UNDERDETERMINED DOA ESTIMATION OF MULTI-PATH SIGNALS BASED ON ICA AND SPARSE RECONSTRUCTION

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ABSTRACT

This paper proposes a method for estimating the direction of arrival (DOA) of multi-path signals for an underdetermined case, where the number of incident signals exceeds the number of sensors. Some DOA estimation methods have already been proposed for multi-path signals. However, most of them cannot be used for an underdetermined case. To deal with this problem, we propose a novel DOA estimation method which combines independent component analysis (ICA) and sparse reconstruction algorithm. It can estimate DOA of $M \times (M-1)$ incident signals when the number of sensors is M . The simulation results demonstrate the promising performance of the proposed algorithm compared to several state of the art methods.

***Index Terms*—**Underdetermined DOA estimation, ICA, multi-path, sparse reconstruction

1. INTRODUCTION

In real communication environment, signal often transmits with multi-path. And due to the impact of the multi-path aspects, the phenomenon that the number of incident signals exceeds the number of sensors is widespread. In such complicated environment, the existing DOA estimation methods will fail. Therefore, we need to find a novel DOA estimation method for this situation.

At present, the DOA estimation methods can be divided into three categories: subspace-based method, ICA-based method, sparse reconstruction based method. Subspace-based method [1][2] can achieve the approximate optimal performance when sensors and time samples are enough. But when the number of incident signals is less than the number of sensors, all of them will be useless because the noise subspace will disappear. ICA-based method [3][4] can solve the problem of DOA estimation of independent signals when the number of incident signals is not more than the number of sensors. The performance of ICA-based method will be seriously deteriorated to multi-path signals. Sparse reconstruction based method can be divided into three categories by different sparse reconstruction algorithms: MP-based method [5], ℓ_p norm-based method [6] [7] and SBL-based method [8][9]. Compare with subspace-based method and ICA-based method, sparse reconstruction based method can achieve better performance under the condition of low signal-to-noise ratio, small sample and multi-path. But it needs the number of incident signals is less than the number of sensors. It still cannot solve the problem of DOA estimation of multi-path signals for underdetermined cases.

Aimed at the deficiency of the existing algorithm, this paper proposes a novel DOA estimation method based on ICA and sparse reconstruction. First, the array manifold is estimated by ICA algorithm. Then, we see the steering vector in this array manifold as a single time sample of array output, and estimate the DOA of multi-path signals by sparse reconstruction. The theoretical analysis and simulation results demonstrate the proposed method can solve the problem of DOA estimation of multi-path signals for underdetermined cases.

2. PROBLEM FORMULATION

Consider a uniform linear array (ULA) with M elements that receives a mixture of K narrowband stationary signal sources, which are located in the far field of the array with different direction. The mixing model can be written as

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{a}_k s_k(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where the signal $s_k(t)$ impinging on the array with direction angles θ_k for $k = 1, 2, \cdots, K$ are unknown waveforms, $\mathbf{x}(t) = \left[x_1(t), x_2(t), ..., x_M(t)\right]^T$, $\mathbf{s}(t) = \left[s_1(t), s_2(t), ..., s_K(t)\right]^T$, $\mathbf{a}_k = [1, \cdots, e(-j2\pi(M-1)d\sin\theta_k / \lambda)]^T$, λ is the wavelength of the signal carrier, $\mathbf{A} = \left[\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_K\right] \in \mathbb{C}^{M \times K}$, and the noise vector $\mathbf{n}(t) = [n_1(t), n_2(t), ..., n_M(t)]^T$. $n_m(t)$ for $m = 1, 2, \cdots, M$ is the additive noise with variance δ_m^2 and uncorrelated with all source signals.

Consider the array data model in multi-path environment. We assume the source signal $s_k(t)$ via different routes to produce L_k multi-path components that impinging on the array from direction angles θ_{l_k} for $l_k = 1, 2, \cdots, L_k$. At this time, the number of incident signals is $L = \sum_{k=1}^{K} L_k$. And we can get the array data model as long as the steering vector $\mathbf{a}_k$ is slightly adjusted in (1).

$$\mathbf{a}_k = \sum_{l_k=1}^{L_k} \mu_{l_k} \mathbf{b}_{l_k} = \mathbf{B}_k \mathbf{u}_k \tag{2}$$

where $\mathbf{b}_{l_k} = [1, \cdots, e(-j2\pi(M-1)d \sin\theta_{l_k} / \lambda)]^T$, $\mathbf{B}_k = [\mathbf{b}_1, \cdots, \mathbf{b}_{L_k}]$, $\mathbf{u}_k = [\mu_1, \cdots, \mu_{L_k}]^T$, μ_{l_k} is the propagation decay coefficient.

In a real environment, it is likely to cause the phenomenon that the number of incident signals is greater than that of array sensors $L > M$. The existing methods couldn't estimate DOA successfully in this situation. Both subspace-based methods and sparse reconstruction based methods are effective only when $L < M$. ICA-based methods is effective only when $L \le M$. Some underdetermined DOA estimation method [10] can estimate DOA successfully when $L > M$, but all of them are based on the assumption that the source signals are sparse. It is a restrictive assumption. In this paper, we consider more practical assumptions that the source signals are independent.

3. DOA ESTIMATION BASED ON ICA AND SPARSE RECONSTRUCTION

3.1. Motivation and strategy

Through analyses, we know that the array data model in multi-path environment is only different in array manifold matrix $\mathbf{A}$. Obviously, if $K \le M$, we can estimate $\mathbf{A}$ by using the ICA method. But $\mathbf{A}$ becomes more complicated because of multi-path effect, we cannot estimate DOA by the conventional methods. We need seek a new DOA estimation method for this problem.

We can find that (2) is similar to array data model. The steering vector $\mathbf{a}_k$ can be seen as single time sample of the array output when L_k multi-path components impinge on the array of M elements. Because there is only a single time sample of the array output, both the subspace-based method and ICA-based method cannot work. Fortunately, sparse reconstruction based method which was proposed recently can still estimate DOA successfully on the condition of single time sample. If $L_k < M$, we can estimate DOA of multi-path components by spatial sparse reconstructing to $\mathbf{a}_k$.

In this paper, we combine ICA with sparse reconstruction to estimate DOA of incident signal in multi-path environment. First, array manifold matrix $\mathbf{A}$ which is consist of steering vectors is estimated by ICA method, then, DOA of incident signals are estimated by spatial sparse reconstruction to each steering vector. The proposed method can estimate DOA of multi-path signals for an underdetermined case. At the same time, because the energy of direct wave is high than other multi-path component, we can easily distinguish the DOA of direct wave by comparing the peak values of spatial spectra.

3.2. DOA estimation

From complex-valued ICA point of view, the received array data is a mixture of independent signals received from different directions. The mixture matrix is $\mathbf{A}$. We can estimate the demixing matrix $\mathbf{W}$ by ICA method. Then, $\mathbf{A}$ can be estimated by $\mathbf{W}$.

Fewer algorithms for separating complex signal mixtures have been described in the scientific literature. Douglas S C [11] proposed a fixed-point algorithm for the blind separation of arbitrary complex-valued non-Gaussian signal mixtures. The performance of this method meets or exceeds that of existing approaches for complex-valued blind source separation, especially for small data-record lengths. In this paper, we adapt this method to handle with array data model. First, we compute the whiten matrix G. Then, let $\mathbf{v}(n) = \mathbf{G}\mathbf{x}(n)$, $\mathbf{w}_q^T$ denotes the q th row of $\mathbf{W}$, $y(n) = \mathbf{w}_q \mathbf{v}(n)$, $\mathbf{P}_{vv} = \frac{1}{N}\sum_{n=1}^{N} \mathbf{v}(n)\mathbf{v}^T(n)$, $n = 1, 2, \cdots, N$. And N is the total number of snapshots. Then the optimal $\mathbf{w}_q$ can be obtained by using the following iterative equation.

$$\begin{aligned}\mathbf{w}_q(i) = {}^1\!/_N \sum_{n=1}^{N} |y(n)|^2 y(n)\mathbf{v}^*(n) - 2\mathbf{w}_q(i-1) \\ - \mathbf{P}_{vv}^* w_q^*(i-1) w_q^T(i-1)\mathbf{w}_q(i-1)\end{aligned} \tag{3}$$

The estimated array manifold matrix $\hat{\mathbf{A}} = (\hat{\mathbf{W}}\mathbf{G})^{\#}$. $^{\#}$ denotes the pseudo-inverse of the matrix. We cannot identify the matrix $\mathbf{A}$ up to scaling and permutation factors. But there is no influence on our method. For the convenience of explaining our basic idea, we assume there is no permutation. Let $\hat{\mathbf{a}}_k$ denotes the k th column of $\hat{\mathbf{A}}$. Our goal is to find $\theta_1, \theta_2, \cdots, \theta_{L_k}$ through the estimated steering vector $\hat{\mathbf{a}}_k$.

To cast this problem as a sparse representation problem, we introduce an overcomplete representation $\mathbf{B}$ in terms of all possible DOAs of incident signals. Let $\Theta = [\varphi_1, \cdots, \varphi_Q]$ be a sampling grid of all DOAs of interest. We construct a matrix composed of steering vectors corresponding to each potential source location as its columns. Steering vectors $\mathbf{b}(\varphi_q) = [1, \cdots, e(-j2\pi(M-1)d \sin\varphi_q / \lambda)]^T$,and the matrix

$\mathbf{B} = [\mathbf{b}(\varphi_1), \mathbf{b}(\varphi_2), \cdots, \mathbf{b}(\varphi_Q)]$. In this framework $\mathbf{B}$ is known and does not depend on the actual source locations. The spatial overcomplete representation of $\hat{\mathbf{a}}_k$ is as follows.

$$\hat{\mathbf{a}}_k = \mathbf{B}\mathbf{v} + \bar{\mathbf{n}} \quad (4)$$

where, $\bar{\mathbf{n}}$ is small noise component. $\mathbf{v} = [v_1, v_2, \cdots, v_Q]$. If $\varphi_q = \theta_{l_k}$, there is $v_q = \mu_{l_k}$. Otherwise, $v_q = 0$. The problem of DOA estimation is transformed to the problem of sparse spectrum estimation of $\mathbf{v}$, which ideally contains dominant peaks at the true DOAs of incident signals. We can solve this inverse problem via regularizing it to favor sparse signal fields using the l_1 methodology. The appropriate objective function is

$$\min \|\hat{\mathbf{a}}_k - \mathbf{B}\mathbf{v}\|_2^2 + \lambda \|\mathbf{v}\|_1 \quad (5)$$

The optimization criterion is a convex optimization problem for complex data, which can be readily handled by SOC programming [6]. The method of choosing λ can refer to the literature [6].

A DOA estimation method based on ICA and sparse reconstruction is summarized as follows.

Step 1: Compute the whiten matrix $\mathbf{G}$, $\mathbf{v}(n) = \mathbf{G}\mathbf{x}(n)$.

Step 2: Using ICA method to estimate the demixing matrix $\mathbf{W}$.Compute $\hat{\mathbf{A}} = (\hat{\mathbf{W}}\mathbf{G})^{\#}$.

Step 3: Let $\hat{\mathbf{A}} = [\hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2, ..., \hat{\mathbf{a}}_K]$, and $k = 1$. We can see $\hat{\mathbf{a}}_k$ as a single time sample of array output, and estimate DOAs of multi-path component $\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_{L_k}$ by ℓ_1 norm sparse reconstruction algorithm.

Step 4: Let $k = k+1$. If there is $k \le K$, go back to step 3. Otherwise, stop.

3.3. Capability of signal resolution

ICA-based method is able to deal with at most M independent source signals. And sparse reconstruction algorithm is able to deal with at most $\lfloor M/2 \rfloor$ incident source signals under a single time sample case [12]. $\lfloor \upsilon \rfloor$ denotes the maximum integer which isn't bigger than υ. Therefore, the proposed method is able to handle with at most $\lfloor M/2 \rfloor \times M$ incident source signals. In fact, if the amplitude of source signals meets random distribution, the number of incident signals what sparse reconstruction algorithm is able to handle with can achieve to $M-1$. So the proposed method is able to handle with at most $(M-1) \times M$ incident signals. For example, if the number of array sensors is three, the proposed method can deal with at most six incident signals.

4. SIMULATION RESULTS

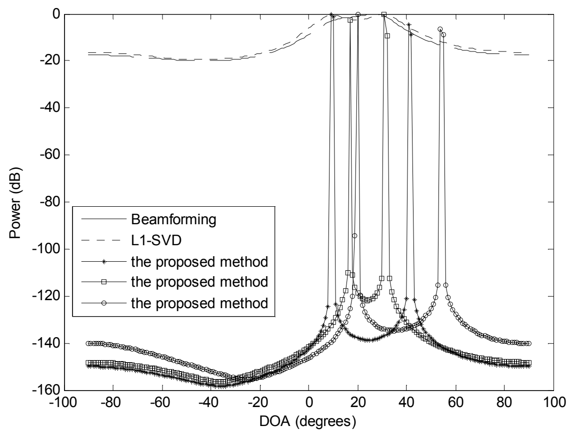
In our experiment, we investigate the performance of the proposed method compared with several state of the art algorithms. First, we compare the spectra of the proposed method to those of $\ell_1 - SVD$ and beamforming for an underdetermined case. Then, we analyze the average root mean square error (RMSE) when different number of incident signals impinging on the array with three sensors.

4.1. Spectra for the proposed method

We assume that there are three independent source signals. Each source signal produces two multi-path components via different transmission paths in the far-field. Six multi-path signals are impinging on a uniform linear array of $M = 3$ sensors from $[10^\circ, 40^\circ]$, $[20^\circ, 50^\circ]$, $[15^\circ, 30^\circ]$. $[\theta_1, \cdots, \theta_k]$ denotes the DOAs of multi-path signals produced by the same source. The space between two adjacent array sensors is one half wavelength. The total number of snapshots is $N = 200$, and the signal-to-noise ratio (SNR) is 20dB. The grid is uniform with 1° sampling $[-90^\circ, 90^\circ]$.

By using ℓ_1 norm sparse reconstruction algorithm to the first column data of estimated array manifold matrix $\hat{\mathbf{A}}$, we can get the spectra of a group of multi-path signals. By dealing with others column data of estimated matrix $\hat{\mathbf{A}}$, we can get the spectra of other signals. In Fig.1, we compare the spectra obtained using the proposed method to spectra obtained using beamforming and $\ell_1 - SVD$. As can be seen from Fig.1, both beamforming and $\ell_1 - SVD$ cannot exhibit peaks at the true DOAs of incident signals, but the proposed method give rise to three effective spectra that exhibit peaks at the true DOAs $[10^\circ, 40^\circ]$, $[20^\circ, 50^\circ]$, $[15^\circ, 30^\circ]$ separately.

Fig.1. Spatial spectra for different methods



4.2. RMSE of DOA estimation

We consider that three independent and narrowband source signals in the far field are impinging on the ULA of $M=3$ via L （$6 \geq L \geq 3$） transmission paths. In the experiment, the number of Monte Carlo trials $\eta = 500$, the grid is uniform with 1° sampling $[-90^{\circ}, 90^{\circ}]$. We use average root mean square error (RMSE) as the performance index

$$\text{RMSE} = \sqrt{\sum_{n=1}^{\eta}\sum_{l=1}^{L}(\hat{\theta}_l(n)-\theta_l)^2 / L \bullet \eta} \qquad (6)$$

We compare the performance of the proposed method with those of method proposed in [4] which based on ICA. As can be seen from table 1, when the number of incident signals is three (that is, there is no multi-path effect), the RMSE of the proposed is similar to those of ICA-based method. When the number of incident signals exceeds three, the ICA-based method cannot work, but the proposed method still has an excellent performance. Experimental results show that the proposed method can deal with at most six incident signals (contain multi-path signals) when the number of array sensors is three.

Table 1. The estimation results of two methods for different number of incident signals

The number of incident signals	DOAs of incident signals	RMSE (method in [4])	RMSE (proposed method)
3	10° 20° 30°	0.6336	0.7135
4	$[10^{\circ},40^{\circ}]$ 20° 30°	——	0.7622
5	30° $[10^{\circ},40^{\circ}]$ $[20^{\circ},50^{\circ}]$	——	0.8015
6	$[10^{\circ},40^{\circ}]$ $[20^{\circ},50^{\circ}]$ $[30^{\circ},15^{\circ}]$	——	0.8221

5. CONCLUSION

This paper proposes a new DOA estimation method based on ICA and sparse reconstruction. Compare to conventional DOA methods, the advantage of the proposed method is that it can solve the problem of DOA estimation of multi-path signals for an underdetermined case. When the number of array sensors is M , the proposed method is able to deal with at most $(M-1)\times M$ incident signals. In addition, we can easily distinguish DOAs of direct wave by peaks value. It is meaningful to source localization. Both theoretical analysis and experimental results demonstrate the effectiveness of the proposed method.

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