheme significantly reduces the CSI feedback dimension while preserving the IA feasibility. Under the same number of feedback bits, more CSI feedback bits can be utilized to reduce the quantization error per dimension and hence, the proposed scheme can achieve less residual interference power from CSI errors. The dramatic performance gain highlights the importance of reducing the feedback dimension in MIMO cellular networks.

## 6. CONCLUSIONS

In this paper, we consider IA processing with partial CSI feedback in MIMO cellular networks. We characterize the feedback cost by the feedback dimension and establish a new set of IA feasibility conditions under the proposed partial CSI feedback scheme. Based on these results, we formulate the problem of feedback dimension minimization subject to IA constraints and propose an asymptotic optimal solution. Simulation results show that the proposed scheme can significantly reduce the CSI feedback cost of IA in MIMO cellular networks.

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