## A COMPUTATIONALLY EFFICIENT SOURCE LOCALIZATION METHOD FOR A MIXTURE OF NEAR-FIELD AND FAR-FIELD NARROWBAND SIGNALS

Weiliang Zuo<sup>†</sup>, Jingmin Xin<sup>†\*</sup>, Jiasong Wang<sup>‡</sup>, Nanning Zheng<sup>†</sup>, and Akira Sano<sup>§</sup>

<sup>†</sup> Institute of Artificial Intelligence and Robotics, Xi'an Jiaotong University, Xi'an 710049, China,
 <sup>‡</sup> State Key Laboratory of Astronautic Dynamics, Xi'an 710043, China
 <sup>§</sup> Department of System Design Engineering, Keio University, Yokohama 223-8522, Japan

## ABSTRACT

In this paper, we consider the source localization for a mixed near-field (NF) and far-field (FF) narrowband signals impinging on a uniform linear array (ULA) with the symmetrical geometric configuration. A computationally efficient direction-of-arrivals (DOAs) and range estimation method for the mixed NF and FF signals is proposed, where the DOAs of the NF and FF signals are estimated separately, and the computationally burdensome eigendecomposition is avoided. Comparing to some existent methods, the proposed method can separate the NF signals from the FF signals more efficiently, and consequently the estimation performance is improved. The effectiveness of the proposed method is verified though numerical examples.

*Index Terms*— Source localization, far-field, near-field, uniform linear array, direction-of-arrival.

### 1. INTRODUCTION

The source localization is a fundamental problem in radar, sonar, wireless communication, seismic exploration and so on (e.g., [1] and references therein), and many algorithms have been developed to deal with either the far-field (FF) signals [2, 3] or the near-field (NF) signals [4, 5], respectively. In many application scenarios of sources localization such as speaker localization using microphone arrays [6, 7], the NF and FF signals may coexist. Hence the source localization and classification of the mixed NF and FF signals have received considerable attention recently.

By utilizing the properties of the higher-order statistics (HOS) (i.e., cumulant), and some methods were developed to localize the mixed NF and FF of non-Gaussian signal-s [8–11]. In these methods, the main step is to form a special cumulant matrix, which only contains the direction-of-arrival (DOA) information of mixed signals, and from this resulting cumulant matrix, the DOA estimates are obtained firstly. Then the range estimation is easily obtained, where an additional DOA association procedure is needed. However, the HOS-based methods require high computational complexity

for constructing cumulant matrix. A new second-order statistics (SOS) based differencing method was developed in [12], where the crux is to eliminate the contribution of the FF signals and additive noise for localizing NF signals. Unfortunately, the structure property of the covariance matrix of incident signals is required, which is only valid for large number of snapshots. By utilizing the advantage of a symmetric uniform linear array (ULA), some SOS-based methods [13, 14] were proposed to localize the mixed NF and FF signals. These methods have low computational burden and use a subjective criterion, which distinguishes the NF and FF signals by DOAs or ranges estimation, to determine the type (i.e., NF or FF) of the incident signals. Additionally, the oblique projection [15], which projects the measurement onto a low-rank subspace along a non-orthogonal subspace, was used to separate the incident signals [13]. Although it outperforms the differencing one, it possesses a "saturation behavior" for localizing the NF signals. Moreover these SOS-based methods aforementioned require a computationally intensive procedure of eigendecomposition.

Therefore, we propose a more computationally efficient method for localizing the mixed NF and FF incident signals impinging on the ULA with the symmetrical geometric configuration. By taking the advantage of the oblique projector, a new covariance matrix which only contains the information of the NF signals is formed, then the anti-diagonal elements of this resulting matrix are used to estimate the DOA of the NF signals. Although the oblique projector was used for separating the mixed signals in [13], in this paper, we present a more efficient way to calculate it. Furthermore in order to overcome the "saturation behavior" encountered by the differencing and the oblique projection based methods for localizing the NF signals, an alternating iterative method is developed. Finally, the effectiveness of the proposed method is verified through some numerical examples.

## 2. DATA MODEL AND BASIC ASSUMPTIONS

As shown in Fig. 1, we consider K noncoherent narrowband signals  $\{s_k(n)\}$  impinging on the ULA consisting of 2M + 1 omnidirectional sensors with spacing d, and

This work was supported in part by the National Natural Science Foundation of China under Grant 61172162.

without loss of generality, we assume the first  $K_1$  incident signals  $\{s_k(n)\}_{k=1}^{K_1}$  are the FF ones with the locations of  $\{(\infty, \theta_k)\}_{k=1}^{K_1}$ , while the other  $K_2$  signals  $\{s_k(n)\}_{k=K_1+1}^{K}$ are the NF ones with the locations of  $\{(r_k, \theta_k)\}_{k=K_1+1}^{K}$ , where  $K = K_1 + K_2$ . By letting the center of the ULA be the phase reference point, the received noisy array data can be expressed in a vector-matrix form as

$$\begin{aligned} \boldsymbol{x}(n) &= \boldsymbol{A}_f \boldsymbol{s}_f(n) + \boldsymbol{A}_n \boldsymbol{s}_n(n) + \boldsymbol{w}(n) \\ &= \boldsymbol{A} \boldsymbol{s}(n) + \boldsymbol{w}(n) \end{aligned} \tag{1}$$

where  $\boldsymbol{x}(n) \triangleq [x_{-M}(n), x_{-M+1}(n), \cdots, x_{M-1}(n),$  $x_M(n)$ <sup>T</sup>,  $\boldsymbol{w}(n) \triangleq [w_{-M}(n), w_{-M+1}(n), \cdots, w_{M-1}(n),$  $w_M(n)]^T$ ,  $s_f(n) \triangleq [s_1(n), s_2(n), \cdots, s_{K_1}(n)]^T$ ,  $s_n(n) \triangleq$  $[s_{K_1+1}(n), s_{K_1+2}(n), \cdots, s_K(n)]^T$ , and  $s(n) \triangleq [s_f(n)^T,$  $\boldsymbol{s}_n(n)^T]^T$ , while  $\boldsymbol{A}$  is the array response matrix given by  $\boldsymbol{A} \triangleq$  $[\mathbf{A}_{f}, \mathbf{A}_{n}], \mathbf{A}_{f} \triangleq [\mathbf{a}_{f}(\theta_{1}), \mathbf{a}_{f}(\theta_{2}), \cdots, \mathbf{a}_{f}(\theta_{K_{1}})], \mathbf{A}_{n} \triangleq$  $[a_n(r_{K_1+1}, \theta_{K_1+1}), a_n(r_{K_1+2}, \theta_{K_1+2}), \cdots, a_n(r_K, \theta_K)],$ and the response vectors for the FF and NF signals are defined as  $\boldsymbol{a}_{f}(\theta_{k}) \triangleq [e^{-jM\omega_{k}}, \cdots, e^{-j\omega_{k}}, 1, e^{j\omega_{k}}, \cdots, e^{jM\omega_{k}}]^{T},$ and  $\boldsymbol{a}_n(r_k,\theta_k) \triangleq [e^{j\tau_{-Mk}},\cdots,e^{j\tau_{-k}},1,e^{j\tau_k},\cdots,e^{j\tau_{Mk}}]^T$ , where  $(\cdot)^T$  denotes transpose. Furthermore the phase delay of the FF signals  $\omega_k$  is defined as  $\omega_k \triangleq -2\pi d \sin \theta_k / \lambda$ , where  $\phi_k \triangleq \pi d^2 \cos^2 \theta_k / (\lambda r_k)$ , and  $\lambda$  is the wavelength of incident signals, while by using the so-called Fresnel approximation, the phase delay of the NF signals  $\tau_{mk}$  is given by  $\tau_{mk} \triangleq \omega_k m + \phi_k m^2$  for  $m = -M, \cdots, -1, 0, 1, \cdots, M$ , where  $\phi_k \triangleq \pi d^2 \cos^2 \theta_k / (\lambda r_k)$ .

In this paper, we make the following assumptions: 1) The array is calibrated and the array response matrix A has full rank. 2) The incident signals  $\{s_k(n)\}$  are zero-mean widesense stationary random processes and are uncorrelated each other. 3) The additive noises  $\{w_i(n)\}$  are temporally and spatially complex white Gaussian random process with zeromean and variance  $\sigma^2$  and are independent to the incident signals  $\{s_k(n)\}$ . 4) The numbers of the incident NF and FF signals  $K_1$  and  $K_2$  are known, and the number of all incident signals K satisfies the relation K < M + 1.

### 3. NEW METHOD FOR SOURCE LOCALIZATION

Under the basic assumptions, from (1), we can obtain the array covariance matrix R of the received data as

$$\boldsymbol{R} \triangleq E\{\boldsymbol{x}(n)\boldsymbol{x}^{H}(n)\}$$
  
=  $\boldsymbol{A}_{f}\boldsymbol{R}_{sf}\boldsymbol{A}_{f}^{H} + \boldsymbol{A}_{n}\boldsymbol{R}_{sn}\boldsymbol{A}_{n}^{H} + \sigma^{2}\boldsymbol{I}_{2M+1}$   
=  $\bar{\boldsymbol{R}} + \sigma^{2}\boldsymbol{I}_{2M+1}$  (2)

where  $\bar{\mathbf{R}}$  is the noiseless array covariance matrix defined by  $\bar{\mathbf{R}} \triangleq \mathbf{A}\mathbf{R}_s\mathbf{A}^H = \mathbf{R} - \sigma^2 \mathbf{I}_{2M+1}, \mathbf{R}_{sf}$  and  $\mathbf{R}_{sn}$  are the covariance matrix of the FF or NF signals defined by  $\mathbf{R}_{sf} \triangleq E\{\mathbf{s}_f(n)\mathbf{s}_f^H(n)\}$  and  $\mathbf{R}_{sn} \triangleq E\{\mathbf{s}_n(n)\mathbf{s}_n^H(n)\}$ , while  $\mathbf{R}_s \triangleq E\{\mathbf{s}(n)\mathbf{s}^H(n)\} = \text{blkdiag}(\mathbf{R}_{sf}, \mathbf{R}_{sn}), E\{\mathbf{w}(n)\mathbf{w}^H(n)\} = \sigma^2 \mathbf{I}_{2M+1}$ , blkdiag $(\cdot)$  and  $(\cdot)^H$  denotes block diagonal matrix operator and the Hermitian transpose, and  $\mathbf{I}_m$  is a  $m \times m$ identity matrix.



**Fig. 1**. The ULA with the symmetrical geometric configuration.

## 3.1. DOA Estimation of FF Signals

Firstly, we can divide the array response matrix A in (1) into two submatrices as

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \end{bmatrix}_{2M+1-K.}^{K}$$
(3)

Under the basic assumptions, we can find that there exist a  $K \times (2M+1-K)$  linear operator P between  $A_1$  and  $A_2$  [3], i.e.,

$$\boldsymbol{A}_2 = \boldsymbol{P}^H \boldsymbol{A}_1 \tag{4}$$

or

$$[\boldsymbol{P}^{H}, -\boldsymbol{I}_{2M+1-K}]\boldsymbol{A} = \boldsymbol{Q}^{H}\boldsymbol{A} = \boldsymbol{O}_{(2M+1-K)\times K}$$
(5)

where  $O_{p \times q}$  denotes a  $p \times q$  null matrix. Then from (5), we easily have

$$\boldsymbol{Q}^{H}\boldsymbol{a}_{n}(r,\theta) = \boldsymbol{0}_{(2M+1-K)\times 1} \tag{6}$$

$$\boldsymbol{Q}^{H}\boldsymbol{a}_{f}(\theta) = \boldsymbol{0}_{(2M+1-K)\times 1}.$$
 (7)

Secondly, to get the noiseless covariance matrix  $\bar{R}$ , we need to estimate the noise variance. By considering the partition

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{K} & 2M+1-K \\ \boldsymbol{R}_{11} & \boldsymbol{R}_{12} \\ \boldsymbol{R}_{21} & \boldsymbol{R}_{22} \end{bmatrix} \begin{bmatrix} K \\ 2M+1-K \end{bmatrix}$$
(8)

then the noise variance  $\sigma^2$  can be obtained by [16]

$$\sigma^2 = \frac{\operatorname{tr}\{\boldsymbol{R}_{22}\boldsymbol{\Pi}\}}{\operatorname{tr}\{\boldsymbol{\Pi}\}} \tag{9}$$

where  $\Pi = I_{2M+1-K} - R_{21}R_{21}^{\dagger}$ , tr{·} denotes the trace operator, and  $(\cdot)^{\dagger}$  denotes the Moore-Penrose pseudoinverse. Furthermore, we can also divide  $\bar{R}$  into two parts as  $\bar{R} = [G, H]$ , in which G and H consist of the first K or the last 2M+1-K columns. Thus, when the number of snapshots is finite, the DOAs  $\{\theta_k\}_{k=1}^{K_1}$  of the FF signals can be estimated by minimizing the following cost function

$$g_f(\theta) = \boldsymbol{a}_f^H(\theta) \boldsymbol{\Pi}_{\hat{Q}} \boldsymbol{a}_f(\theta)$$
(10)

where  $\mathbf{\Pi}_{\hat{Q}} = \hat{Q}(\hat{Q}^{H}\hat{Q})^{-1}\hat{Q}^{H} = \hat{Q}(I_{2M+1-K}-\hat{P}^{H}(\hat{P}\hat{P}^{H}+I_{K})^{-1}\hat{P})\hat{Q}^{H}, \hat{P} = (\hat{G}^{H}\hat{G})^{-1}\hat{G}^{H}\hat{H}, \text{ and } \hat{Q} = [\hat{P}^{T}, -I_{2M+1-K}]^{T}, \text{ while } \mathbf{\Pi}_{\hat{Q}} \text{ is calculated using the matrix inversion lemma implicitly [17].}$ 

#### 3.2. DOA Estimation of NF Signals

By using the DOAs of the FF signals, from (2), we have a  $(2M+1) \times (2M+1)$  matrix as

$$\boldsymbol{R}_{o} \triangleq \bar{\boldsymbol{R}} \boldsymbol{\Pi}_{A_{f}}^{\perp} = \boldsymbol{A}_{n} \boldsymbol{R}_{sn} \boldsymbol{A}_{n}^{H} \boldsymbol{\Pi}_{A_{f}}^{\perp}$$
(11)

where  $\Pi_{A_f}^{\perp} \triangleq I_{2M+1} - A_f (A_f^H A_f)^{-1} A_f^H$ . By employing the QR decomposition of  $R_o$  as  $R_o = \tilde{Q}\tilde{R}$ , we can obtain the oblique projector  $E_{A_f|A_n}$  in the following manner

$$\boldsymbol{E}_{A_f|A_n} = \boldsymbol{A}_f (\tilde{\boldsymbol{Q}}_2 \tilde{\boldsymbol{Q}}_2^H \boldsymbol{A}_f)^{\dagger}$$
(12)

where  $\hat{Q}_2$  is the last  $2M + 1 - K_2$  columns of the  $\hat{Q}$  [18]. Then from (2) and (12), a array covariances matrix  $R_n$  corresponding to the NF signals can be obtained

$$\boldsymbol{R}_{n} \triangleq \boldsymbol{A}_{n}\boldsymbol{R}_{sn}\boldsymbol{A}_{n}^{H}$$
  
=  $(\boldsymbol{I}_{2M+1} - \boldsymbol{E}_{A_{f}|A_{n}})\bar{\boldsymbol{R}}(\boldsymbol{I}_{2M+1} - \boldsymbol{E}_{A_{f}|A_{n}})^{H}(13)$ 

where it's anti-diagonal element  $r_n(i)$  is given by

$$r_n(i) \triangleq (\mathbf{R}_n)_{i,2M+2-i} = \sum_{k=K_1+1}^K r_{s_k}^2 e^{-j2(M+1-i)\omega_k}$$
 (14)

where  $i = 1, 2, \dots, 2M + 1$ , and  $r_{s_k}^2$  is the power of the kth NF signal defined by  $r_{s_k}^2 \triangleq E\{s_k(n)s_k^*(n)\}$ , where  $(\cdot)^*$  denotes the complex conjugate. The element  $r_n(i)$  only contains the DOA information of the NF signals. Then we can construct a new  $(M + 1) \times (M + 1)$  covariance matrix like the FF Toepliz covariance matrix [19] as follows

$$\bar{\boldsymbol{R}}_{n} \triangleq \begin{bmatrix} r_{n}(M+1), \dots, r_{n}(2), r_{n}(1) \\ r_{n}(M+2), \dots, r_{n}(3), r_{n}(2) \\ \vdots & \ddots & \vdots \\ r_{n}(2M+1), \dots, r_{n}(M+2), r_{n}(M+1) \end{bmatrix}$$
(15)

After some simple algebraic manipulations, it can be rewritten in a more compact form as

$$\bar{\boldsymbol{R}}_n = \bar{\boldsymbol{A}}_n(\theta) \boldsymbol{R}_{sn} \bar{\boldsymbol{A}}_n^H(\theta)$$
(16)

where  $\bar{A}_n(\theta) \triangleq [\bar{a}_n(\theta_1), \cdots, \bar{a}_n(\theta_{K_1})]^T$ , and  $\bar{a}_n(\theta_k) \triangleq [1, e^{j\omega_k}, \cdots, e^{j2M\omega_k}]^T$ . In a similar way to the above, we can also divide  $\bar{A}_n(\theta)$  into two submatrices as

$$\bar{\boldsymbol{A}}_{n}(\theta) = \begin{bmatrix} \bar{\boldsymbol{A}}_{n1}(\theta) \\ \bar{\boldsymbol{A}}_{n2}(\theta) \end{bmatrix}_{M+1-K_{2}}^{K_{2}}$$
(17)

where  $\bar{A}_{n1}(\theta)$  is of full rank, and the rows of  $\bar{A}_{n2}(\theta)$  can be expressed as a linear combination of the rows of  $\bar{A}_{n1}(\theta)$ . Hence there exist a  $K_2 \times (M + 1 - K_2)$  linear operator  $\bar{P}$ between  $\bar{A}_{n1}(\theta)$  and  $\bar{A}_{n2}(\theta)$  [3]

$$\bar{\boldsymbol{A}}_{n2}(\theta) = \bar{\boldsymbol{P}}^H \bar{\boldsymbol{A}}_{n1}(\theta) \tag{18}$$

or

$$[\bar{\boldsymbol{P}}^{H}, -\boldsymbol{I}_{M+1-K_{2}}]\bar{\boldsymbol{A}}_{n}(\theta) = \bar{\boldsymbol{Q}}^{H}\bar{\boldsymbol{A}}_{n}(\theta) = \boldsymbol{O}_{(M+1-K_{2})\times K_{2}}$$
(19)

Then we have

$$\bar{\boldsymbol{Q}}^{H}\bar{\boldsymbol{A}}_{n}(\theta) = \boldsymbol{0}_{(M+1-K_{2})\times 1}.$$
(20)

Similarly, we can divide  $\bar{R}_n$  into two parts as  $\bar{R}_n = [\bar{G}, \bar{H}]$ , in which  $\bar{G}$  and  $\bar{H}$  are two submatrices consisting of its first  $K_2$  or the last  $M+1-K_2$  columns. Thus when the number of snapshots is finite, the DOAs  $\{\theta_k\}_{k=K_1+1}^K$  of the NF signals can be estimated by minimizing the following cost function

$$f_n(\theta) = \bar{\boldsymbol{a}}_n^H(\theta) \boldsymbol{\Pi}_{\hat{Q}} \bar{\boldsymbol{a}}_n(\theta)$$
(21)

where  $\Pi_{\hat{Q}} = \hat{Q}(\hat{Q}^{H}\hat{Q})^{-1}\hat{Q}^{H} = \hat{Q}(I_{M+1-K_{2}} - \hat{P}^{H}(\hat{P}\hat{P}^{H} + I_{K_{2}})^{-1}\hat{P})\hat{Q}^{H}, \ \hat{P} = (\hat{G}^{H}\hat{G})^{-1}\hat{G}^{H}\hat{H}, \ \text{and} \ \hat{Q} = [\hat{P}^{T}, -I_{M+1-K_{2}}]^{T}.$ 

# 3.3. Range Estimation of NF Signals

Once we get the DOA estimates of the NF signals, the ranges can be found from (6). In the case of the finite number of snapshots,  $\{r_k\}_{k=K_1+1}^K$  can be estimated in the Fresnel region (i.e.,  $r_f \subset (0.62(D^3/\lambda)^{1/2}, 2D^2/\lambda))$  by minimizing the following cost function

$$\bar{f}_n(r) = \boldsymbol{a}_n^H(r,\hat{\theta}) \boldsymbol{\Pi}_{\hat{Q}} \boldsymbol{a}_n(r,\hat{\theta}).$$
(22)

where *D* is the aperture of the array [5], and the estimated range  $\{\hat{r}_k\}_{k=K_1+1}^K$  and the estimated DOAs  $\{\hat{\theta}_k\}_{k=K_1+1}^K$  are automatically paired without any additional procedure.

## 3.4. Alternating Iterative Scheme for NF Signals

In the DOA estimation of the NF signals, we need to calculate the oblique projector  $E_{A_f|A_n}$ , where the block diagonal structure of signal covariance matrix  $R_s$  and the anti-diagonal elements of  $R_n$  are exploited. Unfortunately, when the number of snapshots is finite,  $R_s$  may be not block diagonal, and the anti-diagonal elements of  $R_n$  contain not only the information of the NF signals but also that of the FF signals. Then from (11), we can see that

$$\hat{\boldsymbol{R}}_{o} = \boldsymbol{A}_{n} \hat{\boldsymbol{R}}_{sn} \boldsymbol{A}_{n}^{H} \boldsymbol{\Pi}_{A_{f}}^{\perp} + \boldsymbol{A}_{f} \hat{\boldsymbol{R}}_{sfn} \boldsymbol{A}_{n}^{H} \boldsymbol{\Pi}_{A_{f}}^{\perp}$$
(23)

where  $\hat{\mathbf{R}}_{sfn} = (1/N) \sum_{n=1}^{N} \mathbf{s}_{f}(n) \mathbf{s}_{n}^{H}(n)$ . It is obvious that the range space of  $\hat{\mathbf{R}}_{o}$  is not strictly equal to the range space of  $\mathbf{A}_{n}$  due to the existence of  $\hat{\mathbf{R}}_{sfn}$ . As a result, we cannot get the exactly estimation of  $\mathbf{E}_{A_{f}|A_{n}}$  by using (12), and  $\hat{\mathbf{R}}_{sfn}$  is not affected by the additive noise, which means that even the the signal-to-noise ratio (SNR) tends to infinity, the influence of  $\hat{\mathbf{R}}_{sfn}$  still exists. Hence in the localization of the NF signals, the estimation error doesn't decrease with the increasing SNR, which is so-called "saturation behavior" [18]. To cope with this problem, we propose an alternating iterative scheme for localizing the NF signals.

When the DOAs of the FF and NF signals and the ranges of the NF signals are estimated from (10), (21), and (22), we can recalculate the oblique projector [15] as

$$\boldsymbol{E}_{A_f|A_n} = \boldsymbol{A}_f (\boldsymbol{A}_f^H \boldsymbol{\Pi}_{A_n}^{\perp} \boldsymbol{A}_f)^{-1} \boldsymbol{A}_f^H \boldsymbol{\Pi}_{A_n}^{\perp}$$
(24)

then we can refine the estimation of the DOAs and ranges of the NF signals with (13), (15), (21) and (22). By repeating these steps serval times, the saturation problem is solved.

## 3.5. Implementation of Proposed Method

When the N snapshots of array data are available, the implementation of the proposed method can be summarized as follows:

1) Calculate the estimates of covariance matrices  $\boldsymbol{R}$  in (2) as

$$\hat{\boldsymbol{R}} = \frac{1}{N} \sum_{n=1}^{N} \boldsymbol{x}(n) \boldsymbol{x}^{H}(n)$$
(25)



Fig. 2. RMSEs of the DOA estimates versus SNR.

 $\dots 32(2M+1)^2N$  flops

## 2) Estimate the DOAs of the FF signals by using (10).

 $\begin{array}{l} .\,.\,8(2M+1)^3+8K(2M+1-K)^2+8K^2(2M+1)\\+8(2M+1)(2M+1-K)^2+16K^2(2M+1-K)\\+2(2M+1-K)^2+2K^2+O(K^3)\\+4M^2+O(8M^3)\;\text{flops} \end{array}$ 

3) Calculate the matrix  $\bar{R}$  in (2) with (8) and (9).

 $\begin{array}{l} \dots 8(2M+1-K)^3+16(2M+1-K)K^2+O(K^3)\\ +8(2M+1-K)^2K+2(2M+1-K)^2\\ +4(2M+1-K)+2(2M+1)^2\, {\rm flops} \end{array}$ 

4) Calculate the oblique projector  $E_{A_f|A_n}$  in (12).

 $\begin{array}{l} \ldots 22(2M+1)^3+32K_1^2(2M+1)+16(2M+1)^2K_1\\ +16(2M+1)^2(2M+1-K_2)+22(2M+1)\\ +32K_1(2M+1)(2M+1-K_2)-5\\ -4(2M+1)^2+O(K_1^3) \; {\rm flops} \end{array}$ 

5) Estimate the new covariance matrix  $\bar{R}_n$  in (15).

6) Estimate the DOAs of the NF signals with (21).

$$\begin{split} 8(M+1)^3 + 8(M+1)(M+1-K_2)^2 + 2K_2^2 + O(K_2^3) \\ + 8K_2(M+1-K_2)^2 + 2(M+1-K_2)^2 \\ + 8K_2^2(M+1) + 16K_2^2(M+1-K_2) \\ + M^2 + O(M^3) \, \text{flops} \end{split}$$

7) Estimate the ranges of the NF signals with (22).

$$\dots 8K_2(M+1)(2M+1)^2 + 8K_2(M+1)^2(2M+1) \\ + K_2M^2 \text{flops}$$

In the above implementation, the computational complexity of each step is roughly indicated in terms of the number of MATLAB flops, and the computational complexity of the proposed method is nearly  $32(2M + 1)^2N + 84(2M + 1)^3 + 16(M + 1)^3$  flops, when  $2M + 1 \gg K$ , which occurs often in application of source localization.



Fig. 3. RMSEs of the DOA estimates versus snapshots.

### 4. NUMERICAL EXAMPLES

In this section, we evaluate the performance of the proposed method in localizing the mixed NF and FF signals by using a ULA consisting of N = 7 sensors with element spacing  $d = \lambda/4$ . Two signals with equal power arrive from the locations  $(\infty, -5^{\circ})$  and  $(1.7\lambda, 25^{\circ})$ , where the first one is the FF signal and the second one is the NF signal. Meanwhile, the behavior of the SOS-based algorithm [13] and the Cramer-Rao lower bound (CRB) [13] are also presented. The results in each of the examples below are obtained from 500 independent Monte Carlo trails, where SNR is defined as the ratio of the signal power to the noise variance at each sensor.

*Example 1–Performance versus SNR:* The number of snapshots is N = 200. In Fig. 2, we can see that the DOA estimates of the mixed signals are obtained separately. The performance of the proposed FF estimator is almost same as the SOS-based algorithm, while the proposed NF estimator is super better than it, especially at the high SNR. Furthermore, the saturation problem is efficiently solved by alternating iterative procedure.

*Example 2–Performance versus Number of Snapshots:* The SNR is fixed at 0dB, and the number of snapshots varies from 10 to 1000. The results are shown in Fig. 3. It can be observed that for both the FF and NF signals, the proposed method behaves better than the SOS-based algorithm.

### 5. CONCLUSION

Based on the SOS and the oblique projector technique, this paper proposes a new method for the mixed FF and N-F sources localization problem without multidimensional search, HOS and eigendecomposition. The examples show that the estimates of the parameters of both FF and NF signals are reasonably good. Meanwhile, the proposed method is computationally efficient, and achieves a classification of the source types without extra procedure.

### 6. REFERENCES

- H. Krim and M. Viberg, "Two decades of array signal processing research: The parametric approach," *IEEE Signal Process. Mag.*, vol. 13, no. 4, pp. 67–94, Jul. 1996.
- [2] R. O. Schmidt, "Multiple emitter location and signal parameters estimation," *IEEE Trans. Antennas Propag.*, vol. 34, no. 3, pp. 267–280, Mar. 1986.
- [3] S. Marcos, A. Marsal, and M. Benider, "The propagator method for sources bearing estimation," *Signal Process.*, vol. 42, no. 2, pp. 121–138, Mar. 1995.
- [4] N. Yuenand B. Friedlander, "Performance analysis of higher order esprit for localization of near-field sources," *IEEE Trans. Signal Process.*, vol. 46, no. 3, pp. 709–719, Mar. 1998.
- [5] W. Zhi and M. Y. W. Chia, "Near-field source localization via symmetric subarrays," *IEEE Signal Process. Lett.*, vol. 14, no. 6, pp. 409–412, Jun. 2007.
- [6] D. B. Ward and G. W. Elko, "Mixed near field/far field beamforming: A new technique for speech acquisition in a reverberant environment," in *Proc. IEEE ASSP Workshop on Applications of Signal Processing to Audio and Acoustics*. New Paltz, NY, Oct. 1997, pp. 19–22.
- [7] P. Tichavsky, K. T. Wong, and M. D. Zoltowski, "Near-field/far-field azimuth elevation angle estimation using a single vector-hydrophone," *IEEE Trans. Signal Process.*, vol. 49, no. 11, pp. 2498–2510, Nov. 2001.
- [8] J. Liang and D. Liu, "Passive localization of mixed nearfield and far-field sources using two-stage music algorithm," *IEEE Trans. Signal Process.*, vol. 58, no. 1, pp. 108–120, Jan. 2010.
- [9] B. Wang, J. Liu, and X. Sun, "Mixed sources localization based on sparse signal reconstruction," *IEEE Signal Process. Lett.*, vol. 19, no. 8, pp. 487–490, Aug. 2012.
- [10] G. Liu, X. Sun, Y. Liu, and Y. Qin, "Low-complexity estimation of signal parameters via rotational invariance techniques algorithm for mixed far-field and near-field cyclostationary sources localisation," *IET Signal Process.*, vol. 7, no. 5, pp. 382–388, Jul. 2013.

- [11] B. Wang, Y. Zhao, and J. Liu, "Mixed-order music algorithm for localization of far-field and near-field sources," *IEEE Signal Process. Lett.*, vol. 20, no. 4, pp. 311–314, Apr. 2013.
- [12] G. Liu and X. Sun, "Efficient method of passive localization for mixed far-field and near-field sources," *IEEE Antennas and Wireless Propag. Lett.*, vol. 12, pp. 902– 905, Jul. 2013.
- [13] J. He, M. N. S. Swamy, and M. O. Ahmad, "Efficient application of music algorithm under the coexistence of far-field and near-field sources," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 2066–2070, Apr. 2012.
- [14] J. Jiang, F. Duan, J. Chen, Y. Li, and X. Hua, "Mixed near-field and far-field sources localization using the uniform linear sensor array," *IEEE Sensors Journal*, vol. 13, no. 8, pp. 3136–3143, Aug. 2013.
- [15] R. T. Behrens and L. L. Scharf, "Signal processing applications of oblique projection operators," *IEEE Trans. Signal Process.*, vol. 42, no. 6, pp. 1413–1424, Jun. 1994.
- [16] J. Sanchez-Araujo and S. Marcos, "Statistical analysis of the propagator method for doa estimation without eigendecomposition," in *Proc. IEEE 8th Workshop on Statistical Signal and Array Processing*. Corfu, Greece, Jun. 1996, pp. 570–573.
- [17] J. Xin and A. Sano, "Computationally efficient subspace-based method for direction-of-arrival estimation without eigendecomposition," *IEEE Trans. Signal Process.*, vol. 52, no. 4, pp. 876–893, Apr. 2004.
- [18] H. Tao, J. Xin, J. Wang, N. Zheng, and A. Sano, "Twodimensional direction of arrival estimation method for a mixture of noncoherent and coherent narrowband signals," in *Proc. IEEE 14th Workshop on Signal Processing Advances in Wireless Communications*. Darmstadt, Germany, Jun. 2013, pp. 440–444.
- [19] H. Liu and W. Zhang, "A novel near-field localization method based on second order statistics," in *Proc. Congress on Image and Signal Processing*. Sanya, China, May 2008, vol. 5, pp. 29–33.