SPARSE INVERSION OF THE RADON COEFFICIENTS IN THE PRESENCE OF ERRATIC NOISE WITH APPLICATION TO SIMULTANEOUS SEISMIC SOURCE PROCESSING

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ABSTRACT

In recent years, efforts have been made in designing simultaneous-source strategies that permit to save seismic acquisition costs. Seismic sources are fired with time overlap producing seismic records that contain a mixture of sources. These records need to be unmixed before seismic imaging. The unmixing process can be written as an inverse problem where one attempts to solve a linear system of equations to estimate the unmixed seismic data. This article describes a source separation process where we assume that source interferences can be modelled via an erratic noise process. In addition, the ideal unmixed data are assumed to be sparse in the Hyperbolic Radon transform domain. Therefore, the source separation problem is posed as an inverse problem where one seeks to retrieve a sparse model from observations contaminated with erratic (sparse) noise. We present a modification of the fast iterative shrinkage-thresholding algorithm that permits to cope with the simultaneous estimation of sparse Radon coefficients that are required to synthesize the unmixed data. The algorithm is also utilized to estimate the erratic noise caused by source interferences.

***Index Terms*—** Geophysical signal processing, Radon Transform, sparsity, simultaneous sources, separation of sources, inversion

1. INTRODUCTION

In conventional seismic acquisition sources are fired with no time overlap leading to a common receiver gather similar to Figure 1(a). In simultaneous source acquisition [1, 2, 3], sources are fired randomly at time intervals that are shorter than the total acquisition length of previous sources. This leads to common receiver gathers that are contaminated by source interferences. Figure 1(b) is an example of a simultaneous source acquisition common receiver gather. Source separation in this particular case becomes a denoising problem [4, 5, 6] where one attempts to remove source interferences to obtain the ideal gather (Figure 1(a)). Simultaneous acquisition permits to save acquisition time and therefore, it permits to decrease the cost of acquiring seismic data. However, conventional signal processing and imaging techniques require data that are composed of records that were acquired with no overlapping of sources.

In this article, we present a method to separate sources from seismic gathers collected via simultaneous source acquisition. We propose to represent the ideal data via a sparse Radon synthesis [7] with the inclusion of a noise term that represent source interferences. The problem is tackled via the simultaneous solution of a set of sparse Radon coefficients that represent the data and the sparse erratic noise representing source interferences. Unlike previous work where the source separation was posed as an l_1-l_1 problem [8, 9], we propose to solve an l_2-l_1 optimization where sparsity is jointly imposed on the unknown vectors of Radon coefficients and source interferences.

Our problem reduces to finding the sparse solution of a system of equations $\mathbf{d} = \mathbf{L}\mathbf{m} + \beta\mathbf{u} + \mathbf{e}$ where the data $\mathbf{d}$ modelled by the linear operator $\mathbf{L}$ acting on coefficients $\mathbf{c}$ is corrupted by erratic noise $\beta\mathbf{u}$ and Gaussian noise $\mathbf{e}$. A similar model was proposed for robust face recognition in [10, 11, 12]. However, the latter does not contain an additive Gaussian noise term $\mathbf{e}$.

2. HYPERBOLIC RADON TRANSFORM

We start with a brief description of the Hyperbolic Radon Transform (HRT) [7, 13]. This is a transform that permits to model seismic reflections via a superposition of hyperbolas. In its basic form, the HRT can be represented via two operators. First we define the adjoint HRT (often also called the analysis operator)

$$\hat{m}(\tau, v) = \sum_x d(t = \sqrt{\tau^2 + \frac{x^2}{v^2}}, x)\,. \tag{1}$$

We also define the Forward Radon operator given by

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$$d(t,x) = \sum_v m(\tau = \sqrt{\tau^2 - \frac{x^2}{v^2}}, v) \tag{2}$$

In these expressions x denotes the source-receiver distance, τ and v are intercept time and velocity, respectively. The goal is to estimate the coefficients $m(\tau, v)$ that honour the data $d(t, x)$. In our case, the data represents a common receiver gather. In other words, a collection of time series acquired by one seismic detector as a result of firing sources at a distance x from the detector.

To avoid notational clutter we will rewrite the operators in matrix-times-vector form as follows

$$\mathbf{m}' = \mathbf{L}'\mathbf{d} \tag{3}$$

and

$$\mathbf{d} = \mathbf{L}\mathbf{m}\,, \tag{4}$$

where $\mathbf{d}$ indicates the data in terms of a vector of length $N \times 1$. Similarly, $\mathbf{m}$ and $\mathbf{m}'$ are the $M \times 1$ vectors of Radon coefficients of the adjoint and synthesis operators, respectively. The linear operator $\mathbf{L}$ represents the forward Radon operator (synthesis operator) and $\mathbf{L}'$ its adjoint (analysis operator). Estimating $\mathbf{m}$ from $\mathbf{d}$ involves solving an ill-posed problem. In addition, one needs to consider that the seismic data are contaminated by a noise term $\mathbf{e}$. Therefore, one needs to solve the problem: $\mathbf{d} = \mathbf{L}\mathbf{m} + \mathbf{e}$. Assuming Gaussian noise, the vector of Radon coefficients is estimated via the solution of the following problem

$$\hat{\mathbf{m}} = \underset{\mathbf{m}}{\operatorname{argmin}} \, \|\mathbf{L}\mathbf{m} - \mathbf{d}\|_2^2 + \lambda R(\mathbf{m}) \tag{5}$$

where $R(\mathbf{m})$ is a regularization term and λ the trade-off parameter of the problem. Equation (5) is often minimized using iterative methods with sparsity promoting regularization terms like the l_1 or the Cauchy norms [7, 14].

3. SPARSE INVERSION OF THE RADON COEFFICIENTS IN THE PRESENCE OF ERRATIC NOISE

Simultaneous source acquisition leads to gathers contaminated with erratic source interferences [9]. We will consider that source interferences can be modelled via an erratic noise term that we will denote $\beta\mathbf{u}$

$$\mathbf{d} = \mathbf{L}\mathbf{m} + \beta\mathbf{u} + \mathbf{e}\,. \tag{6}$$

The latter is rewritten in this form

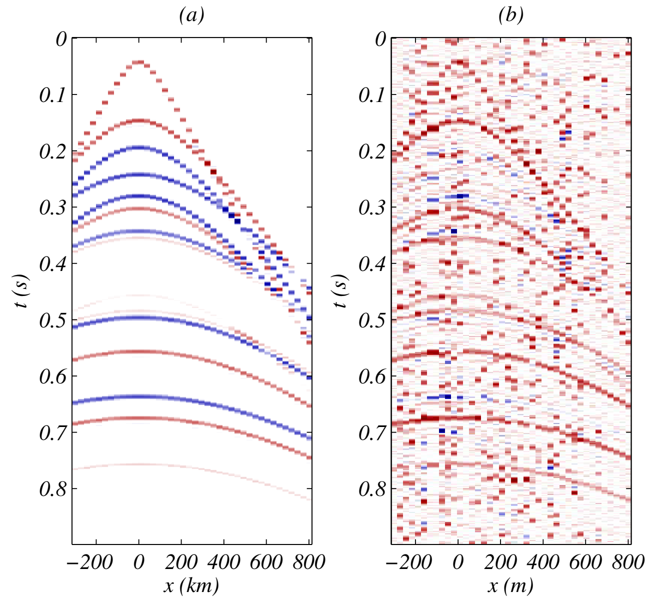

Fig. 1. (a) Ideal common receiver gather. Each column corresponds to the seismogram obtained by a source at distance x from the receiver. (b) Common receiver gather obtained via simultaneous source acquisition.

$$\mathbf{d} = \begin{pmatrix} \mathbf{L} & \beta\mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{m} \\ \mathbf{u} \end{pmatrix} + \mathbf{e}\,. \tag{7}$$

We consider the case where $\mathbf{m}$ and $\mathbf{u}$ are sparse signals. In addition, we consider that $\mathbf{e}$ is a vector of white noise with Gaussian distribution. The parameter β is needed to provide similar scale parameter to the unknown sparse vectors $\mathbf{m}$ and $\mathbf{u}$. In this article β is fixed but it is clear that this parameter should become part of the estimation process.

We propose the joint estimation of $\mathbf{m}$ and $\mathbf{u}$ via minimization of the following cost function (the standard l_2-l_1 problem) of the form

$$\hat{\mathbf{a}} = \underset{\mathbf{a}}{\operatorname{argmin}} \, \|\mathbf{A}\mathbf{a} - \mathbf{d}\|_2^2 + \lambda\|\mathbf{a}\|_1 \tag{8}$$

where

$$\mathbf{A} = \begin{pmatrix} \mathbf{L} & \beta\mathbf{I} \end{pmatrix}, \quad \mathbf{a} = \begin{pmatrix} \mathbf{m} \\ \mathbf{u} \end{pmatrix}. \tag{9}$$

Equation (8) is solved via the Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) [15]. However, one could have also adopted the iterative reweighed least-squares (IRLS) method [16, 17, 18].

Example

Figure 1(a) shows the ideal common receiver gather that one would have computed using conventional seismic acquisition.

Figure 1(b) is the gather one obtains by simultaneously firing 40 sources on one receiver. Figure 2(a) displays the l_2 norm of the error $\mathbf{e}$ (normalized by the noise variance) versus the tradeoff parameter λ. The vertical axis is the χ^2 functional that is used to estimate the optimal tradeoff parameter λ. In our test to estimate the parameter λ we have assumed that the standard error of the additive noise, σ_e, is known. This is needed for the χ^2-test:

$$\chi^2 = \frac{\|\mathbf{d} - \mathbf{L}\hat{\mathbf{m}}\|_2^2}{\sigma_e^2} \quad \text{and} \quad E[\chi^2] = N\,.$$

Figure 2(b) is the Pareto curve representing χ^2 (also the l_2 norm normalized by the variance of the noise) versus the l_1 norm. The optimum value of λ is chosen such that $\chi^2 = N$ where N is the number of observations. In this simulation, we also fixed the parameter $\beta = 5$ and run FISTA [15] to estimate the solution $\hat{\mathbf{a}}$. The solution $\hat{\mathbf{a}}$ is split in the sparse Radon panel, $\hat{\mathbf{m}}$ (Figure 3(a)) and the erratic noise interferences $\hat{\mathbf{u}}$ (Figure 3(b)). The inverted Radon coefficients $\hat{\mathbf{m}}$ were used to synthesize data free of interferences $\hat{\mathbf{d}} = \mathbf{L}\hat{\mathbf{m}}$. The synthesized data are shown in Figure 4(a). Figure 4(b) shows the reconstruction error panel magnified by 1000. We also compute the quality of the reconstruction via the following expression

$$Q = 20\log_{10}\frac{\|\mathbf{d}_{true}\|}{\|\mathbf{d}_{true} - \hat{\mathbf{d}}\|}\,.$$

For this particular example $Q = 43$ dB.

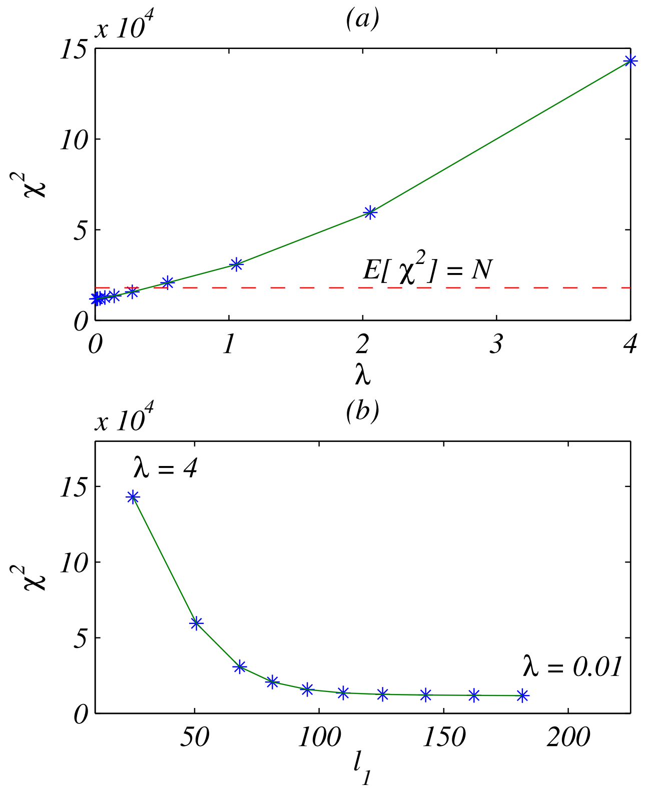

Fig. 2. (a) Misfit versus tradeoff parameter λ. The χ^2 test is used to estimate the optimal trade-off parameter which in this case corresponds to $\lambda \approx 0.5$. (b) Tradeoff curve (Pareto curve) indicating the behaviour of the l_2 norm of the error versus the l_1 norm of the solution for varying trade off parameter λ.

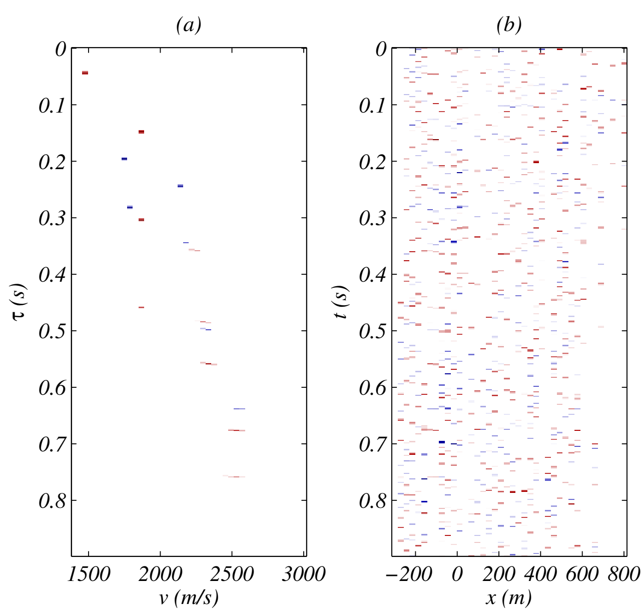

Fig. 3. (a) Estimated Radon coefficients $\hat{\mathbf{m}}$ from the data in Figure 1(b) . Each coefficient in $\tau - v$ represents a hyperbola in $t - x$. (b) Estimated incoherent noise $\hat{\mathbf{u}}$.

4. CONCLUSIONS

We have presented an algorithm for the joint estimation of a sparse model from data corrupted with Gaussian and impulsive noise. The problem was reduced to the joint estimation of a sparse vector of coefficients and a sparse vector of noise using the Fast Iterative Shrinkage-Thresholding Algorithm (FISTA). We have applied the algorithm to a synthetic example pertaining the unmixing of data acquired via a simultaneous seismic source acquisition experiment. The algorithm depends on two hyper-parameters λ (the trade-off parameter of the l_1-l_2 classical mixed norm minimization problem) and β, a parameter that is needed to consider scale differences in the desired Radon coefficients ($\mathbf{m}$) and erratic interferences ($\mathbf{u}$). The parameter λ was estimated using the χ^2 test. For our simulations β was assumed to be known a priori. Clearly, a statistical test to estimate β is also needed.

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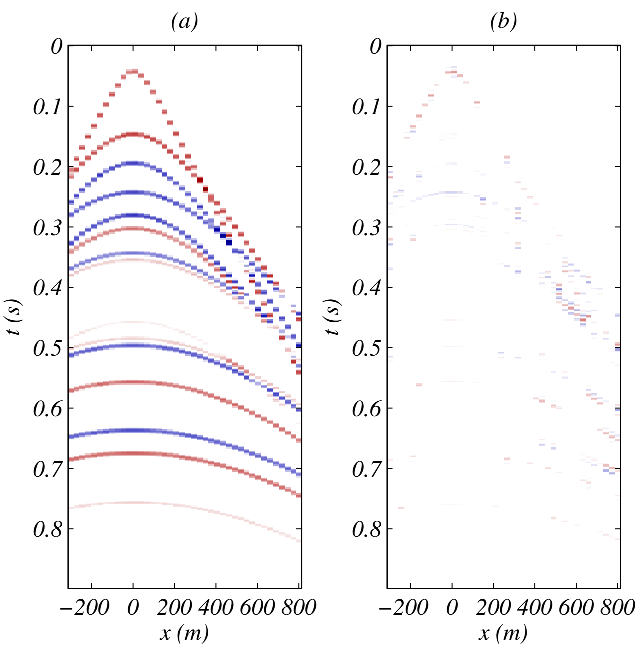

Fig. 4. (a) Noise-free data synthesized from the inverted Radon coefficients $\hat{\mathbf{m}}$ in Figure 3(a). (b) Error panel displaying the synthesized data minus the ideal data (Figure 1(a)), the error was multiplied by 1000 to improve visualization. The quality of the reconstruction is $Q = 43$ dB.

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