Orbit Refinement for Software Defined Radio For Space Applications

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***Abstract*—Digital down-conversion in software defined radios in space-to-Earth and Earth-to-space communications is challenging due to potentially large Doppler frequency offsets. The predictability of satellite orbits suggests that a computed Doppler offset could be used to perform an orbit-informed down-conversion. However, publicly available orbital parameters and orbit propagator codes lack the accuracy required for fine Doppler removal. This paper proposes a technique to adjust the orbital parameters until the computed Doppler profile matches the Doppler profile in the received signal. This technique is applied to a real signal recorded from a satellite overpass and is shown to be able to very accurately estimate frequency offsets.**

***Index Terms*—orbit estimation, two-line orbital elements, software defined radio**

I. Introduction

The design and implementation of Space-to-Earth (StoE) and Earth-to-Space (EtoS) communication systems presents many challenges for which software defined radio (SDR) is ideally suited. One of these challenges is the removal of Doppler frequency offsets that arise due to relative motion between a ground terminal and a satellite in orbit. Consider, for example, a satellite in a 500 km altitude circular low-Earth orbit. With an orbital speed of about 7000 m/s and an S-band carrier frequency of $f_c = 2.4$ GHz, the maximum Doppler shift is

$$f_{d,\max} = -\frac{v_{\max}}{c} f_c \approx 56 \text{ kHz}, \tag{1}$$

where c is the speed of light and $v_{\max}$ is the maximum range rate, or range velocity, which is the time rate of change of the distance between the satellite and the ground terminal. Inter-satellite cross-links can experience even higher Doppler shifts. By comparison, Doppler shifts experienced in most terrestrial applications are much lower. A fast-moving car with a speed of 30 m/s (67 mi/hr) experiences a maximum Doppler shift of $f_{d,\max} = 240$ Hz.

Properly designed phase locked loops (PLL) in software defined radios can track frequency offsets such as those arising from Doppler shifts. The pull-in range of a carrier recovery PLL is the range of frequency offsets for which the loop can acquire lock. When a proportional-plus-integral (PI) loop filter is used in the PLL, the pull-in range is given by [1]

$$\Delta f_{\text{pull-in}} \approx 2\pi\sqrt{2}\zeta B_n, \tag{2}$$

where ζ is the damping factor of the loop, and B_n is the noise bandwidth. Another important PLL characteristic is the variance of the error between the true phase of the incoming signal and the phase of the PLL. This phase error variance is given by [1]

$$\sigma^2_{\theta_e} = \frac{B_n}{C/N_0}, \tag{3}$$

where C/N_0 is the carrier power to noise spectral density ratio. These relations illustrate one tradeoff in PLL design: to track large Doppler frequency offsets, a large pull-in range is desired, and this is achieved by having a large noise bandwidth B_n. However, a large B_n allows excessive noise to circulate in the PLL leading to large phase errors according to (3). To appreciate the seriousness of the problem in space-based applications, let $\zeta = 0.5$ (an underdamped loop) and set the PLL pull-in range equal to the maximum Doppler shift for a LEO orbit $f_{d,\max} = \Delta f_{\text{pull-in}} = 56$ kHz. Solving for the noise bandwidth leads to $B_n = 12.6$ kHz. Substituting this into (3) and using $C/N_0 = 80$ and 46 dB in the case of a high-gain antenna (18 m dish) and an omni-directional antenna, respectively, leads to phase error standard deviations of 1 and 33 degrees, respectively. (The C/N_0 figures are for the case of 1 W of transmitter power, a 500 km orbital altitude, and the satellite at 40 degrees above the horizon.) This example illustrates that excessive phase jitter results when the the PLL is given full responsibility for Doppler compensation in a space-Earth communication link. Very large antennas mitigate the problem but are very expensive to build and operate, and they require accurate pointing. Large antennas may be feasible for fixed ground stations but not for mobile terminals. For systems employing small antennas, other strategies for Doppler compensation must be considered.

Large frequency offsets induced by spacecraft motion can be effectively dealt with using frequency locked loops (FLL) [2], [3]. The main disadvantage of FLLs is that FLLs, like PLLs, require time to estimate the frequency offset. Until the locked condition is reached, reliable communication is not achieved.

A. Contributions of This Paper

In the setting of communication with satellites, orbital information is generally available and can be used to eliminate time-varying Doppler frequency offsets. The Doppler frequency offset depends upon the time-varying range $r(t)$ between the transmitter and receiver. If the time-dependent

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position of the spacecraft and ground terminal are known with exactness, then perfect down-conversion can be achieved.

The orbital parameters of many spacecraft are publicly available [4]. These parameters together with local time and ground station coordinates can be input to orbit propagation codes (commercial [5] and open-source [6] versions readily available) to calculate the Doppler frequency offset for any given overpass. This information can be precomputed and used during the overpass. This is beneficial when the received signal power is low and frequency locked loops may have difficulty locking and tracking.

This paper leverages the predictability of satellite orbits to compute the Doppler profile and use it for digital down-conversion. Our goal is to achieve the most accurate down-conversion possible while expending the least computation, making SDR implementation feasible. Orbit informed down-conversion is only possible in the setting of software defined radio where the current position of the ground terminal and the orbital parameters of the spacecraft can be entered into orbit propagation codes to predict the Doppler profile. However, it is well known that the accuracy of orbital parameters and orbit propagators is limited [7]–[10]. Errors in spacecraft position lead to errors in the Doppler shift as well as residual frequency offsets in the baseband signal. Therefore, some means to refine the orbital parameters is needed. We propose a technique to estimate or refine the orbital parameters to cause the predicted Doppler frequency to match the actual Doppler shift present in measured data.

B. Relation to Prior Work

The fundamentals of orbital determination were worked out long ago. Some of these techniques are based on observing the satellite at two positions or three angles, and use simplified two-body dynamics. Advanced techniques achieve greater accuracy by accounting for positional variations in Earth's gravitational field, the influence of the sun, moon, Venus and Jupiter, solar radiation pressure, and atmospheric drag [11], [12]. Many of these effects are encoded into modern orbit propagators [5], [6]. In the current paper, an orbit propagator is used as a black box. The input orbital parameters are adjusted until predicted frequency offsets agree with observed offsets in measured data.

One approach to StoE/EtoS communications is to embed pilot signals in every transmission so that the receiver can detect and correct for Doppler offsets [13]. This approach requires that frequency estimation be performed for every packet transmitted and adds a significant amount of overhead when there are thousands or millions of packets transmitted during, say, a 10 minute overpass.

Another approach to increasing accuracy of orbital predictions is to refine the TLEs using GPS measurements on-board the spacecraft [7], [14]. While this may be a good solution for large spacecraft, smaller classes of spacecraft (micro-, nano-, and pico-satellites) are always constrained by size, weight, and power so that GPS may not be present on the satellite. A further consideration for small satellites is computation and communication. Even when GPS is present, the computations required for orbit estimation and the requirement to communication updates to the ground may exceed computational resources and communication limits. Investigations into accurate estimation of satellite orbits using software defined radios in ground terminals is therefore of great interest to the small-satellite community.

The rest of the paper is organized as follows. Section II provides some background on orbit propagators and orbital parameterizations, and Section III proposes an iterative technique for orbit estimation and applies the proposed technique to real data measured from a beacon transmitting satellite. The conclusions and future work are summarized in Section IV.

II. Orbit Parameters and Propagators

Decades of research on satellite orbits have led to the development of computer codes [15] capable of predicting the location of a satellite at a given time. Given the location (latitude, longitude, altitude) of an observation point on Earth, these codes can compute range $r(t)$ and range-rate $v(t) = dr(t)/dt$ profiles as a function of time t. Then a Doppler frequency profile can be computed according to

$$f_d(t) = -\frac{v(t)}{c} f_c. \tag{4}$$

A software defined radio could use an orbit propagator code to compute the Doppler profile and the perform an orbit-informed down-conversion. This could be implemented in a SDR as follows. The RF front-end performs a fixed down-conversion from the carrier frequency f_c to an intermediate frequency f_i where the signal is sampled at a rate of f_s samples/second. Assuming Nyquist sampling (as opposed to bandpass sampling [16]), the instantaneous frequency of the sampled signal is

$$F(t) = \frac{f_i}{f_s} - \frac{f_c}{f_s}\frac{v(t)}{c}, \tag{5}$$

where capital F is used to denote normalized frequencies in units of cycles/sample. In the sampled data, the apparent Doppler shift is still related to the range-rate $v(t)$. To shift the signal to baseband, both intermediate frequency and the Doppler frequency offsets must be removed. This is accomplished by mixing the sampled signal with the complex sinusoid

$$\begin{aligned}\exp\left(j2\pi \int_0^t F(\tau)d\tau\right) &= \exp\left(j2\pi \left[\frac{f_i}{f_s}t - \frac{f_c}{f_s}\int_0^t \frac{v(\tau)}{c}d\tau\right]\right) \\ &= \exp\left(j2\pi \left[\frac{f_i}{f_s}t - \frac{f_c}{f_s}\frac{r(t)}{c}\right]\right), \end{aligned} \tag{6}$$

where $r(t) = \int_0^t v(\tau)d\tau$ was used in the last step. Therefore, for down-conversion only the range profile $r(t)$ is needed.

A popular approach to determining the orbit of a particular object is to use the Simplified General Perturbations (SGP4) propagator [15], an orbit propagator whose source code is readily available in a variety of computer languages [17]. SGP4 takes as input a set of parameters describing the orbit of

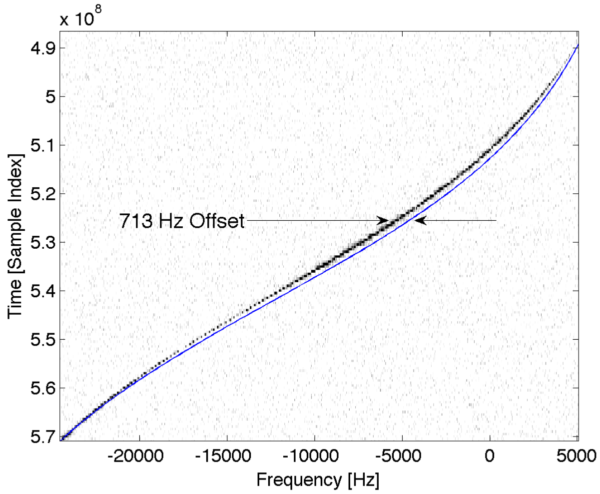

Fig. 1. The spectrogram of the measured signal is the gray scale image in the background. The blue line is the Doppler profile $f_d(t)$ computed by SPG4 using the NORAD TLEs.

the object of interest. The North American Aerospace Defense Command (NORAD) uses radars and electro-optical senors for space surveillance and provides up-to-date ephemeris information to the public [4] in the form of two-line element sets (TLE) for space objects. These TLEs are input to SGP4. SGP4 is a computationally efficient algorithm and could easily run on an SDR processor.

The accuracy of Doppler compensation using a predicted orbit depends on many factors. The SGP4 propagator accounts for disturbances encountered by a spacecraft that perturb its motion away from an ideal Keplerian two-body orbit, which considers only the gravitational attraction between two bodies. To achieve greater accuracy, SGP4 accounts for atmospheric drag, solar radiation, non-spherical Earth, and gravitation from the sun and moon. However, other effects including stochastic phenomena impose additional perturbations and limit the accuracy of SGP4 orbital predictions. Furthermore, the TLEs produced by NORAD are derived from measurements containing noise and uncertainty, and errors in the TLEs lead to errors in orbital predictions. Studies have shown that SGP4 range errors can be as large as 1 km at the starting time (called epoch) and grow at a rate of 1 to 3 km per day [7]–[10]. To maintain accuracy of SGP4 over time, NORAD updates the TLEs on a fairly consistent basis.

As an example, data was recorded during the overpass of a beacon satellite. (For a list of beacon satellites see [18].) The signal was down-converted to a low intermediate frequency and sampled at 500,000 samples/sec. Given the NORAD TLEs for this satellite, the SGP4 propagator was used to compute the range-rate $v(t)$ and the Doppler profile $f_d(t)$, which is plotted on top of the spectrogram of the received signal in Fig. 1. A 713 Hz offset exists between the measured frequency and the calculated frequency. If the computed Doppler profile were used in down-conversion, a 713 Hz residual offset would remain. This offset is due to imprecise TLEs and imperfections in SGP4.

III. TLE Estimation

In the following, the SGP4 orbit propagator is used as a tool to generate Doppler shift $f_d(t)$ profiles. The TLEs are viewed as parameters that can be adjusted to make the SGP4 predicted Doppler profile match the Doppler shift present in received data. Thus, we are searching for TLEs that best explain observed Doppler offsets over an interval of multiple overpasses. This is an approach to orbit determination relying on fitting the predicted frequency to the measured frequency. Conventional approaches to orbit determination rely on measurements of spacecraft position or angle at multiple times [11], [12].

This paper considers a simplified version of the TLE estimation problem by focusing on fitting the Doppler profile for a single overpass. As a further simplification, this paper utilizes data transmitted by a beacon satellite [18] that transmits only a series of radio frequency (RF) tones. Because our ultimate goal is to use communication signals for the estimation, working with beacon signals is a further simplification of the problem. The work reported in this paper provides a foundation upon which to extend to other types of signal waveforms as well as extending to include data from multiple overpasses.

A block diagram of the proposed TLE estimation scheme is shown in Fig. 2. It consists of four steps: (1) the upper branch estimates the frequency of the received signal; (2) the lower branch predicts the frequency of the received signal using SGP4 and the current TLEs; (3) the middle branch computes the mean-squared error (MSE) between the two frequency tables; and (4) a Nelder-Mead optimization step is taken to update the TLEs before returning to step 2. Let us describe each of these steps in turn.

The first operation performed on the sampled signal is frequency estimation. Because the nominal frequency of the signal of interest is known in advance, and because the signal of interest is a pure sinusoid, the chirp z-transform (CZT) [19] is used to efficiently sample the Fourier spectrum over a narrow spectral band. The peak of the Fourier spectrum is taken to be the frequency estimate. To manage the computational complexity, we choose a list of N times $t_1, \cdots, t_N$ during an overpass to apply frequency estimation. In our experiments, $N = 200$ points were chosen uniformly across the overpass.

Using the NORAD TLEs as initial values, SGP4 is used to compute the expected frequencies of the received signal at times $t_1, \cdots, t_N$. The mean-squared value of the error between the two frequency tables is computed.

The Nelder-Mead method [20] is a simple nonlinear optimization technique that is well suited to minimizing the mean-squared frequency error. Only function evaluations are needed, and derivatives are not used. The Nelder-Mead search terminates when the improvement in MSE drops below a threshold. We set this threshold to 10^{-4}. If the final MSE is small, then the SGP4 computed frequency offset is a close match to the frequency in the received signal. The final TLEs from the Nelder-Mead iterations can be used together with SGP4 to compute the range history $r(t)$ and down-convert the signal to baseband using (6). Note that the final

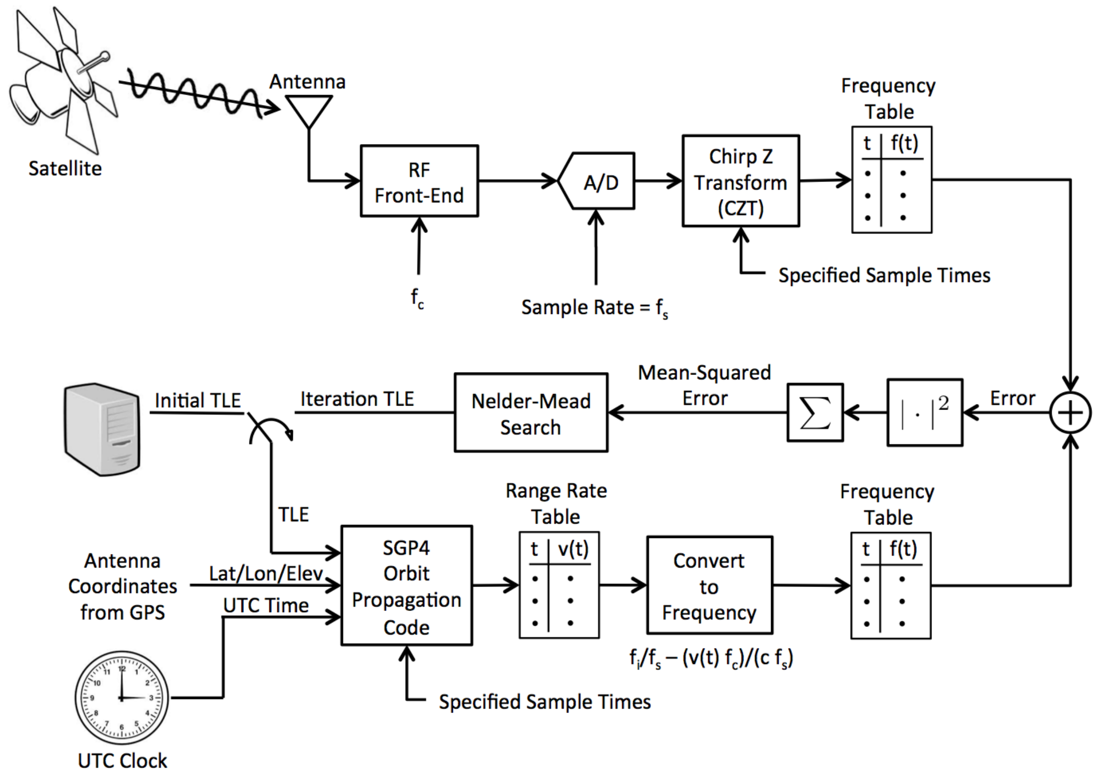

Fig. 2. Block diagram showing the how TLEs are estimated.

TLEs describe a different orbit than the NORAD TLEs that are used for initialization. What this procedure is essentially doing is adjusting the satellite orbit until the SGP4 calculated frequencies match the measured frequencies.

We applied this scheme to the signal whose spectrogram is shown in Fig. 1. The NORAD TLEs were used as initial conditions for the Nelder-Mead search. The final results are shown in Fig. 3, which plots the error between the estimated frequencies of the received signal and the SGP4 computed frequencies. The x-axis represents time and is given in samples from the start of the recorded file. The sample rate is approximately 500,000 samples/second. The overpass is about 10^8 samples in length or 3.5 minutes long. Frequency errors for both the NORAD TLEs and the estimated TLEs are shown. Note the large swings in frequency error for the NORAD TLEs with a maximum error of about 713 Hz. We see that the proposed estimation scheme is effective at finding TLEs whose SGP4 predicted frequencies give an excellent match to the measured frequency profile.

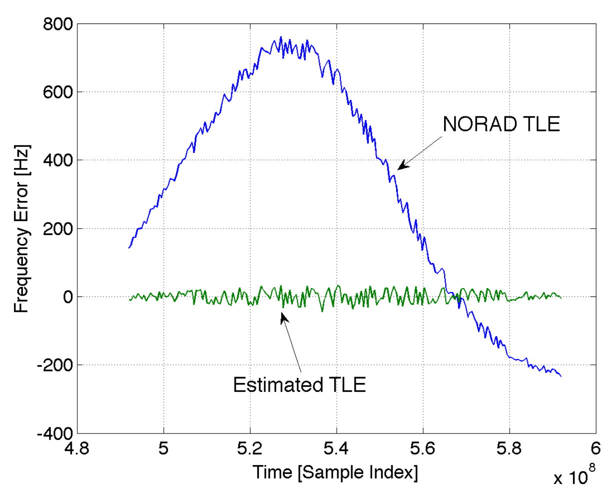

Fig. 3. Plot of the error over time between the estimated frequency of the received signal and the SPG4 computed frequency for the NORAD TLE (blue) and for the estimated TLE (green).

IV. Conclusions

Starting with the NORAD TLEs, the proposed method finds new TLEs that give SGP4 computed frequencies very close to measured frequencies in the received signal. This paper considers the special case of beacon signals and a single overpass. In future work, this will be extended data modulated carrier signals and to incorporate data from multiple overpasses. Down-conversion of data from the current overpass is performed using the TLEs estimated on the previous overpass. On each overpass, improved TLEs are estimated in preparation for the next overpass. The computational capabilities of software defined radio is needed to perform the calculations needed for TLE estimation and to compensate for time-varying frequency offsets for down-conversion.

References

[1] M. D. Rice, *Digital Communications: A Discrete-Time Approach*. Prentice Hall, 2009.

[2] f. harris, E. Venosa, X. Chen, and C. Dick, "Band edge filters perform non data-aided carrier and timing synchronization of software defined radio qam receivers," in *Wireless Personal Multimedia Communications (WPMC), 2012 15th International Symposium on*, 2012, pp. 271–275.

[3] f. harris, "Band edge filters: Characteristics and performance in carrier and symbol synchronization," in *Wireless Personal Multimedia Communications (WPMC), 2010 13th International Symposium on*, Recife, Brazil, Oct. 11-14 2010.

[4] "NORAD two-line element sets: Current data," Available online: http://celestrak.com/NORAD/elements/, accessed June 2013.

[5] Analytical Graphics, Inc., "Systems Tool Kit (STK)," http:\\www.agi.com.

[6] "Pyephem - astronomical computations for python," http://rhodesmill.org/pyephem/.

[7] E. Kahr, O. Montenbruck, and K. P. G. O'Keefe, "Estimation and analysis of two-line elements for small satellites," *Journal of Spacecraft and Rockets*, vol. 50, no. 2, Mar.-Apr. 2013.

[8] D. A. Vallado, "Revisiting STR #3," in *AIAA/AAS Astrodynamics Specialist Conference*. AIAA, Aug. 2006, available online: http://celestrak.com/publications/AIAA/2006-6753/AIAA-2006-6753.pps.

[9] ——, "Revisiting spacetrack report #3," in *AIAA/AAS Astrodynamics Specialist Conference*. AIAA, Aug. 2006.

[10] T. S. Kelso, "Validation of SGP4 and IS-GPS-200D against GPS precision ephemerides," in *17th AAS/AIAA Space Flight Mechanics Conference*, Sedona, AZ, Jan.-Feb. 2007.

[11] D. A. Vallado, *Fundamentals of Astrodynamics and Applications*, 3rd ed. Space Technology Library, Springer, 2007.

[12] O. Montenbruck and E. Gill, *Satellite Orbits: Models, Methods and Applications*. Springer, 2000.

[13] J. A. Larsen, H. P. Mortensen, and J. D. Nielsen, "An SDR based AIS receiver for satellites," in *Recent Advances in Space Technologies (RAST), 2011 5th International Conference on*, 2011, pp. 526–531.

[14] M. R. Green and R. E. Zee, "Increasing the accuracy of orbital position information from NORAD SGP4 using intermittent GPS readings," in *23rd Annual AIAA/USU Conference on Small Satellites*, Logan, UT, Aug. 2009.

[15] F. R. Hoots and R. L. Roehrich, "Spacetrack report no. 3: Models for propagation of NORAD element sets," U.S. Air Force Aerospace Defense Command, Colorado Springs, CO, Tech. Rep., 1980.

[16] R. G. Vaughan, N. L. Scott, and D. R. White, "The theory of bandpass sampling," *IEEE Trans. on Signal Processing*, vol. 39, no. 9, pp. 1973–1984, 1991.

[17] "CSSI downloads," Available online: http://www.centerforspace.com/downloads/, accessed June 2013.

[18] "Zarya: Satellite radio frequencies," Available online: http://www.zarya.info/Frequencies/FrequenciesAll.php, accessed June 2013.

[19] A. V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Processing*, 3rd ed. Prentice Hall, 2010.

[20] N. A. Nelder and R. Mead, "A simplex method for function minimization," *Computer Journal*, vol. 7, pp. 308–313, 1965.