HIGH-RESOLUTION IMAGING OF SEISMIC DATA: HOW TO COMBINE WAVE-THEORY WITH SIGNAL PROCESSING TECHNIQUES

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ABSTRACT

Seismic migration aims to re-locate recorded seismic events to their true locations in the subsurface and requires a velocity model.. To improve the resolution of such a subsurface image, we employ ideas taken from Fresnel-aperture migration which uses low-frequency stationarity to select that part of data that coherently contribute to the final image. Thus, this technique offers an efficient way to window the coherent reflection energy which if being aligned can be input to a window-steered MUSIC approach with the potential of giving high-resolution seismic images.

***Index Terms*—** seismic imaging, MUSIC, resolution

1. INTRODUCTION

One major class of techniques employed to image seismic data is integral-equation (Kirchhoff type) migration (IEM) [1]. Like other imaging techniques its resolution power is diffraction limited. Moreover, the basic inherent assumption is that the larger migration aperture the better resolution. In case of reflections, this is not true in general and the use of a too large aperture may lead to severe degradation of the final seismic reconstruction. That part of the data which constructively contributes to an image point, is denoted the Fresnel aperture. In case of both reflections and diffractions, we use a technique introduced by [2] to identify the corresponding Fresnel apertures in an efficient manner. The Fresnel apertures of diffractions are significantly larger reflecting their higher resolution capabilities. Such windowed data can then be input to a standard IEM technique. However, in this paper we advocate for the use of a windowed-MUSIC approach to imaging. This implies that the Fresnel-aperture selected data are aligned or steered and combined with the high-coherency technique Multiple Signal Classification (MUSIC) introduced by [3].

2. FRESNEL APERTURE

If we consider the basic working principle of IEM, the image of a general subsurface point (acting either as a scatterer or as part of a reflector) is formed by adding seismic data values falling along a *time-diffraction curve* defining the migration operator. The concept is shown schematically in Fig.1 in case of a single reflector.

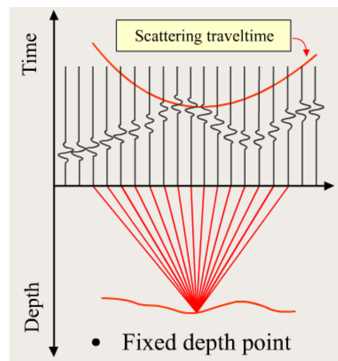

Fig.1 Schematics of the scattering traveltime (time-diffraction curve) representing the migration operator.

It is generally assumed that a large migration aperture, i.e. a migration operator acting on many traces simultaneously, is a needed requirement. However, this is only partly true. In the simple analysis to follow we will demonstrate that in case of seismic reflections, only a small area defined by the so-called Fresnel aperture is needed. However, in case of diffractions, the corresponding Fresnel aperture is much wider.

Figure 2 shows a simple reflection experiment within the zero-offset limit (coinciding source and receivers), where the time-diffraction curve corresponding to reflection point *R* is 'tangent' to the zero-offset reflected energy in a region surrounding the point of emergence of the corresponding specular ray at the receiver array. We will denote this tangential contact the *Fresnel aperture.* Thus the envelope of the cluster of seismic traces receiving a contribution from a common point on the reflector forms what we call here the Fresnel aperture. It is important to stress that the Fresnel aperture is a result of considering several distinct Fresnel zones belonging to different specular reflection points surrounding the actual one (i.e. around *R* in Fig.2 for instance). All these Fresnel zones have in common the actual specular reflection point. This is the only condition to include a neighboring Fresnel zone's trace in the Fresnel

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aperture. If one considers a fixed scattering point in the subsurface, only that part of the migration operator which is represented by the Fresnel aperture will contribute constructively to its image. In the following we will introduce an efficient way to isolate that part of the operator, thus we will window the operator.

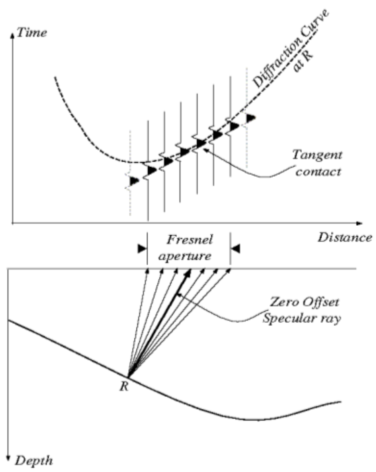


Fig.2 Definition of the Fresnel aperture (ZO case)

To illustrate this procedure, consider the simple zero-offset (ZO) synthetic data set as shown in Fig.3. It consists of a dipping reflector (20 degree) and a nearby scatterer, both embedded in a homogeneous background with a velocity of 2000m/s. A Ricker wavelet with a center frequency of 20Hz was used as the pulse. Fairly strong white-noise with a STD of about 10% of the maximum signal strength was added.

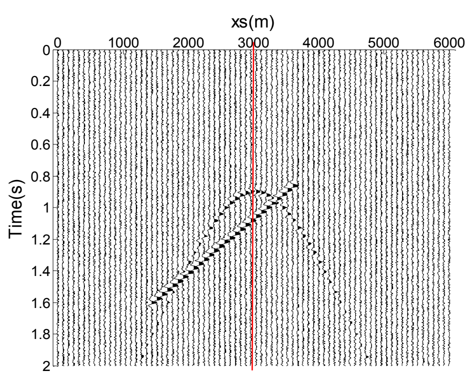


Fig.3 Synthetic ZO data set

Consider now the red vertical line in Fig.3 and let it represent the output position of a migrated trace (note, it also represents the lateral location of the scatterer in this example). Thus for each point along this line, the corresponding migration operator is calculated. If the data that falls along each operator are plotted horizontally for each image point along this line, a migration-operator panel is formed as shown in Fig.4a. Stacking the traces in this panel gives the corresponding image or migrated trace. This stacking process can be regarded as a spatial low-pass filtering of the data. Thus if the data corresponding to each operator are correspondingly low-pass filtered before stacking, the final migrated trace should virtually not change. In [2] a triangular smoothing filter is applied, and it is demonstrated using synthetic and field data that the final migrated image is virtually unchanged if such smoothening is applied or not. We apply the same smoothening approach to the operator panel in Fig.4a and obtain the filtered panel as shown in Fig.4b. This filtering can be regarded as a low-frequency stationarity process, leaving only that part of the migration operator that represents coherent contributions. *But such contributions are simply equivalent to the Fresnel apertures*. This can be easily seen from Fig.4 b where the Fresnel aperture for both the reflected event and the diffracted event are now enhanced. We can also see the difference between the aperture lengths, with that of a diffraction being much wider as expected. Since the Fresnel aperture associated with diffractions is considerably larger, it is another demonstration of the well-known fact that diffractions carry higher-resolution information than reflections.

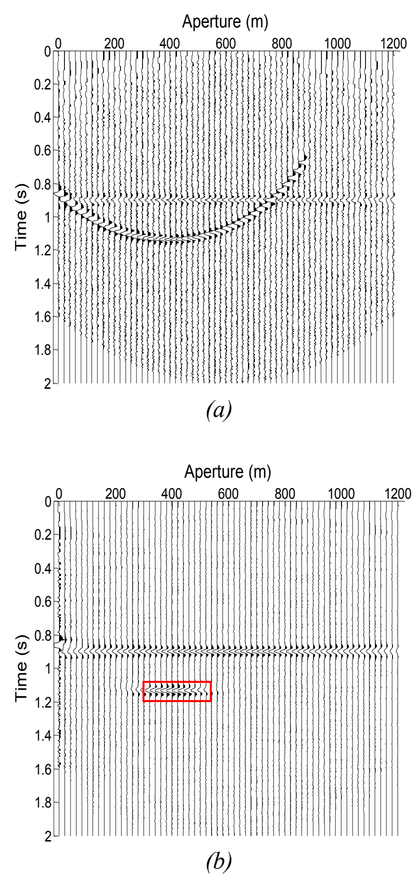


Fig.4 Migration-operator panel. (a) Raw data and (b) after filtering

By considering the full set of migration-operator panels after filtering, and by analogy with a velocity-analysis, the Fresnel apertures can be identified by coherency measures. For more details the reader is referred to [2].

3. WINDOWED-MUSIC APPROACH TO IMAGING

We assume in the following that accurate migration velocities are available. For such a case, both reflections and diffractions are (near) horizontalized inside the corresponding Fresnel aperture in the filtered migration operator panel (cf. Fig.4b). Define now a data window surrounding the aligned event in the operator panel with the length of the Fresnel aperture (N_r traces) and a width of N_t time samples representing a typical pulse length (cf. red rectangle in Fig.4b in case of the reflection). The corresponding aligned or steered data D inside this window can formally be decomposed in three different contributions (all matrices having dimension N_r x N_t)

$$D = A + \Delta A + N = \vec{m}_{av}\vec{u} + \Delta A + N \tag{1}$$

where A represents the average-trace contribution, ΔA the residual traces and N the noise (cf. Fig.5).

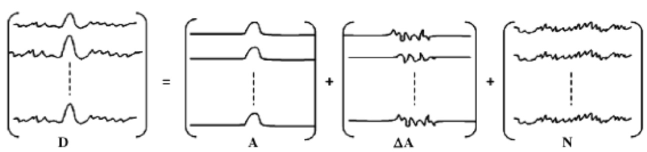


Fig.5 Decomposition of steered data matrix D

In Eq.(1), $\vec{m}_{av}$ is the average measurement vector associated with this window and $\vec{u}$ is a column vector of ones. The covariance matrix can now be calculated as

$$R = \frac{1}{N_t} D^T D \tag{2}$$

Assuming incoherency between the residual traces (each of them having an energy ε) and white noise with a variance σ_n^2, this latter expression can be further simplified (also combined with Eq.(1))

$$R \cong \frac{\|\vec{m}_{av}\|^2}{N_t}\vec{u}\vec{u}^T + \left(\frac{\varepsilon}{N_t} + \sigma_n^2\right) I \ . \tag{3}$$

It is now straightforward to show that the vector $\vec{u} / \sqrt{N_r}$ is an eigenvector of the matrix R. Singular-value decomposition of the correlation matrix R, when computed from the actual data window, gives formally

$$R = V_s \Sigma_s V_s^T + V_n \Sigma_n V_n^T \ , \tag{4}$$

where V_s and V_n are, respectively, the signal and nil subspace singular matrices, while Σ_s and Σ_n are the corresponding singular-value matrices.

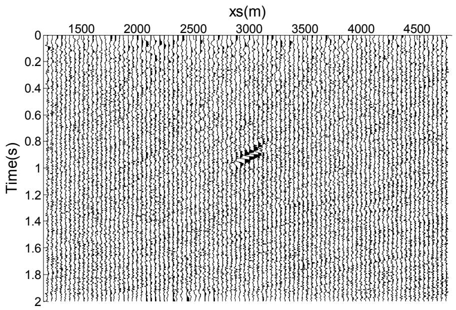
MUSIC is a high-resolution technique that requires a larger number of observations than features to be resolved. If this requirement is fulfilled, the correlation matrix can be singular-value decomposed into two orthogonal spaces (respectively, signal and nil spaces). In the case of steered MUSIC as discussed here, we have N_r multiple measurements of the same single horizontal event (reflection or diffraction within the Fresnel aperture). Thus, the fundamental requirement is well satisfied. Since, the normalized vector $\vec{u}$ is an eigenvector of the correlation matrix R, this corresponds to the use of a horizontal steering vector and thus implying frequency independency. This allows us to handle wideband seismic data as discussed by [4], who applied the concept of window-steered MUSIC to further improve the velocity analysis.
We are now in the position to construct the MUSIC coherency measure (pseudo-spectrum) at a given time t_0 and trace position m_0 (assuming one eigenvalue in signal space)

$$I(m_0, t_0) = \frac{\vec{u}^T \vec{u}}{\vec{u}^T P_n \vec{u}} \cdot w_{sb} \tag{4}$$

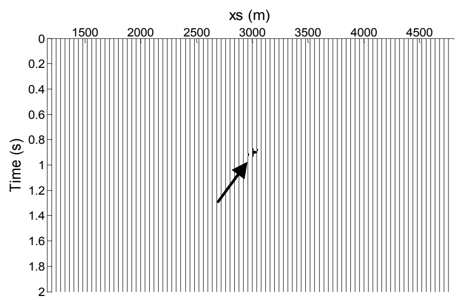
where $P_n = V_n V_n^T$ is the nil-space projection matrix and w_{sb} is a so-called semblance-balancing factor taking into account that MUSIC yields amplitudes (pseudo-spectra) of arbitrary values [5]. For further details the reader is referred to [6].

4. SIMPLE DEMONSTRATION OF PRINCIPLES

In order to demonstrate the main principles, we will employ the same simple test data (cf. Fig.3) as before, but image the reflection and the diffraction separately. This will often be the case in practice, since the diffracted energy is quite low and normally masked by the stronger reflections. However, diffractions can be imaged separately after application of a diffraction-enhancement technique [7],[8],[9]. As discussed before (cf. Fig.4b), the Fresnel aperture of a diffracted event is significantly larger than in case of a reflection. Thus quite a large migration aperture (operator) is needed if a good image is to be obtained employing migration. In the example shown here, we used a Fresnel aperture width corresponding to the reflected event in Fig.4b, thus far too short. The corresponding migrated image obtained is shown in Fig.6a. The focus of the scattering energy is seen to be rather poor as expected due to the limited aperture. In addition, the effect of the noise is quite prominent since the migration operator is rather short. Next, employing the windowed MUSIC approach with data input again limited to the same aperture, gave the image shown in Fig.6b. The effect of noise has been virtually eliminated and the focus is now well reconstructed. *Thus a short Fresnel aperture can also be employed for diffractions in case of MUSIC imaging.* For further discussions about imaging of diffractions the reader is referred to [6] and [10]. In these works, a resolution beyond the classical diffraction limit of Rayleigh has been demonstrated employing the windowed MUSIC approach.



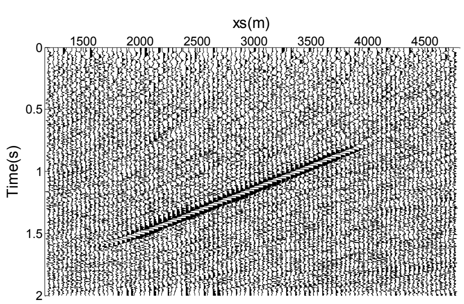
(a)



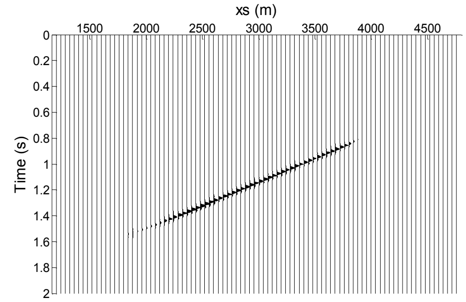
(b)

Fig.6 (a) Kirchhoff migration and (b) MUSIC imaging of diffraction.

Next, we continue to image the reflection response and use the same Fresnel-aperture length as in case of the diffraction, i.e. a length which is now tailored for the event considered. Reflections do not carry the same potential resolution as the diffracted events, since they represent an ensemble response of a series of infinitesimal scatterers. However, use of the windowed MUSIC technique can still improve the S/N as well as the sharpness of the reflector. This is demonstrated in Figs.7a and b, where the MUSIC image is seen to be quite superior in quality and resolution.



(a)



(b)

Fig.7 (a) Kirchhoff migration and (b) MUSIC imaging of reflection.

5. CONCLUDING REMARKS

This paper has demonstrated that the resolution power of standard wave-equation imaging of seismic data can be further enhanced if combined with techniques adapted from the signal processing community. More specifically, recovering the Fresnel apertures corresponding to stationarity in the seismic data makes it possible to reduce distortions during the reconstruction process. In particular, when combined with the high-coherency technique MUSIC considerable improvements in the seismic resolution can be obtained. This observation applies for both reflections and diffractions, but with the latter ones being superior when it comes to the finer details.

6. RELATION TO PRIOR WORK

Improving the resolution of seismic images, has been an area of research for many years. Various approaches have been introduced and investigated, among them 1-D deconvolution techniques [11], 2-D deconvolution technques [12],[13] and Least-Squares migration [14].
Combination of wave-theory and high-resolution techniques like MUSIC has been investigated by several groups [15],[16],[17],[18] where the problem in common has been to image a sparse set of scatterers. The work presented here (in combination with other recent publications [6],[10]) can be regarded as an extension of these works to cover the case of seismic data. For such data, the scattering contributions are not sparse and reflections are major information carriers. The key to success has been to introduce a windowed-MUSIC approach to imaging. This ensures that the sparsity condition is fulfilled. The idea of window-steered MUSIC was first investigated by [4]. However, in this earlier the area of application was not within seismic imaging but velocity analysis.

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