

ML ESTIMATION AND CRB FOR NARROWBAND AR SIGNALS ON A SENSOR ARRAY

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ABSTRACT

This paper considers the exploitation of temporal correlation in incident sources in a narrowband array processing scenario. The MLE and CRB are derived for parameter estimation of spatially uncorrelated first order Gaussian autoregressive source signals with additive Gaussian spatially and temporally uncorrelated sensor noise. These are compared to the MLE and CRB for the usual uncorrelated (WN) sources model. The paper deals with the case where the number of data snapshots is small. Numerical simulations show that (i) there is no significant performance gain in the correlated signal case, and significantly, (ii) the WN MLE performance does degrade in the presence of source correlation, which appears to be in contrast to some recently published work.

Index Terms— array signal processing, direction-of-arrival estimation, autoregressive models, maximum likelihood

1. INTRODUCTION

Conventional likelihood based narrowband sensor array signal estimation generally assumes that the incident signals are either (i) deterministic and unknown, or (ii) realisations of zero-mean temporally uncorrelated wide-sense stationary Gaussian random processes with known (spatial) covariance. Maximum Likelihood estimation (MLE) for the signals' angles-of-arrival (AoA) for these models is a conventional approach, and its performance has been analysed in detail (see e.g. [2]). In particular, there is a well-known "threshold" phenomenon whereby the performance of the MLE deviates markedly from the associated Cramer-Rao bound (CRB) below a certain Signal-to-Noise ratio and/or number of independent data "snapshots". Although the MLE offers superior threshold performance, its computational requirements have led to many other approaches, such as signal subspace (e.g. ESPRIT [3]) or noise subspace (e.g. MUSIC [4]) based methods. These methods offer similar performance to MLE for large number of data snapshots, but their thresholding behaviour is significantly worse. Thus in investigating the performance of various AoA estimators, the criteria used are the CRB associated with the particular signal model (the best attainable "above-threshold" performance), and the SNR, or number of snapshots when thresholding occurs.

1.1. Background and Motivation

In practical scenarios, the signals incident on the array may not be well-modelled at baseband as independent (white noise) samples. Typically, the data collection system selects a passband filter and appropriate sample rate to yield Nyquist sampling for the largest bandwidth signal incident on the array at the specified carrier frequency.

Other signals, which may possess smaller bandwidths, will thus generally yield correlated samples at baseband. It is therefore natural to ask whether this correlation can be exploited in the design of an estimator for all incident signals' AoAs. Studies such as [7] have shown that the MLE designed for independent Gaussian data samples is robust to the presence of temporal correlation in the signals' samples, however there are no detailed studies concerning the performance of the MLE designed specifically for correlated signals. Some results are available concerning the CRB for correlated signals, when the spatio-temporal source correlation matrix is known [5], and these results show that the CRB for correlated sources is lower than that for uncorrelated sources. However for large number of data snapshots, the difference between these CRBs becomes smaller. The thresholding behaviour of the correlated sources MLE has not been studied. *We are specifically interested in those processing scenarios where computational complexity is not a limiting factor and that computationally intensive techniques such as the correlated sources MLE can be justified if better performance can be obtained.* It should be pointed out at this stage, that a signal state-space based approach using the Expectation-Maximisation (EM) algorithm for ML AoA estimation of general linear Gauss-Markov sources with known models, has been proposed in [8]. This algorithm iteratively applies a fixed-interval Kalman smoother to perform estimation of the signals, and a likelihood based method using the resulting signal estimates to find the AoAs.

1.2. Existing Work

Since we are interested in potential exploitation of source correlation, we will focus on this issue in this paper. Source correlation was addressed specifically as its main focus in [6] where it was concluded that covariance based estimators designed for uncorrelated sources (including the MLE) are robust to temporal source correlation when the additive sensor noise is itself temporally uncorrelated (the case considered in this paper). Methods designed to use temporal correlation to advantage are described in [10] and [5], however these papers do not consider the performance attained by an MLE specifically designed for the temporally correlated sources case, but rather an approximation method which intrinsically estimates temporal source correlation lags. However, these papers do present the CRB for the temporally correlated source case, and show that it lower than the corresponding CRB for the temporally uncorrelated (white) source case. In [7], simulation results which include numerical studies of the behaviour of the white sources based MLE when temporal correlation is present, show that this MLE is indeed robust to the presence of source correlation. The paper claims that the performance so attained is similar to that obtained "for techniques specifically designed for the circumstance". In this respect, they refer to [11] which does not address the MLE for the temporally correlated source case,

but rather the MLE for the case where no assumptions are made on the sources (the so-called “conditional” Gaussian data model). Thus there is a need to consider how the correlated sources MLE does indeed perform in comparison with the white sources MLE.

1.3. Contributions of this Paper

This paper makes the following contributions : (i) We derive a more convenient form of the array data model and associated covariance for Gaussian spatially uncorrelated, but possibly temporally correlated sources. The additive Gaussian sensor noise is considered to be both spatially and temporally white, with known variance. These assumptions are made because we want to focus solely on the issue of temporal source correlation. This model is very similar to that of [5], but we order the source data in a different way to highlight the temporal correlation of the sources. (ii) We derive the CRB for the case when the sources are autoregressive processes of order one, with unknown parameters (variance and AR parameter). The CRB term for AOA is equivalent to that previously presented in [10] and [5]. The CRB terms for the AR parameters in this setting are new. (iii) We derive the form of the MLE for the AR(1) source signals case, and examine specifically the cases where there are one, and two incident signals. In these cases, reduction in numerical complexity can be obtained by use of the matrix inversion lemma and Kronecker calculus. This is important because the data covariance matrix is of size NT , rather than N for the white sources case, and we do not want computational complexity of $O(N^3T^3)$ as would generally be required to evaluate the log likelihood and its derivatives. (iv) We conduct numerical experiments to compare the performance of the AR(1) based MLEs with their white noise source counterparts. In particular, noting the numerical experiments reported in [7], we provide a “missing piece” of the puzzle. We can report that our results support the conjecture that the performance of “techniques specifically designed” for source correlation, in this case the proper (i.e. matched) MLE for AR(1) sources, does not yield significantly better performance than the case when the correct MLE is applied to uncorrelated sources. However, our results don’t conform with those presented in [7] which appear to show that the uncorrelated sources (WN) MLE is robust to the presence of source correlation. We did observe some degradation in performance of the WN MLE when the sources were AR(1) signals. This paper thus suggests that additional work may be required in this area.

2. CORRELATED SOURCE SIGNAL MODEL

In this section, we introduce a baseband complex array data model for Gaussian temporally correlated narrowband plane waves incident on a sensor array for the case when the source signals are spatially uncorrelated AR(1) signals with unknown signal parameters. For reasons of simplicity, we assume in each case that the additive Gaussian sensor noise is spatially and temporally uncorrelated with known variance, although this assumption is easily relaxed with little modification. This model is similar to that used in [5] but with the source terms ordered in a different way to highlight their temporal correlation. The associated covariance model is also derived.

Suppose we have a uniformly spaced linear sensor array with $N > 1$ sensors, with sensor n located at position $x = nd$, $n = 0, \dots, N - 1$, along the x -axis in the (x, y) plane. Here d denotes the sensor spacing in metres. Narrowband plane waves with centre frequency f Hz are incident on the array. Suppose there are K such incident waves making angles θ_k , $k = 1, \dots, K$ with respect to the

positive y direction, measured in the clockwise direction. Then the relative phase between for signal k received at sensor n and sensor 0 is given by $2\pi nd \sin(\theta_k)/\lambda$ where $\lambda = f/c$ is the wavelength and c denotes the speed of the wave propagation. If we take a set of sensor measurements simultaneously at some time t (often termed a data “snapshot”), then the vector of complex baseband signals measured on the sensors can be written as

$$x(t) = \sum_{k=1}^K a(\theta_k) s_k(t) + n(t), \quad (1)$$

where $x(t) \in \mathbb{C}^N$, $s_k(t) \in \mathbb{C}$ is the sample of complex source signal k at time t , and $n(t) \in \mathbb{C}^N$ denotes the vector of additive noise on the sensor array. Element n of the “steering” vector $a(\theta) \in \mathbb{C}^N$ is the relative phase between sensor n and sensor 0 for signal AoA θ . Following [5], we stack the data snapshots vertically to create a NT dimensional vector, but in contrast to [5], we stack the sources in temporal order first. Let $\mathbf{s}_k = [s_k(t_1) \dots s_k(t_T)]^T$ denote the $T \times 1$ vector of temporal samples corresponding to source signal k , then in vector form,

$$\mathbf{x} = \sum_{k=1}^K (I_T \otimes a(\theta_k)) \mathbf{s}_k + \mathbf{n}, \quad (2)$$

where I_T denotes the identity matrix of size T , \otimes denotes Kronecker product. We emphasise that the model (2) contains exactly the same information as the model in [5], but with the source signal samples arranged in a different way. We now assume that the sensor noise is temporally and spatially white with known variance σ^2 (an assumption which is readily relaxed), and that the sources are spatially uncorrelated with source k having $T \times T$ temporal covariance matrix R_k . The $NT \times NT$ array data covariance matrix then has the form

$$R_{\mathbf{x}} = \sum_{k=1}^K R_k \otimes (a(\theta_k) a(\theta_k)^H) + \sigma^2 I_{NT}. \quad (3)$$

It is easily seen that this model can be reduced to the usual form when the sources are temporally uncorrelated (i.e. when the matrices R_k are diagonal). The covariance model (3) will form the basis for the MLE and CRB to be derived in the next two sections.

3. MAXIMUM LIKELIHOOD ESTIMATION

In this section, we derive the MLEs for the AoAs and AR(1) source signal parameters for K incident signals on a ULA. The derivation uses the signal model from section 2 and is relatively straightforward. The special case of a single incident AR(1) signal gives some additional insight. We shall model the source signals $s(t)$ and array noise $n(t)$ by zero-mean, stationary, circular complex Gaussian random processes. In particular, this implies that the source covariance matrices R_k are real. In the sequel, we will sometimes use the shorthand notation a_k to denote the steering vector $a(\theta_k)$ for source k . We use the notation α to refer to any one of the $3K$ model parameters, the K AoA θ_k , and the variances σ_k^2 , and AR parameters ρ_k for signal $k = 1, \dots, K$, and $\Theta \in \mathbb{R}^{3K}$ to denote the vector $\Theta = [\theta_1, \sigma_1^2, \rho_1, \dots, \theta_K, \sigma_K^2, \rho_K]$ to be a vector containing all these unknown parameters. From (3), the data log-likelihood is (neglecting a constant term)

$$\log \mathbf{P}\{\mathbf{x}\} = -\log \det R_{\mathbf{x}}(\Theta) - \mathbf{x}^H R_{\mathbf{x}}(\Theta)^{-1} \mathbf{x}. \quad (4)$$

where $R_{\mathbf{x}}$ depends on the parameters Θ through (3). Thus for any of the scalar parameters α , we have that

$$\frac{\partial \log \mathbf{P} \{\mathbf{x}\}}{\partial \alpha} = \text{Tr} \left\{ R_{\mathbf{x}}(\Theta) \frac{\partial R_{\mathbf{x}}(\Theta)^{-1}}{\partial \alpha} \right\} - \mathbf{x}^H \frac{\partial R_{\mathbf{x}}(\Theta)^{-1}}{\partial \alpha} \mathbf{x}. \quad (5)$$

We can use this result to define a gradient-based maximisation approach for finding (local) maxima of the log likelihood. In all case, we avoid the calculation of products, determinants and inverses of matrices of the dimension of the data vector \mathbf{x} (i.e. NT). This can be done using knowledge of the special structure of the covariance matrix $R_{\mathbf{x}}$ in (3) as we subsequently show.

3.1. Single Source Case

A circular AR(1) source $s_1 \in \mathbb{C}^T$, with variance σ_1^2 and AR parameter ρ_1 has $T \times T$ covariance matrix with elements $[R_1]_{i,j} = \mathbf{E} \{s_1(t_i) s_1(t_j)\} = \sigma_1^2 \rho_1^{|i-j|}$. Its inverse is tridiagonal of known form. In the case ($K = 1$) where there is a single AR(1) source signal incident on the array at angle θ , the data covariance (3) is

$$R_{\mathbf{x}} = R_1(\rho_1, \sigma_1^2) \otimes (a(\theta) a(\theta)^H) + \sigma^2 I_{NT}. \quad (6)$$

We can then show, using the matrix inversion lemma (MIL) [1], that

$$R_{\mathbf{x}}^{-1} = \sigma^{-2} \left(I_{NT} - \left(P_1 \otimes (a(\theta) a(\theta)^H) \right) \right), \quad (7)$$

where

$$P_1^{-1} = N I_T + \sigma^2 R_1^{-1} \quad (8)$$

is a tridiagonal matrix which depends only on the AR(1) signal parameters and has simple derivatives with respect to these parameters. We can then work out the derivatives of the log likelihood via (5) to develop a gradient based numerical MLE scheme, as well as the terms required for the CRB. Indeed, we have the $NT \times NT$ derivative matrices

$$\begin{aligned} \frac{\partial R_{\mathbf{x}}^{-1}}{\partial \theta} &= -\sigma^{-2} P_1 \otimes (d(\theta) a(\theta)^H + a(\theta) d(\theta)^H), \\ \frac{\partial R_{\mathbf{x}}^{-1}}{\partial \rho_1} &= \sigma^{-2} \left(P_1 \frac{\partial P_1^{-1}}{\partial \rho_1} P_1 \right) \otimes (a(\theta) a(\theta)^H), \\ \frac{\partial R_{\mathbf{x}}^{-1}}{\partial \sigma_1^2} &= \sigma^{-2} \left(P_1 \frac{\partial P_1^{-1}}{\partial \sigma_1^2} P_1 \right) \otimes (a(\theta) a(\theta)^H), \end{aligned} \quad (9)$$

where $d(\theta) = da(\theta)/d\theta$. The log-likelihood can be evaluated in $O(T^3) + O(N^2T^2)$ calculations because the eigenvalues of $R_{\mathbf{x}}$ in (6) have a simple form, so $\log \det R_{\mathbf{x}}$ has a closed form in terms of the eigenvalues of R_1 . The three derivatives of the log-likelihood can also be evaluated in $O(T^3) + O(N^2T^2)$ from (5)-(9) using Kronecker calculus, which avoids multiplication of large matrices.

3.2. Two Source Case

In the two source case, we can again avoid inversion or determinant evaluation of $R_{\mathbf{x}}$ by using the matrix inversion lemma twice. Write

$$\begin{aligned} A_1 &= (I_T \otimes a_1) R_1 (I_T \otimes a_1^H) + \sigma^2 I_{NT}, \text{ and} \\ R_{\mathbf{x}} &= A_1 + (I_T \otimes a_2) R_2 (I_T \otimes a_2^H). \end{aligned}$$

Then A_1^{-1} is computed in $O(T^3) + O(N^2T^2)$ operations as in 3.1, where P_1 is specified by (8). Applying the MIL again gives

$$\begin{aligned} R_{\mathbf{x}}^{-1} &= \sigma^{-2} (I_{NT} - Q_1 - Q_2 + Q_{2,1} + Q_{1,2} - Q_{1,2,1}), \\ Q_j &= P_j \otimes (a_j a_j^H), \quad j = 1, 2, \\ Q_{j,k} &= (P_j P_k) \otimes \left((a_j^H a_k) a_j a_k^H \right), \quad j, k = 1, 2, \\ Q_{1,2,1} &= (P_1 P_2 P_1) \otimes (|a_2^H a_1|^2 a_1 a_1^H), \end{aligned}$$

and $P_2^{-1} = N I_T - |a_2^H a_1|^2 P_1 + \sigma^2 R_2^{-1}$. Thus $R_{\mathbf{x}}^{-1}$ can be computed in $O(T^3) + O(N^2T^2)$ operations.

4. CRAMER-RAO BOUND FOR AR(1) SIGNALS CASE

The Cramer-Rao bound (CRB) for AoA estimation with general temporally correlated sources was specifically derived in [10] and is based on a result from [12]. This result, in turn, is based on a general result for parameter estimation for complex Gaussian data which appears in general form in [13]. We shall apply the latter formula for the elements of the Fisher information matrix FI

$$[\text{FI}(\Theta)]_{i,j} = \text{Tr} \left\{ \frac{\partial R_{\mathbf{x}}}{\partial \Theta_i} R_{\mathbf{x}}^{-1} \frac{\partial R_{\mathbf{x}}}{\partial \Theta_j} R_{\mathbf{x}}^{-1} \right\}, \quad (10)$$

where the array data covariance $R_{\mathbf{x}}$, and consequently FI is regarded as a function of the parameter vector Θ . In the general case, evaluation of (10) requires $O(N^3T^3)$ operations. although in the one and two source cases, we can use the simplified forms presented in section 3 to reduce this computation to $O(T^3) + O(N^2T^2)$. However, unlike the numerical maximisation of the log-likelihood which involves multiple evaluations of the log-likelihood (and its derivatives, if a gradient based scheme is used), we need only evaluate (10), a total of $3K(3K+4)/2$ times, and then perform the inverse of a $3K \times 3K$ matrix to get the CRB terms for each specified values of the model parameters Θ . The values of the derivatives follow (3), where the derivatives of R_k are easily found from its elements $[R_k]_{i,j} = \sigma_k^2 \rho_k^{|i-j|}$.

5. NUMERICAL EXAMPLES

In these examples, we use the scenario of [7] for comparison purposes. In this scenario, we have a $N = 20$ element, half-wavelength spaced ULA with $K = 2$ spatially uncorrelated, zero-mean circular complex Gaussian sources incident on the array at angles $\theta_1 = 16^\circ$ and $\theta_2 = 18^\circ$ embedded in zero-mean, spatially and temporally uncorrelated additive circular complex Gaussian sensor noise with known variance σ^2 . Signal-to-Noise ratio (SNR) (for signal k) is defined as the ratio of the source signal variance σ_k^2 to the additive noise variance σ^2 . We are uncertain as to the definition of SNR used in [7]. The number of data snapshots was fixed at $T = 40$.

5.1. Properties of the AR(1) based MLE - single signal case

In the first example, we considered only a single source incident from $\theta = 16^\circ$. We computed the CRBs for the 3 parameters (AoA θ , AR co-efficient ρ_1 and noise variance parameter σ_1^2). Using 500 independent trials, we estimated the standard deviation of the MLEs under each model. Figure 1 shows the AoA MLE estimation errors and CRBs for an AR(1) source with $\rho = 0.9$, and an uncorrelated source. We observe a slightly smaller CRB for the AR(1) sources

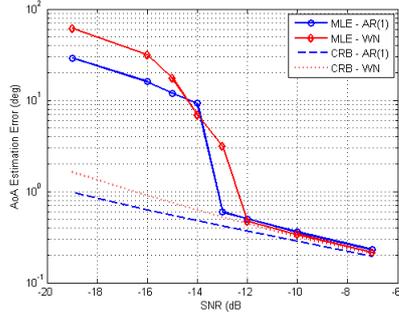


Fig. 1. Figure shows the MLE performance and CRBs for a single source incident for the AR(1) and WN source cases.

case and a small SNR threshold extension of approximately 1 dB for this example.

We were then motivated to examine robustness issues associated with ML estimation. It was reported in [7], that the uncorrelated signals MLE appeared robust to the presence of temporal correlation (see figures 3(a) and 3(b) of [7]). We decided to try and verify this observation as it has bearing on the applicability of the AR(1) MLE, given the behaviour shown in figure 1. Figure 2 shows the performance of the uncorrelated signal MLE on both the uncorrelated signal (as shown in figure 1) and when the same MLE is applied to an AR(1) signal of identical power, with parameter $\rho = 0.9$. A reduction in thresholding performance of about 2 dB is observed. This doesn't support the observations of [7], however our assessment here is for a *single* signal. We'll see in the following section, that for 2 sources, the uncorrelated signal MLE is also sensitive to source correlation, however it is difficult to compare with [7] as the latter doesn't plot the two curves on the same graph. Our results do suggest however, that when combining the results evident from figures 1 and 2, the use of the AR(1) MLE yields approximately 3 dB threshold extension for a single incident signal compared to using the uncorrelated signal MLE, when indeed the signal is AR(1) with (unknown) $\rho = 0.9$. Of course, the AR MLE requires significantly more computation, so shouldn't be used if it is known *a priori* that the source is uncorrelated.

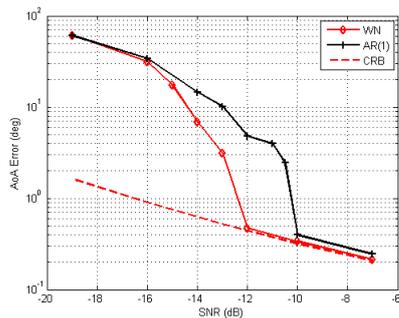


Fig. 2. Figure shows the estimated standard deviation for the uncorrelated signal ML estimates for the cases when the source signal is uncorrelated (matched), and when it is an AR(1) signal with $\rho = 0.9$ (mismatched). Figure also shows the CRB for the uncorrelated source case.

5.2. Properties of the AR(1) based MLE - two sources

In this section, we compare the performance of the correlated sources MLE to the uncorrelated sources MLE when there are two incident source signals as described above. Figure 3 shows the AoA angle accuracy (averaged for both signals) for the MLE, and CRB for each model. Performance of each is similar, although it appears that the AR(1) MLE is not efficient for this number of snapshots over the SNR range considered.

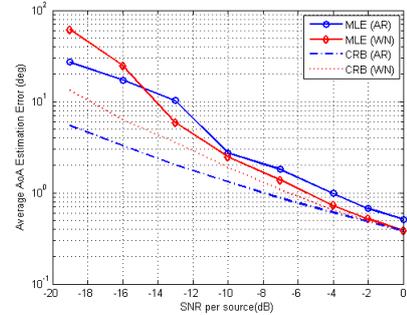


Fig. 3. Figure shows the average estimated standard deviation for the WN and AR(1) signal ML estimates for two source signals having identical SNR. The figure also shows the CRBs for each model.

Figure 4 examines the robustness of the WN MLE in the presence of correlated signals. A degradation in performance when the sources are correlated is noted.

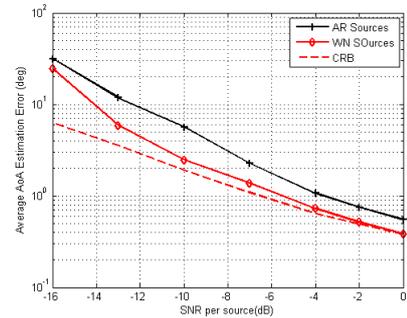


Fig. 4. Figure shows the average estimated standard deviation for WN ML estimates for the cases when the two source signals are (i) WN (matched), and (ii) AR(1) signals having identical SNR (mismatched).

6. CONCLUSION

In this paper, we have proposed a signal model for temporally correlated Gaussian narrowband signals incident on a sensor array. The CRB and MLE for spatially uncorrelated first-order AR signals have been derived, and compared to those for the uncorrelated sources (WN) case. Numerical simulations show that whilst the AR based MLE does not achieve significant improvement over the WN case when the correct MLEs are used, the WN MLE loses performance when the signals are correlated, which appears to differ to that reported recently in [7]. More investigation is required on this point. Various extensions to this approach are possible including studying temporal correlation in the sensor noise [6], higher order AR sources, spatial signal/noise correlation and threshold analyses.

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