

# A Sparse MLE Approach for Joint Interference Mitigation and Data Recovery

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**Abstract**—Consider the scenario where a receiver acquires information (data) corrupted by interference and noise. Both the information and interference have a sparse structure. To fully exploit the *individual sparse structure* of the information and interference, the joint interference mitigation and data recovery is formulated as a sparse maximum likelihood estimation (MLE) problem which maximizes the associated likelihood function under *individual sparsity levels* (ISLs) constraints. We propose an *alternating optimization* (AO) recovery algorithm to solve the *non-convex* sparse MLE problem. Under certain restricted isometry property (RIP) conditions, we show that the proposed AO algorithm converges to the optimal solution of the sparse MLE problem. We also derive an upper bound of the corresponding estimation error for the information. Simulations show that the proposed solution achieves significant gain over various baselines.

**Index Terms**—Compressive sensing, Interference mitigation

## I. INTRODUCTION

In many applications, a receiver needs to recover information (or data) in the presence of both interference and noise. For example, in cellular systems, a receiver usually suffers from the intra-cell interference as well as the inter-cell interference. As a result, interference is a fundamental bottleneck in wireless systems. In cellular systems, the intra-cell interference is usually mitigated using various multi-access techniques [1], and the inter-cell interference is usually mitigated using frequency reuse techniques [2]. However, in these interference mitigation techniques, the signal structure of the interference and information is not exploited. In many applications, both the interference and information have a sparse structure. For instance, in cellular systems, there are usually a few dominating interferences across the entire signal space seen at a receiver and hence, there is inherent sparsity in the interference. Moreover, wireless systems are not always fully loaded. As a result, the information signal usually does not occupy all the available signal dimensions. Some other applications involving sparse interference and data can be found in [3]. Hence, there is a potential advantage of exploiting the sparse structures of both the interference and information to design efficient joint interference mitigation and data recovery schemes.

In this paper, we propose a compressive sensing (CS) based joint interference mitigation and data recovery framework which can fully exploit the *individual sparse structure* of the information and interference to achieve huge SNR gain over the existing interference mitigation and data recovery schemes.

Specifically, we formulate a sparse maximum likelihood estimation (MLE) problem which maximizes the likelihood function [4] under *individual sparsity levels* (ISLs) constraints. The optimal solution of this sparse MLE problem is then used as a “good” estimation of the information.

There are a few CS-based interference cancellation schemes in the literature. In [3], [5], CS techniques were deployed to recover a sparse signal from compressed measurements in the presence of sparse interference. However, the methods in [3], [5] require that the support of interference is known at the receiver. The proposed sparse MLE framework does not require such restrictive assumption and thus can be applied to more general applications. Moreover, we propose a new *alternating optimization* (AO) recovery algorithm to solve the sparse MLE problem which achieves better performance than the conventional CS recovery algorithms in [6], [7], [8], [9]. However, there are several first order technical challenges.

- **Non-convexity of the Sparse MLE Problem:** The sparse MLE problem is non-convex due to the ISLs constraints.
- **Convergence of the AO Algorithm:** In each iteration of the proposed AO algorithm, we need to solve an *information vector optimization subproblem*, and an *interference vector optimization subproblem*. However, each subproblem is still non-convex. It is highly non-trivial to prove the convergence of such AO algorithm [10].

To address the above challenges, we first show that the conventional CS recovery algorithms such as CoSaMP [8] can be used to find the optimal solution of the aforementioned information/interference vector optimization subproblems under some RIP conditions. Based on this, we propose an AO algorithm with CoSaMP as a building block and show that this AO algorithm converges to the global optimal solution of the non-convex sparse MLE problem under certain RIP conditions. We further derive an upper bound of the error between the optimal solution found by the AO algorithm and the true information vector.

*Notations:* The superscripts  $(\cdot)^T$ ,  $(\cdot)^H$  and  $(\cdot)^\dagger$  denote transpose, Hermitian, and Pseudo inverse respectively. The notation  $\circ$  denotes the Hadamard product. Let  $\mathbf{x}(i)$  denote the  $i$ -th element of a vector  $\mathbf{x} \in \mathbb{C}^N$ . The  $l_0$ -norm,  $l_1$ -norm and  $l_2$ -norm of  $\mathbf{x}$  are respectively denoted by  $\|\mathbf{x}\|_0$ ,  $\|\mathbf{x}\|_1$  and  $\|\mathbf{x}\|_2$ . For a  $K$ -sparse vector  $\mathbf{x}$  with  $K < N$  (i.e.,  $\|\mathbf{x}\|_0 = K$ ), let  $T_{\mathbf{x}}$  denote the support of  $\mathbf{x}$  (i.e.,  $T_{\mathbf{x}}$  is the index set corresponding to the  $K$  non-zero entries of  $\mathbf{x}$ ). For a given index set  $\mathcal{T} \subseteq \{1, \dots, N\}$ , let  $|\mathcal{T}|$  denote its

cardinality, let  $\mathcal{T}^c = \{1, \dots, N\} \setminus \mathcal{T}$ , let  $\mathbf{x}(\mathcal{T}) \in \mathbb{C}^{|\mathcal{T}|}$  denote the subvector consisting of the elements of  $\mathbf{x}$  indexed by the set  $\mathcal{T}$ , let  $\mathbf{x}(\mathcal{T}^c) \in \mathbb{C}^{|\mathcal{T}^c|}$  denote the subvector consisting of the elements of  $\mathbf{x}$  indexed by the set  $\mathcal{T}^c$ , and let  $\Phi(\mathcal{T}) \in \mathbb{C}^{M \times |\mathcal{T}|}$  denote the matrix consisting of the columns of  $\Phi \in \mathbb{C}^{M \times N}$  indexed by the set  $\mathcal{T}$ .

## II. SYSTEM MODEL

Consider the following CS model

$$\mathbf{y} = \Phi \mathbf{x} + \mathbf{n} = \begin{bmatrix} \Phi_S & \Phi_I \end{bmatrix} \begin{bmatrix} \mathbf{x}_S \\ \mathbf{x}_I \end{bmatrix} + \mathbf{n}, \quad (1)$$

where  $\mathbf{x} \in \mathbb{C}^N$  is an unknown signal,  $\Phi \in \mathbb{C}^{M \times N}$  is the measurement matrix,  $\mathbf{y} \in \mathbb{C}^M$  is the received signal, and  $\mathbf{n} \in \mathbb{C}^M$  is the noise. The unknown signal  $\mathbf{x} = [\mathbf{x}_S^T, \mathbf{x}_I^T]^T$  contains two subvectors, namely, the information vector  $\mathbf{x}_S \in \mathbb{C}^{N_S}$  and the interference vector  $\mathbf{x}_I \in \mathbb{C}^{N_I}$ , where  $N_S + N_I = N$ . Correspondingly, the measurement matrix  $\Phi = \begin{bmatrix} \Phi_S & \Phi_I \end{bmatrix}$  contains two submatrices, where  $\Phi_S \in \mathbb{C}^{M \times N_S}$  and  $\Phi_I \in \mathbb{C}^{M \times N_I}$ . The information vector  $\mathbf{x}_S$  is  $K_S$ -sparse (i.e.,  $\|\mathbf{x}_S\|_0 = K_S$ ) and the interference vector  $\mathbf{x}_I \in \mathbb{C}^{N_I}$  is  $K_I$ -sparse (i.e.,  $\|\mathbf{x}_I\|_0 = K_I$ ). The total sparsity level of  $\mathbf{x}$  is  $K = K_S + K_I$ . We assume that the receiver has the knowledge of the measurement matrices  $\Phi_S$ ,  $\Phi_I$ , and the individual sparsity levels (ISLs)  $K_S$  and  $K_I$  of the information and interference respectively<sup>1</sup>. The problem is to recover the sparse information vector  $\mathbf{x}_S$  in the presence of the sparse interference  $\mathbf{x}_I$  and noise  $\mathbf{n}$  at the receiver. Note that the CS model in (1) is different from the conventional CS model because the information vector and interference vector have ISLs. As a result, we can exploit this sparse signal structure with ISLs to design recovery algorithms that outperform the conventional CS recovery algorithms in [6], [7], [8], [9]. The above system model covers many interesting application scenarios in wireless communications [11] and other fields involving CS signal processing [3], [5].

## III. CONVENTIONAL CS RECOVERY: A SPARSE MLE VIEW

When there is no interference, we have  $\mathbf{x} = \mathbf{x}_S$ ,  $\Phi = \Phi_S$  and the received signal model in (1) reduces to the conventional CS model. The conventional CS recovery algorithms such as CoSaMP [8], OMP [7], and subspace pursuit [9] aim at finding a stable estimation  $\hat{\mathbf{x}}$  of  $\mathbf{x}$  (i.e.,  $\|\hat{\mathbf{x}} - \mathbf{x}\|_2 \leq C \|\mathbf{n}\|_2$  for some constant  $C$ ) from  $M < N$  compressive measurements  $\mathbf{y}$ . For example, for a given CS model  $\mathbf{y} = \Phi \mathbf{x} + \mathbf{n}$  and sparsity level  $K$ , the CoSaMP uses  $\mathbf{y}, \Phi, K, \eta$  as the input and calculates an approximation  $\hat{\mathbf{x}}$  of  $\mathbf{x}$ , where  $\eta$  is a precise parameter. Please refer to [8] for the details of the CoSaMP algorithm. In order to achieve stable recovery, the measurement matrix  $\Phi$  must satisfy certain conditions. One of the most important conditions is the restricted isometry property (RIP) introduced by Candes and Tao [12].

<sup>1</sup>The proposed solution can also be extended to the case when the receiver does not have the knowledge of  $K_S, K_I$  as discussed in Remark 2.

**Definition 1** (Restricted Isometry Property). The measurement matrix  $\Phi$  satisfies the restricted isometry property (RIP) of order  $K$  ( $K$ -RIP) with constant  $\delta_K \in (0, 1)$  if

$$(1 - \delta_K) \|\mathbf{x}\|_2^2 \leq \|\Phi \mathbf{x}\|_2^2 \leq (1 + \delta_K) \|\mathbf{x}\|_2^2. \quad (2)$$

holds for all  $\mathbf{x} \in \Sigma_{(N,K)} \triangleq \{\mathbf{z} : \mathbf{z} \in \mathbb{C}^N, \|\mathbf{z}\|_0 \leq K\}$ .

The conventional CS recovery can be interpreted as a solution to the following sparse MLE problem

$$\mathcal{P}_A : \min_{\mathbf{z}} \|\Phi \mathbf{z} - \mathbf{y}\|_2, \text{ s.t. } \|\mathbf{z}\|_0 = K. \quad (3)$$

Specifically, we show that under certain RIP conditions, CoSaMP is able to find the optimal solution  $\mathbf{x}^*$  of  $\mathcal{P}_A$ , despite that  $\mathcal{P}_A$  is non-convex. Moreover,  $\mathbf{x}^*$  is a stable estimation of  $\mathbf{x}$ . These results provide a basis for solving the more complicated joint interference mitigation and data recovery problem in Section IV.

**Theorem 1** (Conditions for the Optimality of CoSaMP). *Suppose that  $\Phi$  satisfies the  $4K$ -RIP with constant  $\delta_{4K} \leq 0.1$ . Apply CoSaMP with input  $\mathbf{y}, \Phi, K, \eta$  to obtain  $\hat{\mathbf{x}}$  as an approximation of  $\mathbf{x}$ . Then  $\hat{\mathbf{x}}$  is the unique global optimal solution of Problem  $\mathcal{P}_A$  if the following conditions are satisfied:*

$$\min_{j \in T_{\hat{\mathbf{x}}}} |\mathbf{x}(j)| > \eta + 15 \|\mathbf{n}\|_2. \quad (4)$$

Moreover, we have  $T_{\hat{\mathbf{x}}} = T_{\mathbf{x}}$  and

$$\|\hat{\mathbf{x}} - \mathbf{x}\|_2 \leq \frac{2\sqrt{10}}{3} \|\mathbf{n}\|_2, \quad (5)$$

*Proof:* By [8, Theorem 4.1], we have  $\|\hat{\mathbf{x}} - \mathbf{x}\|_2 \leq \eta + 15 \|\mathbf{n}\|_2$ . Suppose  $T_{\mathbf{x}} \neq T_{\hat{\mathbf{x}}}$ , then we have  $\|\hat{\mathbf{x}} - \mathbf{x}\|_2 \geq \min_{j \in T_{\hat{\mathbf{x}}}} |\mathbf{x}(j)|$ . It follows that  $\min_{j \in T_{\hat{\mathbf{x}}}} |\mathbf{x}(j)| \leq \eta + 15 \|\mathbf{n}\|_2$ , which contradicts with the condition in (4). Hence, we must have  $T_{\mathbf{x}} = T_{\hat{\mathbf{x}}}$  and  $\hat{\mathbf{x}} = \Phi^\dagger(T_{\mathbf{x}}) \mathbf{y}$ . Note that the optimal solution  $\mathbf{x}^*$  of  $\mathcal{P}_A$  satisfies  $\|\Phi \mathbf{x}^* - \mathbf{y}\|_2 \leq \|\Phi \mathbf{x} - \mathbf{y}\|_2 = \|\mathbf{n}\|_2$ . Then it follows from the triangle inequality and RIP that  $\sqrt{1 - \delta_{2K}} \|\mathbf{x}^* - \mathbf{x}\|_2 \leq \|\Phi \mathbf{x}^* - \Phi \mathbf{x}\|_2 \leq 2 \|\mathbf{n}\|_2$ . Using the fact that  $\delta_{2K} \leq \delta_{4K} = 0.1$ , we have  $\|\mathbf{x}^* - \mathbf{x}\|_2 \leq \frac{2\sqrt{10}}{3} \|\mathbf{n}\|_2 \leq \eta + 15 \|\mathbf{n}\|_2$ . Following similar analysis as for  $\hat{\mathbf{x}}$ , we have  $T_{\mathbf{x}} = T_{\mathbf{x}^*}$  and  $\mathbf{x}^* = \Phi^\dagger(T_{\mathbf{x}}) \mathbf{y} = \hat{\mathbf{x}}$ . ■

*Remark 1.* We can obtain similar optimality conditions for other CS recovery algorithms such as OMP [7] and subspace pursuit [9]. In this paper, we focus on the design of AO algorithm with CoSaMP as a basic building block because it achieves very competitive numerical performance with robust recovery guarantees and low computation complexity.

## IV. JOINT INTERFERENCE MITIGATION AND DATA RECOVERY

### A. Sparse MLE Formulation with ISLs

Since the receiver has the knowledge of the ISLs  $K_S$  and  $K_I$ , we can exploit this side information and formulate the following sparse MLE problem with ISLs constraints for joint interference mitigation and data recovery:

$$\mathcal{P}_B : \min_{\mathbf{z}} \|\Phi \mathbf{z} - \mathbf{y}\|_2, \text{ s.t. } \|\mathbf{z}_S\|_0 = K_S \text{ and } \|\mathbf{z}_I\|_0 = K_I, \quad (6)$$

where  $\mathbf{z} = [\mathbf{z}_S^T, \mathbf{z}_I^T]^T$  with  $\mathbf{z}_S \in \mathbb{C}^{N_S}$  and  $\mathbf{z}_I \in \mathbb{C}^{N_I}$ . We show that the optimal solution  $\mathbf{x}^*$  of  $\mathcal{P}_B$  is a stable estimation of  $\mathbf{x}$  if  $\Phi$  satisfies the following RIP with ISLs.

**Definition 2** (ISLs-RIP). The measurement matrix  $\Phi$  satisfies the  $(N_{S,I}, K_{S,I})$ -RIP<sup>2</sup> with constant  $\delta_{(N_{S,I}, K_{S,I})} \in (0, 1)$  if for all  $\mathbf{x} \in \Sigma_{(N_{S,I}, K_{S,I})} \triangleq \left\{ \mathbf{z} = [\mathbf{z}_S^T, \mathbf{z}_I^T]^T : \mathbf{z}_S \in \Sigma_{(N_S, K_S)}, \mathbf{z}_I \in \Sigma_{(N_I, K_I)} \right\}$ , we have  $(1 - \delta_{(N_{S,I}, K_{S,I})}) \|\mathbf{x}\|_2^2 \leq \|\Phi \mathbf{x}\|_2^2 \leq (1 + \delta_{(N_{S,I}, K_{S,I})}) \|\mathbf{x}\|_2^2$ . (7)

**Theorem 2** (Error Bound for Sparse MLE  $\mathcal{P}_B$ ). Suppose that  $\Phi$  satisfies the  $(N_{S,I}, 2K_{S,I})$ -RIP with constant  $\delta_{(N_{S,I}, 2K_{S,I})} \in (0, 1)$ . Let  $\mathbf{x}^*$  denote any optimal solution of Problem  $\mathcal{P}_B$ . Then we have

$$\|\mathbf{x}^* - \mathbf{x}\|_2 \leq \frac{2}{\sqrt{1 - \delta_{(N_{S,I}, 2K_{S,I})}}} \|\mathbf{n}\|_2. \quad (8)$$

Moreover, the optimal solution of  $\mathcal{P}_B$  is uniquely given by  $\mathbf{x}^*(T_{\mathbf{x}}) = \Phi^\dagger(T_{\mathbf{x}}) \mathbf{y}$  and  $\mathbf{x}^*(T_{\mathbf{x}}^c) = \mathbf{0}$  if

$$\min_{j \in T_{\mathbf{x}}} |\mathbf{x}(j)| > \frac{2}{\sqrt{1 - \delta_{(N_{S,I}, 2K_{S,I})}}} \|\mathbf{n}\|_2. \quad (9)$$

The proof is similar to Theorem 1.

### B. Alternating Optimization Algorithm to Solve $\mathcal{P}_B$

Unlike  $\mathcal{P}_A$ , the existing CS recovery algorithms cannot be directly used to solve  $\mathcal{P}_B$ . One possible solution is the model-based CoSaMP proposed in [13]. However, the model-based CoSaMP cannot fully exploit the benefits due to the side information of ISLs as will be demonstrated in Section V.

Note that for fixed  $\mathbf{z}_S$  ( $\mathbf{z}_I$ ),  $\mathcal{P}_B$  reduces to  $\mathcal{P}_A$  with measurement matrix  $\Phi_S$  ( $\Phi_I$ ), which can be efficiently solved using CoSaMP as discussed in Section III. This motivates us to propose an alternating optimization (AO) algorithm to solve  $\mathcal{P}_B$  as summarized below.

*Algorithm AO-SMLE* (for solving  $\mathcal{P}_B$ ):

**Initialization:** Choose a proper initial<sup>3</sup>  $\mathbf{x}_I^{(0)}$ . Let  $i = 1$ .

**Step 1:** Let  $\mathbf{y}_S^{(i)} = \mathbf{y} - \Phi_I \mathbf{x}_I^{(i-1)}$ . Use CoSaMP with input  $\mathbf{y}_S^{(i)}$ ,  $\Phi_S$ ,  $K_S$ ,  $\eta$  to find an approximate solution  $\mathbf{x}_S^{(i)}$  of

$$\mathcal{P}_S^{(i)} : \min_{\mathbf{z}_S} \left\| \mathbf{y}_S^{(i)} - \Phi_S \mathbf{z}_S \right\|, \text{ s.t. } \|\mathbf{z}_S\|_0 = K_S.$$

**Step 2:** Let  $\mathbf{y}_I^{(i)} = \mathbf{y} - \Phi_S \mathbf{x}_S^{(i)}$ . Use CoSaMP with input  $\mathbf{y}_I^{(i)}$ ,  $\Phi_I$ ,  $K_I$ ,  $\eta$  to find an approximate solution  $\mathbf{x}_I^{(i)}$  of

$$\mathcal{P}_I^{(i)} : \min_{\mathbf{z}_I} \left\| \mathbf{y}_I^{(i)} - \Phi_I \mathbf{z}_I \right\|, \text{ s.t. } \|\mathbf{z}_I\|_0 = K_I.$$

**Step 3:** If  $i > 1$  and  $\left| \varepsilon^{(i-1)} - \varepsilon^{(i)} \right| < \epsilon$ , where  $\varepsilon^{(i)} = \left\| \Phi_S \mathbf{x}_S^{(i)} + \Phi_I \mathbf{x}_I^{(i)} - \mathbf{y} \right\|_2$  and  $\epsilon > 0$  is a small number, or if  $i > i_{\max}$ , where  $i_{\max}$  is the maximum allowable number of iterations, then terminate the algorithm and output  $\hat{\mathbf{x}} = \begin{bmatrix} \mathbf{x}_S^{(i^*)} \\ \mathbf{x}_I^{(i^*)} \end{bmatrix}$ , where  $i^* = \arg \min_{i \in [1, i_{\max}]} \varepsilon^{(i)}$ . Otherwise, let  $i = i + 1$  and return to Step 1.

<sup>2</sup>Note that  $(N_{S,I}, K_{S,I})$  is an abbreviation for the four parameters  $(N_S, N_I, K_S, K_I)$  required to define the ISLs-RIP.

<sup>3</sup>For example, we can use CoSaMP to generate the initial point.

Under some conditions, Algorithm AO-SMLE converges to the optimal solution of  $\mathcal{P}_B$ .

**Theorem 3** (Conditions for the Optimality of Algorithm AO-SMLE). Suppose that the following conditions are satisfied.

- 1)  $\Phi$  satisfies the  $(N_{S,I}, 4K_{S,I})$ -RIP with constant  $\delta_{(N_{S,I}, 4K_{S,I})} \leq 0.1$ ;
- 2)  $\min_{j \in T_{\mathbf{x}}} |\mathbf{x}(j)| > 15\sqrt{1.1} \left\| \mathbf{x}_I^{(0)} - \mathbf{x}_I \right\|_2 + \eta + 15 \|\mathbf{n}\|_2$ .

Then as  $i \rightarrow \infty$ , Algorithm AO-SMLE monotonically decreases  $\varepsilon^{(i)}$  and converges to a point  $\hat{\mathbf{x}}$  that satisfies

$$\|\hat{\mathbf{x}} - \mathbf{x}\|_2 \leq \frac{2\sqrt{10}}{3} \|\mathbf{n}\|_2.$$

Moreover,  $\hat{\mathbf{x}}$  is the unique optimal solution of Problem  $\mathcal{P}_B$ .

*Proof:* In Step 1 of the first iteration, the CoSaMP is applied on the CS model  $\mathbf{y}_S^{(1)} = \Phi_S \mathbf{x}_S + \mathbf{n}_S^{(1)}$ , where  $\mathbf{n}_S^{(1)} = \mathbf{y} - \Phi_I \mathbf{x}_I^{(0)} - \Phi_S \mathbf{x}_S = \Phi_I \mathbf{x}_I - \Phi_I \mathbf{x}_I^{(0)} + \mathbf{n}$ . Since  $\Phi$  satisfies  $(N_{S,I}, 4K_{S,I})$ -RIP with constant  $\delta_{(N_{S,I}, 4K_{S,I})} \leq 0.1$ ,  $\Phi_I$  must satisfy  $4K_I$ -RIP with constant  $\delta_{4K_I} \leq 0.1$ , from which it follows that  $\left\| \mathbf{n}_S^{(1)} \right\|_2 \leq \left\| \Phi_I \mathbf{x}_I - \Phi_I \mathbf{x}_I^{(0)} \right\|_2 + \|\mathbf{n}\|_2 \leq \sqrt{1.1} \left\| \mathbf{x}_I^{(0)} - \mathbf{x}_I \right\|_2 + \|\mathbf{n}\|_2$ . From Theorem 1 and condition 2 in Theorem 3, we have  $T_{\mathbf{x}_S^{(1)}} = T_{\mathbf{x}_S}$  and  $\mathbf{x}_S^{(1)}(T_{\mathbf{x}_S}) = \Phi_S^\dagger(T_{\mathbf{x}_S}) \mathbf{y}_S^{(1)}$ . In Step 2 of the first iteration, the CoSaMP is applied on the CS model  $\mathbf{y}_I^{(1)} = \Phi_I \mathbf{x}_I + \mathbf{n}_I^{(1)}$ , where  $\mathbf{n}_I^{(1)} = \Phi_S \mathbf{x}_S - \Phi_S \mathbf{x}_S^{(1)} + \mathbf{n}$ . Consider thin SVD  $\Phi_S(T_{\mathbf{x}_S}) = \mathbf{U} \Delta \mathbf{V}^H$ , where  $\mathbf{U} \in \mathbb{C}^{M \times K_S}$ ,  $\Delta \in \mathbb{R}^{K_S \times K_S}$  and  $\mathbf{V} \in \mathbb{C}^{K_S \times K_S}$ . Using the fact that  $\Phi_S(T_{\mathbf{x}_S}) \Phi_S^\dagger(T_{\mathbf{x}_S}) = \mathbf{U} \mathbf{U}^H$  and  $\mathbf{x}_S^{(1)}(T_{\mathbf{x}_S}) = \Phi_S^\dagger(T_{\mathbf{x}_S}) \mathbf{y}_S^{(1)}$ , it can be shown that  $\mathbf{n}_I^{(1)} = -\mathbf{U} \mathbf{U}^H \mathbf{n}_I^{(0)} + (\mathbf{I} - \mathbf{U} \mathbf{U}^H) \mathbf{n}$ , where  $\mathbf{n}_I^{(0)} = \Phi_I \mathbf{x}_I - \Phi_I \mathbf{x}_I^{(0)}$ . Hence, we have  $\left\| \mathbf{n}_I^{(1)} \right\|_2 \leq \left\| \mathbf{n}_I^{(0)} \right\|_2 + \|\mathbf{n}\|_2 \leq \sqrt{1.1} \left\| \mathbf{x}_I^{(0)} - \mathbf{x}_I \right\|_2 + \|\mathbf{n}\|_2$ . Then from Theorem 1 and condition 2 in Theorem 3, we have  $T_{\mathbf{x}_I^{(1)}} = T_{\mathbf{x}_I}$ . Similarly, it can be shown that  $T_{\mathbf{x}_S^{(i)}} = T_{\mathbf{x}_S}$  and  $T_{\mathbf{x}_I^{(i)}} = T_{\mathbf{x}_I}$  for  $i = 2, 3, \dots$ . As a result, AO-SMLE can be viewed as an AO algorithm for solving the following convex optimization problem:

$$\min_{\mathbf{z}_S \in \mathbb{C}^{K_S}, \mathbf{z}_I \in \mathbb{C}^{K_I}} \left\| \Phi_S(T_{\mathbf{x}_S}) \mathbf{z}_S + \Phi_I(T_{\mathbf{x}_I}) \mathbf{z}_I - \mathbf{n} \right\|_2. \quad (10)$$

According to the AO convergence result in [10], as  $i \rightarrow \infty$ , AO-SMLE monotonically decreases  $\varepsilon^{(i)}$  and converges to a point  $\hat{\mathbf{x}}$  such that  $\hat{\mathbf{x}}(T_{\mathbf{x}_S})$ ,  $\hat{\mathbf{x}}(T_{\mathbf{x}_I})$  is the optimal solution of (10). Since the conditions in Theorem 3 imply the conditions in Theorem 2,  $\hat{\mathbf{x}}$  is also the unique optimal solution of Problem  $\mathcal{P}_B$ . ■

*Remark 2.* In some applications, we may not have the knowledge of  $K_S$  and  $K_I$ . However, the CoSaMP can be modified to solve  $\mathcal{P}_S^{(i)}$  or  $\mathcal{P}_I^{(i)}$  without the knowledge of  $K_S$  and  $K_I$ . Please refer to [8], [14] for the details. Hence, by using the modified CoSaMP algorithms in [8], [14] as the building block for solving the subproblems  $\mathcal{P}_S^{(i)}$  or  $\mathcal{P}_I^{(i)}$ , Algorithm AO-SMLE can also be generalized to solve  $\mathcal{P}_B$  without the knowledge of  $K_S$  and  $K_I$ .

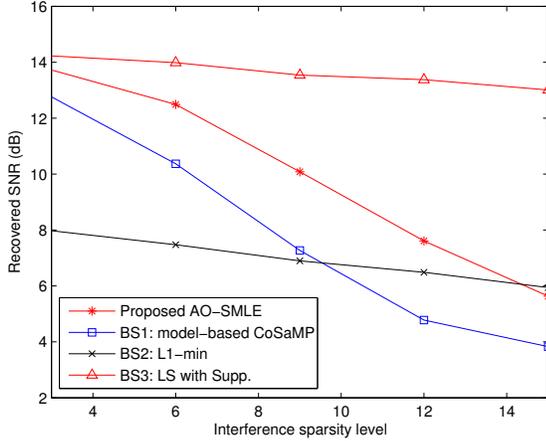


Figure 1: Recovered SNR versus the interference sparsity level  $K_I$  for QPSK modulation. The other simulation parameters are set as  $K_S = 6$ ,  $P_S = 15\text{dB}$  and  $P_I = 15\text{dB}$  (i.e.,  $\text{SNR} = 15\text{dB}$ ,  $\text{SIR} = 0\text{dB}$ ).

## V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed joint interference mitigation and data recovery algorithm. The system parameters are chosen as  $M = 64$ ,  $N_S = 60$ ,  $N_I = 60$ . We consider AWGN noise vector  $\mathbf{n} \sim \mathcal{CN}(0, \mathbf{I})$ . The measurement matrix  $\Phi$  is assumed to have i.i.d. complex Gaussian entries of zero mean and variance  $\frac{1}{\sqrt{M}}$ . Both data  $\mathbf{x}_S$  and interference  $\mathbf{x}_I$  are generated from a digital modulator. Two different modulation schemes are considered, namely, QPSK and 16QAM. Let  $\mathbb{E}[|\mathbf{x}_S(j)|^2] = P_S, \forall j \in T_{x_S}$  and  $\mathbb{E}[|\mathbf{x}_I(j)|^2] = P_I, \forall j \in T_{x_I}$  respectively denote the data power and interference power. Similar to [3], [5], we use the recovered SNR, defined as  $P_S/\sigma_e^2$ , as the performance metric, where  $\sigma_e^2$  is the variance of the effective noise after recovery. We compare the proposed algorithms with the following three baselines: **Baseline 1 (model-based CoSaMP)**, the Algorithm 1 in [13]; **Baseline 2 (L1-min)**, the well known  $l_1$ -norm minimization recovery algorithm [6]; **Baseline 3 (LS with Supp.)**, the conventional least square recovery algorithm with the knowledge of the support of  $\mathbf{x}$  (performance upper bound).

In Fig. 1 to 3, we plot the recovered SNR of different algorithms versus the interference sparsity level  $K_I$  under different combinations of SNR/SIR and modulation schemes. In all cases, the proposed algorithm AO-SMLE achieves significant performance gains over baseline 1 (model-based CoSaMP) and baseline 2 ( $l_1$ -norm minimization). Moreover, for small  $K_I$ , the performance of AO-SMLE approaches the upper bound achieved by baseline 3. These simulation results verified that the proposed algorithm has superior performance compared to the existing CS recovery algorithms under various system parameters.

## VI. CONCLUSION

We propose a sparse MLE framework for joint interference mitigation and data recovery when both information (data) and interference are sparse. In this framework, the estimate

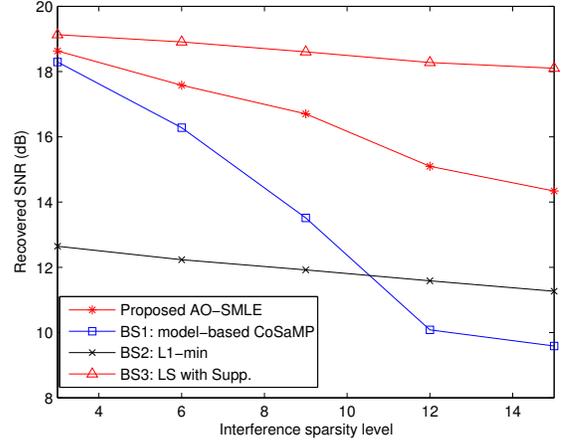


Figure 2: Recovered SNR versus the interference sparsity level  $K_I$  for QPSK modulation. The other simulation parameters are set as  $K_S = 6$ ,  $P_S = 20\text{dB}$  and  $P_I = 10\text{dB}$  (i.e.,  $\text{SNR} = 20\text{dB}$ ,  $\text{SIR} = 10\text{dB}$ ).

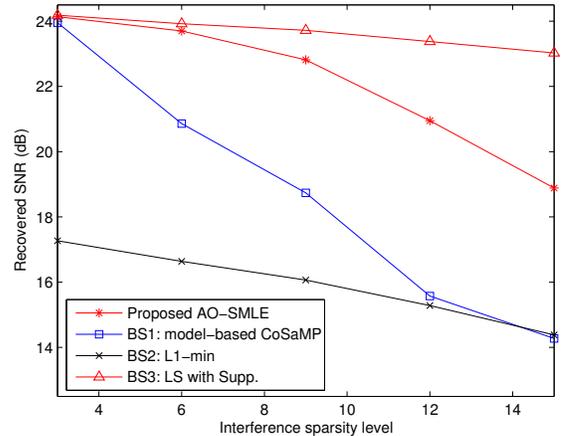


Figure 3: Recovered SNR versus the interference sparsity level  $K_I$  for 16QAM modulation. The other simulation parameters are set as  $K_S = 6$ ,  $P = 25\text{dB}$  and  $P_I = 20\text{dB}$  (i.e.,  $\text{SNR} = 25\text{dB}$ ,  $\text{SIR} = 5\text{dB}$ ).

of the information is obtained by solving the associated likelihood maximization problem under individual sparsity levels (ISLs) constraints on the information and interference vectors respectively. The proposed framework can fully exploit the individual sparse structure of the information and interference to significantly improve the data recovery performance. We propose an alternating optimization (AO) algorithm to solve this non-convex sparse MLE problem and establish the global convergence conditions. Simulations show that the proposed algorithm achieves a significant gain over the existing compressive sensing recovery algorithms under various system parameters.

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