

ROBUST OPTIMIZATION FOR MULTI-CELL INTERFERING MIMO-MAC UNDER LIMITED FEEDBACK

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ABSTRACT

We consider a multi-cell interfering MIMO-MAC system, where each user transmits a single data stream to its desired base station (BS). First, we study the multiplexing gain of the system using two interference cancellation (IC) schemes: the coordinated zero-forcing receiver (CZFR) and the extended grouping method based IA (EGM-IA) with the perfect channel state information at transmitters (CSIT). Then, we propose two algorithms to find the IC transceivers maximizing the rate of each user. Under limited feedback, we additionally propose an algorithm to maximize the minimum worst-case rate of the users by adaptively allocating the feedback bits. Numerical results illustrate the performance of the proposed algorithms.

Index Terms— Interference Alignment, interfering MIMO-MAC, limited feedback bits allocation

1. INTRODUCTION

Interference mitigation has become an important issue of wireless network design. An interference alignment (IA) technique proposed in [1] is to align the interference signals into a lower dimensional subspace at each user so that an orthogonal subspace can be allocated for interference-free transmission.

Focusing on the multiple-input multiple-output (MIMO) system, IA has been studied in several network configurations including interference channels [1, 2, 3, 4] and interfering broadcast channel [5, 6, 7]. Most of the previous works on IA are based on the assumption of perfect channel state information at the transmitter (CSIT) for all users. However, this assumption is unrealistic in practice. Therefore, we assume perfect channel state information at the receiver (CSIR) and unknown CSIT. Each transmitter must require the CSIT to optimize/generate precoding strategies in order to control the performance dynamically, while the resulting rate performance depends heavily on the quality of CSIT via feedback. However, *explicit feedback* of the MIMO channels requires very much overhead such that it is challenging in practice especially for frequency-division duplex (FDD) system. In these systems, the feedback link is usually a rate-limited channel. Therefore, robust IC transceiver design by optimizing limited feedback bits allocation in the practical *limited feedback* scenario should be considered. Several performance metrics for feedback bits allocation have been studied. For instance, a lower bound and an upper bound of the average rate loss for two-cell interfering MIMO-MAC [10] and for multi-user MIMO IC in [8] are minimized; In [11], a lower bound of the sum of average residual interference for the multi-user interference channels is minimized. The sum degrees of freedom (DoF) of interference

channels with respect to the allocation of feedback bits is analyzed in [12], [13].

In this paper, we consider a multi-cell interfering MIMO multiple access channel (MIMO-MAC) under limited feedback, which is well-matched with the multi-cell multi-user uplink scenario. Our main contributions are listed as follows:

1) With the perfect CSIT, we analyze the relation between transmit/receive antennas and the multiplexing gain for the multi-cell interfering MIMO-MAC using two schemes: the coordinated zero-forcing receiver (CZFR) and the extended grouping method based IA (EGM-IA, extended from the two-cell case in [6]), respectively; Different from the traditional IA design, we propose two algorithms, which simultaneously maximize the rate of each user;

2) Under limited feedback, we propose an algorithm to *adaptively* allocate the finite feedback bits to quantize each transmit beamforming vector, which aims at maximizing the minimum worst-case rate of the users to guarantee the fairness. Finally, numerical simulations show the performance of the proposed algorithms with and without CSIT.

2. SYSTEM MODEL

Consider a K -cell interfering MIMO-MAC, where each BS k with N_{R_k} antennas serves L_k users in its own cell. To simplify the notation, we define the set of cells as $\mathcal{K} = \{1, 2, \dots, K\}$, and the indexes of the L_k users in cell k are set as $\mathcal{L}_k = \{\sum_{\ell=1}^{k-1} L_\ell + 1, \sum_{\ell=1}^{k-1} L_\ell + 2, \dots, \sum_{\ell=1}^k L_\ell\}$, and all the users in K cells form the set $\mathcal{L} = \{1, 2, \dots, L\}$ where $L = \sum_{\ell=1}^K L_\ell$. In cell $k \in \mathcal{K}$, each user $i \in \mathcal{L}_k$ with N_i antennas desires to send a single data symbol s_i with $\mathbb{E}[s_i s_i^*] = 1$ to BS k via a linear precoding vector $\mathbf{v}_i \in \mathbb{C}^{N_i \times 1}$. Each user i is assumed to have a transmit power constraint. Without loss of generality (w.l.o.g.), we set $\mathbf{v}_i^H \mathbf{v}_i \leq 1$.

We assume that the CSIR is perfectly estimated at BS side. The received signal at BS $k \in \mathcal{K}$ is given by

$$\mathbf{y}_k = \underbrace{\mathbf{H}_{k,i} \mathbf{v}_i s_i}_{\text{desired signal}} + \underbrace{\sum_{j \in \mathcal{L} \setminus \{i\}} \mathbf{H}_{k,j} \mathbf{v}_j s_j}_{\text{interference}} + \mathbf{n}_k, \quad (1)$$

where $\mathbf{n}_k \in \mathbb{C}^{N_{R_k} \times 1}$ is the additive white Gaussian noise vector with zero mean and variance $\sigma_k^2 \mathbf{I}_{N_{R_k}}$, and $\mathbf{H}_{k,j} \in \mathbb{C}^{N_{R_k} \times N_j}$ is the channel matrix of the link from user j to BS k . Each channel is assumed to satisfy Rayleigh fading channel and be quasi-static and frequency flat fading.

Each BS k *independently* decodes each data stream coming from different users in its own cell. By the receive beamforming vector $\mathbf{u}_i \in \mathbb{C}^{N_{R_k} \times 1}$ for user i , s_i is decoded as $\hat{s}_i = \mathbf{u}_i^H \mathbf{y}_{k,i}$. Under the assumption of *single-stream decoding* and treating the interference

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as noise, the achievable rate of the link of user i to BS k is given by

$$R_i = \log_2 \left(1 + \frac{|\mathbf{u}_i^H \mathbf{H}_{k,i} \mathbf{v}_i|^2}{\sigma_k^2 \|\mathbf{u}_i\|^2 + \sum_{j \in \mathcal{L} \setminus \{i\}} |\mathbf{u}_i^H \mathbf{H}_{k,j} \mathbf{v}_j|^2} \right). \quad (2)$$

Observe R_i is independent of $\|\mathbf{u}_i\|^2$. W.l.o.g., we set $\|\mathbf{u}_i\|^2 = 1$.

3. IC TRANSCEIVER DESIGN WITH PERFECT CSI

We propose two algorithms to compute the IC transceiver with perfect CSI, which *not only mitigates all the interference but also maximizes the rate of each user*. This is different from the traditional IC transceiver design. To achieve the IC with perfect CSI, the following conditions should be satisfied:

$$|\mathbf{u}_i^H \mathbf{H}_{k,i} \mathbf{v}_i| > 0, \quad \forall k \in \mathcal{K} \quad (3a)$$

$$\mathbf{u}_i^H \mathbf{H}_{k,j} \mathbf{v}_j = 0, \quad \forall j \in \mathcal{L} \setminus \{i\}. \quad (3b)$$

According to the number of transmit/receive antennas and multiplexing gain, two IC algorithms are studied as follows.

3.1. Coordinated Zero-Forcing Receiver (CZFR)

In the cellular networks, each BS usually has much more antennas than each user, i.e., $N_{R_k} > N_i$. At BS $k \in \mathcal{K}$, the receive beamforming vector \mathbf{u}_i for user i should be designed to mitigate the interference signals from all the other users $m \in \mathcal{L} \setminus \{i\}$, i.e.,

$$\mathbf{u}_i^H \mathbf{H}_{-i} = \mathbf{0}_{1 \times (L-1)}. \quad (4)$$

$\mathbf{H}_{-i} = [\mathbf{H}_{k,1} \mathbf{v}_1 \cdots \mathbf{H}_{k,i-1} \mathbf{v}_{i-1} \mathbf{H}_{k,i+1} \mathbf{v}_{i+1} \cdots \mathbf{H}_{k,L} \mathbf{v}_L]$ with the size of $N_{R_k} \times (L-1)$ denotes the interference signal space of user i to BS k .

Sufficient condition for CZFR¹: A sufficient condition for \mathbf{u}_i satisfying (4) is $N_{R_k} \geq L$. This leads to a at least $L-1$ -dimensional null space to perfectly eliminate $L-1$ interference signals and simultaneously achieve sum multiplexing gain of L .

Remark 1 *The CZFR only employs the spatial DoF of receivers to eliminate the interferences. In fact, it rarely happens that all the other users $\mathcal{L} \setminus \{i\}$ in K cells, especially for the inter-cell users, strongly interfere the BS k . Additionally, appropriate multi-cell multi-user scheduling methods can also decrease the number of interference signals such that the demanding requirement of the BS antennas, i.e., $N_{R_k} \geq L$, can be relaxed.* \square

Given the transmit beamforming vectors, any feasible \mathbf{u}_i to (4) can be formulated as

$$\mathbf{u}_i = \mathcal{N}(\mathbf{H}_{-i}) \mathbf{w}_i \quad (5)$$

where $\mathcal{N}(\mathbf{H}_{-i})$ denotes the null space of \mathbf{H}_{-i} , which is defined as the space spanned by the eigenvectors corresponding to *all zero-valued* eigenvalues of the matrix $\mathbf{H}_{-i} \mathbf{H}_{-i}^H$. \mathbf{w}_i is a normalized weighted vector.

Here, we compute the optimal \mathbf{u}_i satisfying (4) while maximizing its own rate R_i . Substituting (5) into (2), we formulate the *single-*

user's rate maximization (SURM) problem by the IC transceiver as

$$\max_{\|\mathbf{w}_i\|=1, \|\mathbf{v}_i\|^2 \leq 1} \log_2 \left(1 + \frac{|\mathbf{w}_i^H \mathcal{N}^H(\mathbf{H}_{-i}) \mathbf{H}_{k,i} \mathbf{v}_i|^2}{\sigma_k^2 \|\mathbf{w}_i^H \mathcal{N}^H(\mathbf{H}_{-i})\|^2} \right) \quad (6a)$$

$$\implies \mathbf{w}_i^* = \bar{\mathbf{u}}_1^{svd}(\mathcal{N}^H(\mathbf{H}_{-i}) \mathbf{H}_{k,i}) \quad (6b)$$

$$\mathbf{v}_i^* = \bar{\mathbf{v}}_1^{svd}(\mathcal{N}^H(\mathbf{H}_{-i}) \mathbf{H}_{k,i}) \quad (6c)$$

$$\mathbf{u}_i^* = \mathcal{N}(\mathbf{H}_{-i}) \mathbf{w}_i^* \quad (6d)$$

where $(\bar{\mathbf{u}}_1^{svd}(\mathbf{X}), \bar{\mathbf{v}}_1^{svd}(\mathbf{X}))$ denotes the *principle* right and left singular vector pair of matrix \mathbf{X} by the singular value decomposition (SVD). The strategies of user i in (6b)-(6d) depend on that of other users. Therefore, the SURM problem (6a) is *optimally* solved after iterations. This Algorithm can be included in following Algorithm 1 (for the users in single-user groups).

3.2. Extended Grouping Method based IA (EGM-IA)

When a certain BS k has not sufficient antennas to nullify all the interference signals, i.e., $N_{R_k} < L$, the CZFR design may be infeasible. In this case, if some/all users have large number of transmit antennas (the condition will be shown in the following proposition), we extend the grouping method based IA in [6] *to arrange some or all users in a cell into one or multiple groups*. By jointly optimizing transmit beamforming vectors, multiple interference signals from a group to a undesired BS are aligned in a particular space such that the interference dimension is reduced. Then, the BS employs ZF receive beamforming to eliminate all the interference signals with less antennas than the CZFR.

Assume that all the users in K cells are divided into G groups. If a group contains only one user, it is called as *single-user group*. Otherwise, we call it *multi-user group*. Consider a multi-user group $g \in \mathcal{G} \triangleq \{1, 2, \dots, G\}$ contains $M_g > 1$ users forming the set $\mathcal{M}_g \triangleq \{g_1, \dots, g_{M_g}\}$, and we set the user g_1 as the *group leader*². Let \mathcal{G}_{su} , \mathcal{G}_{mu} and \mathcal{G}_{lead} denote the set of users in single-user groups, the set of users in multi-user groups and the set of all group leaders, respectively, where $\mathcal{G}_{su} \cup \mathcal{G}_{mu} = \mathcal{L}$. We define \mathcal{G}_k as a set of groups in cells $\mathcal{K} \setminus \{k\}$ feasible to align interference to BS k into one dimension. We have the following results.

Proposition 1 1) *For a multi-user group $g \in \mathcal{G}_k$, it is sufficient to help BS k to reduce $M_g - 1$ antennas compared with the CZFR if $\sum_{m \in \mathcal{M}_g} N_m \geq (M_g - 1)N_{R_k} + 1$ and $N_{R_k} \geq L - (M_g - 1)$;*

2) *For the set of groups \mathcal{G}_k , it is sufficient to help BS k to reduce $\sum_{g \in \mathcal{G}_k} (M_g - 1)$ antennas compared with the CZFR if $\sum_{m \in \mathcal{M}_g} N_m \geq (M_g - 1)N_{R_k} + 1$, $\forall g \in \mathcal{G}_k$ and $N_{R_k} \geq L - \sum_{g \in \mathcal{G}_k} (M_g - 1)$.*

It is sufficient for the EGM-IA to save $\sum_{k \in \mathcal{K}} \sum_{g \in \mathcal{G}_k} (M_g - 1)$ receive antennas at BSs in total and simultaneously achieve multiplexing gain of L if $\sum_{m \in \mathcal{M}_g} N_m \geq (M_g - 1)N_{R_k} + 1$, $\forall g \in \mathcal{G}_k$, $\forall k \in \mathcal{K}$ and $N_{R_k} \geq L - \sum_{g \in \mathcal{G}_k} (M_g - 1)$, $\forall k \in \mathcal{K}$. \square

Proof: Take a multi-user group $g \in \mathcal{G}_k$ for instance. Group g aligns M_g -dimensional interference to BS k into one dimension, i.e.,

$$\mathbf{h}_{k,g} \triangleq \text{span} \{ \mathbf{H}_{k,g_m} \mathbf{v}_{g_m} \}, \quad m = \{1, \dots, M_g\} \quad (7)$$

¹If the interference matrix \mathbf{H}_{-i} in (4) has full rank, i.e., $\text{rank}(\mathbf{H}_{-i}) = \min(N_{R_k}, L-1)$, this sufficient condition is also the necessary condition.

²The group leader can be selected as the one with maximum antennas in each group.

which is necessarily achieved by solving the following equation

$$\mathbf{A}_{k,g} \tilde{\mathbf{v}}_{k,g} = \mathbf{0}_{M_g N_{R_k} \times 1} \quad (8)$$

where $\mathbf{A}_{k,g}$ is defined as

$$\begin{pmatrix} \mathbf{I}_{N_{R_k}} & \mathbf{H}_{k,g_1} & \mathbf{0}_{N_{R_k} \times N_{g_2}} & \cdots & \mathbf{0}_{N_{R_k} \times N_{g_{M_g}}} \\ \mathbf{I}_{N_{R_k}} & \mathbf{0}_{N_{R_k} \times N_{g_1}} & \mathbf{H}_{k,g_2} & \cdots & \mathbf{0}_{N_{R_k} \times N_{g_{M_g}}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{I}_{N_{R_k}} & \mathbf{0}_{N_{R_k} \times N_{g_1}} & \mathbf{0}_{N_{R_k} \times N_{g_2}} & \cdots & \mathbf{H}_{k,g_{M_g}} \end{pmatrix}$$

with the size of $M_g N_{R_k} \times (N_{R_k} + \sum_{m \in \mathcal{M}_g} N_m)$ and $\tilde{\mathbf{v}}_{k,g} \triangleq [\mathbf{h}_{k,g}; \mathbf{v}_{g_1}; \mathbf{v}_{g_2}; \cdots; \mathbf{v}_{g_{M_g}}]_{(N_{R_k} + \sum_{m \in \mathcal{M}_g} N_m) \times 1}$.

A sufficient condition for the existence of $\tilde{\mathbf{v}}_{k,g}$ satisfying (8) is $(N_{R_k} + \sum_{m \in \mathcal{M}_g} N_m) \geq M_g N_{R_k} + 1$ such that $\mathbf{A}_{k,g}$ has a null space for $\tilde{\mathbf{v}}_{k,g}$. If all the groups in \mathcal{G}_k are *feasible* to align the interference to BS k , the dimension of interference to BS k decreases with $\sum_{g \in \mathcal{G}_k} (M_g - 1)$. Therefore, Proposition 1 can be concluded.

Remark 2 If each channel matrix $\mathbf{H}_{k,g_m}, \forall m \in \mathcal{M}_g$ is of full rank, thus $\mathbf{A}_{k,g}$ has full rank. Then, the proposed sufficient condition in Proposition 1 is also necessary. \square

If $\sum_{m \in \mathcal{M}_g} N_m \geq (M_g - 1)N_{R_k} + 2$, $\mathbf{A}_{k,g}$ has a multi-dimensional null space, and thus there exist infinite feasible vectors satisfying (8). Thus, we need to find the optimal $\tilde{\mathbf{v}}_{k,g}$ not only satisfying (8) but also maximizing each user's rate. Following the same line with (5), the feasible $\tilde{\mathbf{v}}_{k,g}$ satisfying (8) is formulated as $\tilde{\mathbf{v}}_{k,g} = \mathcal{N}(\mathbf{A}_{k,g}^H) \mathbf{a}_{k,g}$, where $\mathbf{a}_{k,g} \in \mathbb{C}^{(N_{R_k} + \sum_{m \in \mathcal{M}_g} N_m) \times 1}$ is the weight vector. The normalized transmit beamforming vectors of the users in \mathcal{G}_k can be expressed as

$$\mathbf{v}_{g_m} = \frac{\mathbf{T}_{g_m} \mathcal{N}(\mathbf{A}_{k,g}^H) \mathbf{a}_{k,g}}{\|\mathbf{T}_{g_m} \mathcal{N}(\mathbf{A}_{k,g}^H) \mathbf{a}_{k,g}\|}, \quad m \in \{1, \dots, M_g\} \quad (9)$$

where $\mathbf{T}_{g_m} = [\mathbf{0}_{N_{g_m} \times (N_{R_k} + \sum_{n=1}^m N_{g_n})}, \mathbf{I}_{N_{g_m}}, \mathbf{0}]$ with the size of $N_{g_m} \times (N_{R_k} + \sum_{n \in \mathcal{M}_g} N_n)$.

To simultaneously maximize the rate of each user, the IA transceiver based on the EGM-IA are designed as follows.

1) The IA design of each user $\forall i \in \mathcal{G}_{su}$ is similar to the solutions of the CZFR in (6d) and (6c). The only difference is the interference matrix, denoted by $\tilde{\mathbf{H}}_{-i}$, may have lower dimension than \mathbf{H}_{-i} .

2) For the users in a multi-user group g . The rate of group leader R_{g_1} is maximized by

$$\max_{\mathbf{w}_{g_1}, \mathbf{a}_{k,g}} \frac{|\mathbf{w}_{g_1}^H \mathcal{N}^H(\tilde{\mathbf{H}}_{-g_1}) \mathbf{H}_{k,g_1} \mathbf{T}_{g_1} \mathcal{N}(\mathbf{A}_{k,g}^H) \mathbf{a}_{k,g}|^2}{\|\mathbf{w}_{g_1}^H \mathcal{N}^H(\tilde{\mathbf{H}}_{-g_1})\|^2 \|\mathbf{T}_{g_1} \mathcal{N}(\mathbf{A}_{k,g}^H) \mathbf{a}_{k,g}\|^2} \quad (10a)$$

$$\implies \mathbf{w}_{g_1}^* = \bar{\mathbf{u}}_1^{svd}(\mathcal{N}^H(\tilde{\mathbf{H}}_{-g_1}) \mathbf{H}_{k,g_1} \mathbf{T}_{g_1} \mathcal{N}(\mathbf{A}_{k,g}^H)) \quad (10b)$$

$$\mathbf{a}_{k,g}^* = \bar{\mathbf{v}}_1^{svd}(\mathcal{N}^H(\tilde{\mathbf{H}}_{-g_1}) \mathbf{H}_{k,g_m} \mathbf{T}_{g_1} \mathcal{N}(\mathbf{A}_{k,g}^H)) \quad (10c)$$

$$\mathbf{u}_{g_1}^* = \mathcal{N}(\tilde{\mathbf{H}}_{-g_1}) \mathbf{w}_{g_1}^* \quad (10d)$$

where (10a) is independent of $\|\mathbf{w}_{g_1}\|$ and $\|\mathbf{a}_{k,g}\|$. For each group g , if $\mathbf{a}_{k,g}$ is determined by the optimization of each group leader, the transmit beamforming vectors of other users in group g are obtained by (9) and (10c), and their receivers are

$$\mathbf{u}_{g_m}^* = \frac{\mathcal{N}(\tilde{\mathbf{H}}_{-g_m}) \mathcal{N}^H(\tilde{\mathbf{H}}_{-g_m}) \mathbf{H}_{k,g_m} \mathbf{v}_{g_m}}{\|\mathcal{N}^H(\tilde{\mathbf{H}}_{-g_m}) \mathbf{H}_{k,g_m} \mathbf{v}_{g_m}\|}, \quad m \neq 1. \quad (11)$$

The SURM algorithm with the EGM-IA is shown as follows.

Algorithm 1 The SURM Algorithm based on the EGM-IA

Initialization: set a feasible $\{\mathbf{v}_j^{(0)}\}_{j \neq 1}$. $\ell = 0$.

while some convergence criterion is not satisfied **do**
 $\ell + +$.

for $j = 1 \rightarrow L$ **do**

if $j \in \mathcal{G}_{su}$ **then**

 Compute $\mathbf{w}_j \forall j$ as (6b);

 Obtain $(\mathbf{v}_j^{(\ell)}, \mathbf{u}_j^{(\ell)}) \forall j$ as (6c) and (6d);

else if $j \in \mathcal{G}_{lead}$ **then**

 Compute $\mathbf{w}_j \forall j$ and $\mathbf{a}_{k,j} \forall j$ as (10b) and (10c);

 Obtain $\mathbf{v}_j^{(\ell)}$ as (9) and $\mathbf{u}_j^{(\ell)}$ as (10d).

else

 Obtain $\mathbf{v}_j^{(\ell)}$ as (9) and $\mathbf{u}_j^{(\ell)}$ as (11).

4. FEEDBACK BITS ALLOCATION UNDER LIMITED FEEDBACK

We assume perfect information exchange among the BSs via backhaul links and BSs can compute the IA transceiver with perfect CSIR. Under limited feedback, it is more efficient to feed back transmit beamforming vectors from BSs to users than the complete full channel matrices.

4.1. Transmit Beamforming Quantization and Feedback

We use the vector quantization scheme to quantize each transmit beamforming vector, i.e., $\mathbf{v}_i \in \mathbb{C}^{N_i \times 1}$ by B_i bits. The quantization codebook \mathcal{C} contains 2^{B_i} unit-norm vectors, i.e., $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_{2^{B_i}}\}$, which are chosen as the solution of the *Grassmannian complex line-packing problem* [9]: Find the packing of 2^{B_i} lines in \mathbb{C}^{N_i} that has maximum minimum chordal distance between any pair of lines.

The quantization policy is to find the codeword closed to \mathbf{v}_i by

$$\hat{\mathbf{v}}_i = \arg \max_{\mathbf{c}_\ell \in \mathcal{C}} |\mathbf{c}_\ell^H \mathbf{v}_i|. \quad (12)$$

with the minimum chordal distance defined as

$$d_i = \sqrt{1 - |\hat{\mathbf{v}}_i^H \mathbf{v}_i|^2} \quad (13)$$

Based on the quantization in (12), the relation of the actual transmit beamforming vector \mathbf{v}_i and the quantized beamforming vector $\hat{\mathbf{v}}_i$ can be expressed by

$$\hat{\mathbf{v}}_i = \sqrt{1 - d_i^2} \mathbf{v}_i + d_i \mathbf{e}_i \quad (14)$$

where $\mathbf{e}_i \in \mathbb{C}^{N_i \times 1}$ with $\|\mathbf{e}_i\| = 1$ denotes the quantization error in the null space of \mathbf{v}_i .

The quantization distance d_i in (13) is upper bounded by [14]

$$d_i \leq 2^{-\frac{B_i}{2(N_{R_k} N_i - 1)}}, \quad (15)$$

where the upper bound in (15) denotes the minimum *radius* of the centered metric ball of a complex line packing.

4.2. Feedback Bits Allocation

Each user i reconstructs the quantized beamforming vector $\hat{\mathbf{v}}_i$ according to the received index of the closest codeword of \mathbf{v}_i , which is fed back by B_i bits. Then, the achievable rate of user i to BS k under limited feedback can be expressed as

$$\hat{R}_i = \log_2 \left(1 + \frac{\bar{P}_i |\mathbf{u}_i^H \mathbf{H}_{k,i} \hat{\mathbf{v}}_i|^2}{\sigma_k^2 + \sum_{j \in \mathcal{L} \setminus \{i\}} \bar{P}_j |\mathbf{u}_i^H \mathbf{H}_{k,j} \hat{\mathbf{v}}_j|^2} \right). \quad (16)$$

To guarantee each user's QoS and make the limited feedback bits allocate efficiently, we formulate a problem to maximize the minimum rate of users under a sum feedback bits constraint, i.e.,

$$(P0) \quad \max_{\{B_i\}_{i \in \mathcal{L}}} \min_{i \in \mathcal{L}} \hat{R}_i \quad (17a)$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{L}} B_i \leq B. \quad (17b)$$

Since each quantized transmit beamforming vector $\hat{\mathbf{v}}_i$ in (16) is with the error \mathbf{e}_i , the interference to the BS k cannot be cancelled perfectly by \mathbf{u}_i . Substituting (14) into (16), \hat{R}_i becomes (18) (see the bottom of this page). In the transformation (a), the lower bound is the worst-case rate achieved by $\mathbf{e}_i = \frac{\mathbf{H}_{k,i}^H \mathbf{u}_i}{\|\mathbf{H}_{k,i}^H \mathbf{u}_i\|} e^{-\angle \mathbf{u}_i^H \mathbf{H}_{k,i} \mathbf{v}_i}$ and $\mathbf{e}_j = \frac{\mathbf{H}_{k,j}^H \mathbf{u}_i}{\|\mathbf{H}_{k,j}^H \mathbf{u}_i\|} e^{\angle \mathbf{u}_i^H \mathbf{H}_{k,j} \mathbf{v}_j} \forall j \neq i$ and $\mathbf{u}_i^H \mathbf{H}_{k,j} \hat{\mathbf{v}}_j = 0 \forall j \neq i$. The inequality (b) is because the worst-case rate monotonically decreases with d_i^2 and $d_j^2, \forall j \neq i$, and d_i is upper bounded by (15).

By maximizing the lower bound (18) instead of (16), Problem (P0) becomes

$$(P1) \quad \max_{\{B_i\}_{i \in \mathcal{L}}} \gamma \quad (19a)$$

s.t. RC (See the equation at the bottom of this page) (19b)

$$B_i \geq \left\lceil (N_{R_k} N_i - 1) \log_2 \left(1 + \frac{\|\mathbf{u}_i^H \mathbf{H}_{k,i}\|^2}{|\mathbf{u}_i^H \mathbf{H}_{k,i} \mathbf{v}_i|^2} \right) \right\rceil \forall i \in \mathcal{L} \quad (19c)$$

$$\sum_{i \in \mathcal{L}} B_i \leq B \quad (19d)$$

where RC (below) denotes rate constraints. Since it is meaningless to optimize the feedback bits of the zero-valued rate, it is required $|\mathbf{u}_i^H \mathbf{H}_{k,i} \mathbf{v}_i|^2 - 2^{-\frac{B_i}{N_{R_k} N_i - 1}} (|\mathbf{u}_i^H \mathbf{H}_{k,i} \mathbf{v}_i|^2 + \|\mathbf{u}_i^H \mathbf{H}_{k,i}\|^2) > 0$, i.e., the constraint (19c).

Observe that Problem (P1) is a jointly convex optimization prob-

lem of $\{B_i\}_{i \in \mathcal{L}}$ with a fixed R . Then, Problem (P1) can be solved with different R updated based on the bisection search.

4.3. Illustrations

We consider a two-cell interfering MIMO-MAC system sharing spectrum 1 MHz, where each cell has two users. We set the parameters: $N_i = 2, P_i = 1, \sigma_i^2 = 10 \log(-SNR/10)$ for each user i and $N_{R_1} = 3, N_{R_2} = 4$ for BSs. The two users in cell 2 are grouped. In Fig. 1, the proposed algorithm outperforms the "FDMA" denotes the performance of the FDMA scheme with perfect CSI, where each cell has separate equal spectrum 0.5 MHz and each BS applies the minimum mean square error and successive interference cancellation (MMSE-SIC) receiver to cancel the intra-cell interference. The minimum rate of the users increase as SNR increases and the feedback bits budget increases.

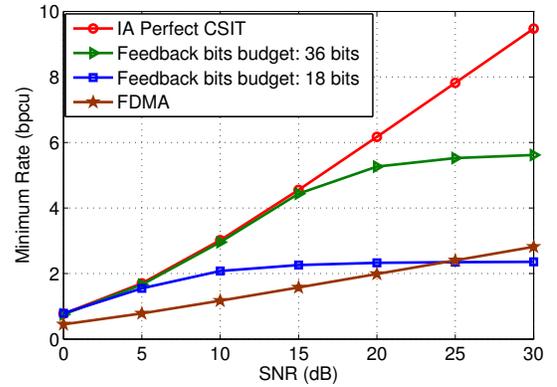


Fig. 1. Minimum-rate performance vs. SNR.

5. CONCLUSIONS

We study the multiplexing gain for the multi-cell interfering MIMO-MAC using the CZFR and the EGM-IA. We propose two algorithms to compute the IC transceivers simultaneously maximizing the rate of each user. For the limited feedback scenario, we proposed a feedback bits allocation algorithm to maximize the minimum worst-case rate of the users subject to a sum feedback bits budget.

$$\begin{aligned} & \log_2 \left(1 + \frac{\bar{P}_i |\mathbf{u}_i^H \mathbf{H}_{k,i} (\sqrt{1-d_i^2} \hat{\mathbf{v}}_i + d_i \mathbf{e}_i)|^2}{\sigma_k^2 + \sum_{j \in \mathcal{L} \setminus \{i\}} \bar{P}_j |\mathbf{u}_i^H \mathbf{H}_{k,j} (\sqrt{1-d_j^2} \hat{\mathbf{v}}_j + d_j \mathbf{e}_j)|^2} \right) \stackrel{(a)}{\geq} \log_2 \left(1 + \frac{\bar{P}_i ((1-d_i^2) |\mathbf{u}_i^H \mathbf{H}_{k,i} \mathbf{v}_i|^2 - d_i^2 \|\mathbf{u}_i^H \mathbf{H}_{k,i}\|^2)_+}{\sigma_k^2 + \sum_{j \in \mathcal{L} \setminus \{i\}} \bar{P}_j d_j^2 \|\mathbf{u}_i^H \mathbf{H}_{k,j}\|^2} \right) \\ & \stackrel{(b)}{\geq} \log_2 \left(1 + \frac{\bar{P}_i \left(|\mathbf{u}_i^H \mathbf{H}_{k,i} \mathbf{v}_i|^2 - 2^{-\frac{B_i}{N_{R_k} N_i - 1}} (|\mathbf{u}_i^H \mathbf{H}_{k,i} \mathbf{v}_i|^2 + \|\mathbf{u}_i^H \mathbf{H}_{k,i}\|^2) \right)_+}{\sigma_k^2 + \sum_{j \in \mathcal{L} \setminus \{i\}} \bar{P}_j 2^{-\frac{B_j}{N_{R_k} N_j - 1}} \|\mathbf{u}_i^H \mathbf{H}_{k,j}\|^2} \right) \end{aligned} \quad (18)$$

$$RC: |\mathbf{u}_i^H \mathbf{H}_{k,i} \hat{\mathbf{v}}_i|^2 - 2^{-\frac{B_i}{N_{R_k} N_i - 1}} (|\mathbf{u}_i^H \mathbf{H}_{k,i} \hat{\mathbf{v}}_i|^2 + \|\mathbf{u}_i^H \mathbf{H}_{k,i}\|^2) \geq \frac{2^\gamma - 1}{P_i} \left(\sigma_k^2 + \sum_{j \in \mathcal{L} \setminus \{i\}} \bar{P}_j 2^{-\frac{B_j}{N_{R_k} N_j - 1}} \|\mathbf{u}_i^H \mathbf{H}_{k,j}\|^2 \right) \forall i \in \mathcal{L}$$

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