SEARCH-FREE DECENTRALIZED DIRECTION-OF-ARRIVAL ESTIMATION USING COMMON ROOTS FOR NON-COHERENT PARTLY CALIBRATED ARRAYS

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ABSTRACT

We consider decentralized direction-of-arrival (DoA) estimation for large partly calibrated arrays composed of multiple fully calibrated uniform linear subarrays. Due to the difficulty of maintaining coherence between signals received in widely separated subarrays, the practical case of non-coherent subarrays is investigated. Our novel approach for decentralized and non-coherent DoA estimation is based on finding the common roots (CRs) of multiple univariate polynomials corresponding to individual subarrays. We propose two algorithms using generalized Sylvester matrix to find the CRs and to estimate the DoAs. The proposed algorithms substantially reduce communication and computation costs compared to traditional centralized DoA estimation methods. Moreover, simulation results demonstrate that our algorithms outperform existing decentralized methods and can deal with possible DoA estimation ambiguities caused by subarray geometries.

Index Terms— decentralized DoA estimation, generalized Sylvester matrix, root-MUSIC.

1. INTRODUCTION

Direction-of-arrival (DoA) estimation using sensor arrays is essential for many applications such as radar, sonar, underwater surveillance, and seismic exploration [1], [2]. Centralized subspace-based algorithms such as MUSIC [3], MODE [4], and WSF [5] exhibit super resolution property and efficient performance. However, these algorithms must process all sensor outputs coherently. Moreover, in these algorithms, the precise knowledge of all sensor locations is required, i.e., the array has to be fully calibrated. In the case of partly calibrated arrays with unknown subarray displacements, other subspace-based algorithms, such as those developed in [6]-[9], can be applied. These algorithms also perform centralized coherent processing. In large sensor systems, decentralized computation is preferable, as it significantly reduces the computational load at the fusion center (FC), or even eliminates the need for FC. Decentralized DoA estimation using matrix completion is presented in [10] for the case of coherent fully calibrated arrays. Considering the same case, algorithms based on consensus operations are introduced in [11]-[13] to estimate sample covariance matrix and to perform subspace tracking without requiring FC. For partly calibrated arrays, a decentralized-ESPRIT approach is introduced in [14] to perform coherent DoA estimation.

However, in large arrays, coherent processing may not be feasible as it becomes very difficult to maintain coherence between signals received in widely separated sensors. Hence, it is necessary to resort to decentralized processing where non-coherent large arrays are split into a number of smaller coherent fully calibrated subarrays. Each subarray locally processes its own measured data coherently and sends the results to FC. The FC uses these results in a non-coherent manner to estimate the DoAs. In [15], a decentralized version of MUSIC algorithm is used for localization. In this algorithm, each subarray sends its locally estimated signal- and noise-subspace matrices to the FC, where the decentralized MUSIC spectrum is computed as the summation of individual subarray spectra. Generalized MUSIC method in [16] performs similar processing only to estimate DoAs. In [17], another version of decentralized MUSIC is analyzed. In this approach, the subarrays send the locally estimated DoAs and their estimated variances to the FC, which linearly combines the local estimates after weighting them by their estimated variances. A similar method which is robust against distributional uncertainties in noise model is presented in [18].

In this paper, a novel decentralized DoA estimation method based on common root (CR) finding is proposed. It is assumed, in Section 3.1, that each subarray computes a polynomial and communicates it to the FC, where the signal roots are estimated from the CRs of these polynomials. Based on Sylvester matrix introduced in Section 3.2, we propose two algorithms for computing the CRs in Sections 3.3 and 3.4. These algorithms reduce computational complexity as discussed in Section 4. The simulation results in Section 5 display that, the proposed algorithms have enhanced resolution capabilities and their performance stays close to the Cramér Rao bound (CRB) asymptotically. Moreover, these algorithms, as shown in Section 5, are able to overcome ambiguities in estimation resulting from ambiguous subarrays (e.g., subarrays with inter-sensor spacing larger than half-

This work was supported by the LOEWE Priority Program Cocoon (http://www.cocoon.tu-darmstadt.de).

wavelength of narrowband signals) as long as at least one subarray is able to estimate the DoAs unambiguously.

2. SIGNAL MODEL

Consider a sensor array composed of K identically-oriented non-overlapping uniform linear subarrays. The inter-sensor spacing at the kth subarray d_k is assumed to be an integer multiple of a smaller distance d, i.e., $d_k = n_k d$ for $n_k \in \mathbb{N}$, where d is measured in wavelength λ. The number of sensors in the kth subarray is M_k. The displacements between the subarrays are assumed to be unknown. Consider L narrowband uncorrelated far-field sources impinging on the array from directions $\theta_1, \ldots, \theta_L$. We assume that each subarray can identify at least L sources. Under these assumptions, the response of the kth subarray (i.e., its manifold vector) to a source at direction θ relative to array broadside is $\boldsymbol{a}_k(z) = \left[1, z^{n_k}, \ldots, z^{n_k M_k - 1}\right]^T$ where $z \triangleq e^{j2\pi d \sin\theta}$. The output of the kth subarray can be formulated as

$$\boldsymbol{y}_k(t) = \boldsymbol{A}_k \boldsymbol{x}(t) + \boldsymbol{n}_k(t), \tag{1}$$

where $\boldsymbol{A}_k = [\boldsymbol{a}_k(\theta_1), \ldots, \boldsymbol{a}_k(\theta_L)]$ is the $M_k \times L$ steering matrix, $\boldsymbol{x}(t)$ is the $L \times 1$ baseband signal vector of L sources, and $\boldsymbol{n}_k(t)$ is the $M_k \times 1$ noise vector. The noise vector $\boldsymbol{n}_k(t)$ is assumed to be white Gaussian with variance $\sigma^2 \boldsymbol{I}_{M_k}$ where $\boldsymbol{I}_{M_k}$ is $M_k \times M_k$ identity matrix.

The covariance matrix for the kth subarray is defined as

$$\boldsymbol{R}_k \triangleq E[\boldsymbol{y}_k(t)\boldsymbol{y}_k^H(t)] = \boldsymbol{A}_k \boldsymbol{R}_{\mathrm{xx}} \boldsymbol{A}_k^H + \sigma^2 \boldsymbol{I}_{M_k}, \tag{2}$$

where $E[\cdot]$ stands for the statistical expectation, $(\cdot)^H$ denotes the conjugate transpose, and $\boldsymbol{R}_{\mathrm{xx}} \triangleq E[\boldsymbol{x}(t)\boldsymbol{x}^H(t)]$ is the signal covariance matrix. After eigendecomposition of $\boldsymbol{R}_k$, its eigenvalues in decreasing order are $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_L \geq \lambda_{L+1} = \ldots = \lambda_{M_k} = \sigma^2$, and their corresponding eigenvectors are $\boldsymbol{e}_1, \ldots, \boldsymbol{e}_{M_k}$. Then, we can write

$$\boldsymbol{R}_k = \boldsymbol{E}_k \boldsymbol{\Lambda}_k \boldsymbol{E}_k^H + \boldsymbol{G}_k \boldsymbol{\Gamma}_k \boldsymbol{G}_k^H, \tag{3}$$

where the diagonal matrices $\boldsymbol{\Lambda}_k = \mathrm{diag}\{\lambda_1 \ldots, \lambda_L\}$ and $\boldsymbol{\Gamma}_k = \mathrm{diag}\{\lambda_{L+1} \ldots, \lambda_{M_k}\}$ contain the so-called signal and noise eigenvalues, respectively, $\boldsymbol{E}_k = [\boldsymbol{e}_1, \ldots, \boldsymbol{e}_L]$ and $\boldsymbol{G}_k = [\boldsymbol{e}_{L+1}, \ldots, \boldsymbol{e}_{M_k}]$ are the signal- and noise-subspace matrices, respectively.

In practice, the covariance matrix is not available and its finite sample estimate $\hat{\boldsymbol{R}}_k = \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{y}_k(t)\boldsymbol{y}_k^H(t)$ is used where N is the number of the available snapshots. Let $\hat{\boldsymbol{E}}_k, \hat{\boldsymbol{G}}_k, \hat{\boldsymbol{\Lambda}}_k$, and $\hat{\boldsymbol{\Gamma}}_k$, obtained from the eigendecomposition of the sample covariance matrix $\hat{\boldsymbol{R}}_k$, be the estimates of $\boldsymbol{E}_k, \boldsymbol{G}_k, \boldsymbol{\Lambda}_k$, and $\boldsymbol{\Gamma}_k$, respectively.

3. DECENTRALIZED DOA ESTIMATION

In this section, we propose a novel decentralized rooting-based DoA estimation method. Our approach is divided into two steps. First, each subarray computes a polynomial locally, and sends it to the FC. Second, at the FC, the CRs of K polynomials are computed using generalized Sylvester matrix (described in more details in Section 3.2) and the DoAs are, then, estimated from the CRs.

3.1. Computing Local Polynomials

Using its locally estimated noise-subspace, the kth subarray can compute, as in root-MUSIC [19], the following polynomial

$$\hat{P}_k(z) = \boldsymbol{a}_k^H(z) \hat{\boldsymbol{G}}_k \hat{\boldsymbol{G}}_k^H \boldsymbol{a}_k(z). \tag{4}$$

In analogy to root-MUSIC, $\hat{P}_k(z)$ has $2n_k(M_k - 1)$ roots which occur in pairs. More precisely, if $\hat{z}_i$ is a root of $\hat{P}_k(z)$, then its conjugate reciprocal $1/\hat{z}_i^*$ is also a root of $\hat{P}_k(z)$. Thus, half of the roots of $\hat{P}_k(z)$ are inside the unit circle (UC) and half are outside the UC. Note that, for $n_k = 1$, $\hat{P}_k(z)$ reduces to the root-MUSIC polynomial. If $n_k > 1$ then $\hat{P}_k(z)$ has n_k times more roots than conventional root-MUSIC, resulting in ambiguities in DoA estimation. The conventional decentralized MUSIC can not deal with these ambiguities. However, we show in the following sections that our CR-finding approach is able to eliminate the extra roots and to estimate the DoAs if at least one subarray is able to estimate unambiguously.

The polynomial in (4) can be written as a multiplication of two polynomials

$$\hat{P}_k(z) = \hat{\hat{P}}_k(z) \hat{\underset{\sim}{P}}_k(z), \tag{5}$$

where $\hat{\hat{P}}_k(z)$ and $\hat{\underset{\sim}{P}}_k(z)$ are formed from the outside and the inside the UC roots of $\hat{P}_k(z)$, respectively. Due to the conjugate reciprocal property of the root pairs, both polynomials contain the same spatial information. Therefore, one of the polynomials is sufficient to estimate the DoAs. We choose $\hat{\hat{P}}_k(z)$, since its (outside the UC) roots are more distant hence more distinguishable than the (inside the UC) roots of $\hat{\underset{\sim}{P}}_k(z)$. Let

$$\hat{\hat{P}}_k(z) = \hat{c}_{k,0} + \hat{c}_{k,1} z + \ldots + z^{D_k}, \quad k = 1, \ldots, K \tag{6}$$

where $\hat{c}_{k,i}$ for $i = 0, \ldots, D_k - 1$ is the complex coefficient of $\hat{\hat{P}}_k(z)$ scaled such that $\hat{c}_{k,D_k} = 1$, and $D_k \triangleq n_k(M_k - 1)$ is the degree of $\hat{\hat{P}}_k(z)$. Only these coefficients are sent to the FC, thus the communication cost per subarray is $D_k = n_k(M_k - 1)$. In centralized processing where all the measurements are sent to FC, the communication cost per subarray is $M_k N$. Thus, the proposed communication scheme reduces the communication cost, since normally $N \gg n_k$.

Sorting the roots of $\hat{\hat{P}}_k(z)$ such that $|\hat{z}_1| \leq \ldots \leq |\hat{z}_{D_k}|$, the smallest L roots are called the signal roots (containing the source DoA information) and the remaining $D_k - L$ roots are called the noise roots. Since each subarray can identify the L sources, all the subarrays share the same signal roots. Thus, the CRs of all local polynomials in the set $\mathcal{P} \triangleq \{\hat{\hat{P}}_k(z)\}_{k=1}^K$ contain L signal roots, and can be used for DoA estimation.

Having received all the K local polynomials from the subarrays, the FC uses generalized Sylvester matrix to estimate the CRs.

3.2. Generalized Sylvester Matrix

Originally, Sylvester matrix is defined for two polynomials [20], [21]. In [22]-[24] generalizations for Sylvester matrix are introduced for more than two polynomials. In the following, we use generalized matrix from [24] as it has the smallest size compared to generalizations defined in [22] and [23].

Assume without loss of generality that $D_1 \geq D_2 \geq \ldots \geq D_K$. Generalized Sylvester matrix $\boldsymbol{S}$ consists of K blocks, i.e.,

$$\boldsymbol{S} = \left[\boldsymbol{S}_1^T, \boldsymbol{S}_2^T, \ldots, \boldsymbol{S}_K^T\right]^T \in \mathbb{C}^{r\times h}, \tag{7}$$

where $r = (K-1)D_K + D_1$, and $h = D_1 + D_K$. The first $K-1$ blocks correspond to the first $K-1$ polynomials such that, for $k = 1, \ldots, K-1$

$$\boldsymbol{S}_k = \left[\tilde{\boldsymbol{c}}_{k,D_K-1}^T, \tilde{\boldsymbol{c}}_{k,D_K-2}^T, \ldots, \tilde{\boldsymbol{c}}_{k,0}^T\right]^T \in \mathbb{C}^{D_K\times h}, \tag{8}$$

where each row $\tilde{\boldsymbol{c}}_{k,m} \triangleq [0, \ldots, 0, \hat{c}_{k,0}, \ldots, \hat{c}_{k,D_k}, 0, \ldots, 0]$ contains zeros at the first $h-m-D_k-1$ and the last m entries. The Kth block corresponds to the polynomial with the smallest degree D_K,

$$\boldsymbol{S}_K = \left[\tilde{\boldsymbol{c}}_{K,D_1-1}^T, \tilde{\boldsymbol{c}}_{K,D_1-2}^T, \ldots, \tilde{\boldsymbol{c}}_{K,0}^T\right]^T \in \mathbb{C}^{D_1\times h}. \tag{9}$$

Sylvester matrix $\boldsymbol{S}$ is of rank h and it drops rank if and only if the set of polynomials $\mathcal{P}$ has at least one CR [24]. More precisely, if polynomials in the set $\mathcal{P}$ have n CRs then

$$\text{rank}(\boldsymbol{S}) = h - n. \tag{10}$$

If the polynomials in $\mathcal{P}$ were exact (or in array processing context $N \to \infty$), then equation (10) implies that $\boldsymbol{S}$ would have exactly n zero singular values. In such case, the CRs can be computed using triangularization [24]. However, the polynomials are not exact and the signal roots are only approximately similar, thus, $\boldsymbol{S}$ will have n small but non-zero singular values. Based on our simulations (not shown in here), the use of triangularization method for computing the CRs and, consequently, estimating the DoAs results in poor asymptotic performance. Therefore, in the following, we propose two algorithms to estimate the CRs of polynomials in $\mathcal{P}$.

3.3. Algorithm I

Let z_i for $i = 1, \ldots, n$ be one CR of polynomials in $\mathcal{P}$ and $\mathcal{N}(\boldsymbol{S})$ be the null space of $\boldsymbol{S}$. Then, the Vandermonde vector $\boldsymbol{z}_i = \left[1, z_i, z_i^2, \ldots, z_i^{h-1}\right]^T$ belongs to $\mathcal{N}(\boldsymbol{S})$, i.e., $\boldsymbol{S}\,\boldsymbol{z}_i = \boldsymbol{0}$. Thus, for all the CRs $z_1, \ldots, z_n$, matrix

$$\boldsymbol{Z}_n = [\boldsymbol{z}_1, \ldots, \boldsymbol{z}_n] \in \mathbb{C}^{h\times n}, \tag{11}$$

forms a set of basis for $\mathcal{N}(\boldsymbol{S})$. Let $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$ be the first n right singular vectors of $\boldsymbol{S}$ (corresponding to the n smallest singular values), then matrix $\boldsymbol{V}_n = [\hat{\boldsymbol{v}}_1, \ldots, \hat{\boldsymbol{v}}_n]$ also forms a set of basis for $\mathcal{N}(\boldsymbol{S})$. Therefore, from equation (10) it can be inferred that $\boldsymbol{V}_n$ and $\boldsymbol{Z}_n$ span the same subspace. Moreover, $\boldsymbol{Z}_n$ has a Vandermonde structure, thus ESPRIT algorithm [6] can be used to estimate the n CRs from $\boldsymbol{V}_n$ [25]. In analogy to [25], we form two matrices $\overline{\boldsymbol{V}_n}$ and $\underline{\boldsymbol{V}_n}$ by deleting the first and the last rows of $\boldsymbol{V}_n$, respectively. The n sought roots are, then, the eigenvalues of matrix

$$\boldsymbol{\Phi} = \left(\underline{\boldsymbol{V}_n}^H \underline{\boldsymbol{V}_n}\right)^{-1} \underline{\boldsymbol{V}_n}^H \overline{\boldsymbol{V}_n}. \tag{12}$$

Note that if the subarrays are not identical, then as discussed in Section 3.1, the polynomials in $\mathcal{P}$ have at least L CRs. However, in low SNRs some of the noise roots corresponding to the subarray(s) with the largest number of sensors may cause the matrix $\boldsymbol{S}$ to be rank deficient. Consequently, we suggest to expand the estimated $\mathcal{N}(\boldsymbol{S})$ for $n = L, L+1, \ldots, h$. For each value of n, we compute n roots from the eigenvalues of $\boldsymbol{\Phi}$ in equation (12). Let the remainders of the polynomials in $\mathcal{P}$ at each of the n roots be

$$\text{Rem}_{n,i} = \sum_{k=1}^{K} \left|\hat{\tilde{P}}_k(z_i)\right|_2^2, \quad i = 1, \ldots, n. \tag{13}$$

$\text{Rem}_{n,i}$ quantifies the quality of each estimated root for different values of n. We define the remainder Rem_n as the sum of the L smallest $\{\text{Rem}_{n,i}\}_{i=1}^n$ at each value of n. The L roots that minimize the remainder Rem_n are chosen as the estimates of the L common signal roots. We should point out that if all the subarrays are identical, then the set of CRs contains not only the signal roots but also the noise roots, i.e., $n = D_1 = D_2 = \ldots = D_K$. In this special case, the L roots which are closer to the UC are chosen.

Having estimated L signal roots as $\hat{z}_l$ for $l = 1, \ldots, L$ at the FC, the DoAs can then be computed from

$$\hat{\theta}_l = \sin^{-1}\left(\frac{\arg(\hat{z}_l)}{2\pi d}\right), \quad l = 1, \ldots, L. \tag{14}$$

3.4. Algorithm II

Let $\boldsymbol{z} \triangleq \left[1, z, z^2, \ldots, z^{h-1}\right]^T$, then the following function

$$f(z) = \|\boldsymbol{S}\,\boldsymbol{z}\|_2^2 = \boldsymbol{z}^H \boldsymbol{S}^H \boldsymbol{S}\,\boldsymbol{z} \tag{15}$$

is exactly zero whenever z is equal to one of the CRs of polynomials in $\mathcal{P}$, since $\boldsymbol{S}\,\boldsymbol{z}\,|_{z=z_i} = \boldsymbol{0}$ for $i = 1, \ldots, L$. Therefore, the CRs of polynomials in $\mathcal{P}$ can be estimated by minimizing $f(z)$. This can be achieved by rooting $f(z)$. However, $f(z)$ has $2(h-1)$ roots, and similar to equation (4), these roots occur in pairs as we explained in Section 3.1. Therefore, the L signal roots are chosen in two steps. First, the $h-1$ roots of $f(z)$ which are outside the UC is selected. Second, the remainder defined in equation (13) is calculated for the $h-1$ roots, and the L roots with the smallest remainder are chosen. We should remark that, choosing the L roots closest to the UC yield almost the same performance. Having estimated the L signal roots, the DoAs can be estimated from equation (14).

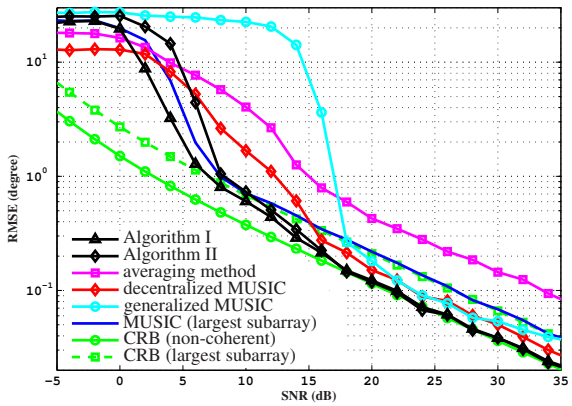

Fig. 1. DoA estimation performance (RMSE) vs. SNR in unambiguous scenario.

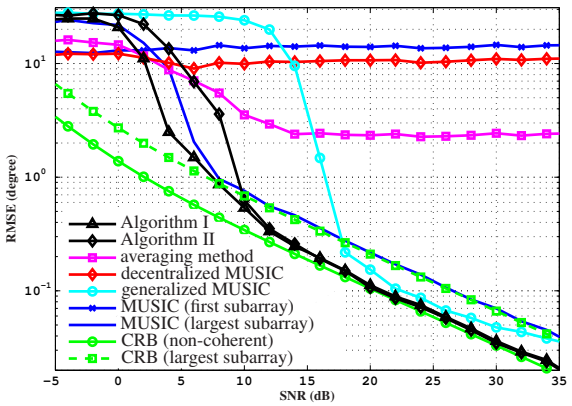

Fig. 2. DoA estimation performance (RMSE) vs. SNR in ambiguous scenario.

4. COMPLEXITY ANALYSIS

We investigate the computational cost of the proposed algorithms at the FC and their dependency on the number of subarrays K. Our focus on the number of subarrays in complexity analysis is due to the fact that large arrays can be build using large number of subarrays each containing moderate number of sensors. To simplify the comparison, it is assumed that the subarray sizes are not widely different. The computational cost of Algorithm I is dominated by calculation of singular value decomposition of Sylvester matrix $\boldsymbol{S}$ which according to [26, p. 215] costs $O(rh^2) = O(KD_K^3)$. The remaining steps depends only on the estimated $\mathcal{N}(\boldsymbol{S})$ which in the worst case has the size of $h \times h$ and does not depend on K. In Algorithm II, the cost is dominated by the multiplication $\boldsymbol{S}^H\boldsymbol{S}$ which is $O(KD_K^3)$. The other step in Algorithm II is rooting a polynomial of degree $2(h-1)$ and does not depend on K. In centralized processing, computing the covariance matrix of size $D_K K \times D_K K$ and its eigendecomposition cost $O(D_K^2 K^2 N)$ and $O(K^3 D_K^3)$, respectively. It can be seen that the complexity of the proposed algorithms scales linearly with the number of subarrays K, while the complexity of centralized processing increases by K^3.

5. SIMULATION RESULTS

Assume a sensor array consisting of 6 uniform linear subarrays with 4, 5, 6, 7, 7, and 7 sensors. The positions of reference sensors of 6 subarrays measured in wavelength are $(0,0)$, $(0.3,0.5)$, $(-0.4,0.4)$, $(1.1,0.91)$, $(1.2,0.61)$, and $(1.5,0.9)$, respectively. The inter-sensor spacing in all the subarrays is taken to be the signal half-wavelength $d_1 = \cdots = d_K = \frac{\lambda}{2}$. Three uncorrelated Gaussian equal-power sources impinge on the array from directions $-3.32°$, $1.41°$, and $17.85°$. A number of $N = 50$ snapshots are collected at each subarray. We compare the two proposed algorithms with the following algorithms: (1) averaging method, where the subarrays use root-MUSIC method to estimate the DoAs and the FC average these estimates, (2) decentralized MUSIC method of [17] which uses averaging of the local estimates weighted by their estimated variances as defined in [17], (3) generalized spectral MUSIC method in [16] using a search grid with granularity of $0.1°$. All algorithms are compared to the CRB for non-coherent processing defined as $\text{CRB} = \left(\sum_{k=1}^{K} \boldsymbol{C}_k^{-1}\right)^{-1}$ [17], where matrix $\boldsymbol{C}_k$ is the CRB matrix corresponding to the kth subarray as defined in [27]. In our simulation, 200 Monte Carlo runs are used.

Fig. 1 shows the root-mean-square-error (RMSE) of the DoA estimates versus SNR. It is obvious that the performance of the two proposed algorithms stay close to the CRB and outperform the three competing methods at high SNRs as well. The proposed methods also have better threshold performance when compared to other algorithms. The resolution probability percentage as defined in [8] is also considered (the related figure is not shown due to lack of space). We found that in the threshold region, the proposed algorithms have the best performance. For example, at SNR=5dB both proposed algorithms have resolution probability over 95%. The next best algorithm is decentralized MUSIC method with resolution probability of around 65%.

Fig. 2 shows the RMSE for the same setup as described above except that $d_1 = \lambda$. Thus, the first subarray is unable to identify the sources unambiguously. As it can be seen in this figure, the ambiguous estimation of the first subarray affects the performance of averaging and decentralized MUSIC methods. However, our proposed algorithms are still able to resolve the sources, since the CRs of all the subarray polynomials remain unchanged.

In the previous scenarios, Algorithm I displays better threshold performance than Algorithm II. However, Algorithm II performs slightly better in the asymptotic region.

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