

AN ADAPTIVE MMSE-SIC SOFT DETECTOR WITH ERROR REGULARIZATION FOR ITERATIVE MIMO RECEIVERS

Alexander Krebs, Michael Joham, and Wolfgang Utschick

Associated Institute for Signal Processing, Technische Universität München, Germany

E-mail: {krebs, joham, utschick}@tum.de

ABSTRACT

We present a low complexity soft detector for multiple-input multiple-output (MIMO) channels. Our proposed minimum mean square error successive interference cancellation (MMSE-SIC) detector is based on a regularization mechanism which reduces error propagation in the channel iterative decoder. Although our proposed detector is easy to implement and has a complexity order that is cubic in the number of transmit antennas, it can reach the performance of the soft max-log maximum-likelihood detector (MLD) under realistic system assumptions, as demonstrated in our simulations.

1. INTRODUCTION

It is widely acknowledged that the achievable data rate over a point-to-point link can be increased significantly, when multiple antennas are used for both transmission and reception of a signal [1]. Compared to conventional single antenna systems, signal processing is more complex and requires more computational effort. In real applications, optimal transmission techniques can often not be applied due to cost and complexity constraints.

As an alternative to optimal detection, linear receive filters can be used to equalize the MIMO channel under low computational cost. Linear filtering is especially advantageous w.r.t. soft detection in MIMO receivers, since the computation of the soft information in terms of the log-likelihood ratios (LLRs) is performed symbol-wise. As a consequence, the complexity order of the LLR computation rises only linearly with the number of transmit antennas.

A straight-forward extension of linear filters is known as successive interference cancellation (SIC) and usually comes with a moderate increase in computational complexity. The idea of SIC is to detect and cancel the interfering signals successively. During the SIC, the linear filter is adapted to the residual interference signal. As a result, the performance of the detection of the data is improved in comparison to stand-alone linear filtering. The application of this principle is manifold, it, e.g., has been applied to remove inter-cell interference in a cellular network [2]-[3], interference from neighboring sub-carriers in orthogonal-frequency-division-multiplexing (OFDM) systems [4]-[5], relay systems [6]-[7], or to obtain channel estimates in the presence of interferers [8].

In this paper, we focus on the soft MIMO MMSE-SIC detection of a coded point-to-point MIMO signal in the presence of additive white Gaussian noise (AWGN). For *uncoded* MIMO detection, the application of SIC in this case is most commonly known under the name VBLAST [9], which has been found inapplicable in its plain version to iterative receiver designs due to error propagation in the transmit code block [10]. The application of the MMSE-SIC principle to multi-user detection in a coded DS-CDMA system has been presented in [11] and [12]. Both approaches use the MMSE filter to

regularize the detection error by using a statistical model of the error. This technique applied to point-to-point MIMO detection has been presented in [13] and [14] for non-iterative receivers, using a table lookup technique for 4-QAM and 16-QAM alphabets. A method to prevent error propagation using LLRs has been discussed in [15].

In this work, we take a different approach to the problem, i.e., we delay the occurrence of the first detection error during the SIC using an adaptive MMSE filter. The filter takes into account the distribution of the first detection error, which we derive for the AWGN case. The algorithm we propose is independent of the modulation order, the channel code, and the code length, and can be applied to all regular QAM alphabets. Furthermore, the detector we present requires no a-posteriori information from the channel decoder and allows the design of a very low complexity soft MIMO receiver. The simulation results provided in this paper show, that a significant gain in terms of reduced number of bit errors can be achieved, even when only imperfect channel state information (CSI) is available at the receiver.

We use lowercase-bold and capital-bold characters for vectors and matrices, respectively. We denote the complex conjugate as $(\cdot)^*$, the transpose as $(\cdot)^T$, the conjugate-transpose as $(\cdot)^H$, \mathbf{e}_i denotes the unit norm vector that is all 0 but 1 at the i -th position, $[\cdot]_i$ gives the i th diagonal element of a square matrix, $E[\cdot]$ denotes the expectation, and $\Pr(\cdot)$ denotes the probability measure.

2. SYSTEM MODEL

We consider a MIMO-OFDM system with N_T transmit antennas and N_F orthogonal sub-carriers. The coded binary data vector corresponding to the i -th transmission slot in time domain is denoted as $\mathbf{c}_i \in \mathbb{B}^{N_T N_F \log_2 |\mathbb{M}|}$ where $\mathbb{B} = \{-1, +1\}$. The length of \mathbf{c}_i is further determined by the modulation order $|\mathbb{M}|$, where \mathbb{M} denotes the quadrature amplitude modulation (QAM) alphabet. In order to generate a transmit waveform signal, \mathbf{c}_i is divided into smaller binary vectors $\mathbf{c}_{t,f,i} \in \mathbb{B}^{\log_2 |\mathbb{M}|}$. The binary vector $\mathbf{c}_{t,f,i}$ at the time instance i is associated to the t -th transmit antenna and the f -th OFDM sub-carrier, with $t \in \{1, \dots, N_T\}$ and $f \in \{1, \dots, N_F\}$.

Each binary vector $\mathbf{c}_{t,f,i}$ is mapped to a complex scalar transmit symbol $s_{t,f,i} \in \mathbb{M}$ by the invertible function $m(\cdot)$, i.e.,

$$s_{t,f,i} := m(\mathbf{c}_{t,f,i}) \in \mathbb{M}. \quad (1)$$

For every OFDM sub-carrier f , $\mathbf{s}_{f,i} = [s_{1,f,i}, \dots, s_{N_T,f,i}]^T \in \mathbb{M}^{N_T}$ denotes the vector of symbols transmitted over the N_T antennas that is obtained via

$$\mathbf{s}_{f,i} := \mathbf{m}(\mathbf{c}_{f,i}) = [m(\mathbf{c}_{1,f,i}), \dots, m(\mathbf{c}_{N_T,f,i})]^T \in \mathbb{M}^{N_T} \quad (2)$$

where $\mathbf{c}_{f,i} = [\mathbf{c}_{1,f,i}, \dots, \mathbf{c}_{N_T,f,i}]^T$ is the binary vector corresponding to sub-carrier f and time slot i .

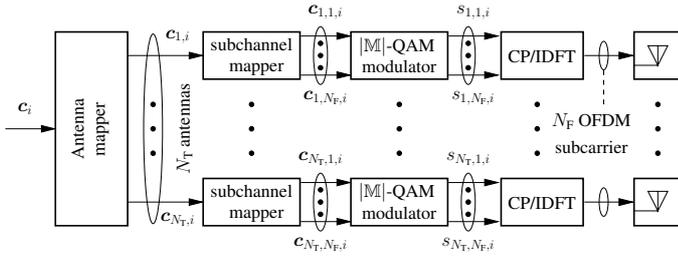


Fig. 1: Transmitter signal flow from the coded binary information c_i at time instance i to the MIMO antenna backend.

The time domain transmission signals are computed by the inverse discrete Fourier (IDFT) transform of the symbols $s_{t,1,i}, \dots, s_{t,N_T,i}$ and a cyclic prefix (CP) is inserted. Fig. 1 depicts the described signal flow from the binary input data vector c_i to the transmit antenna array. We employ the usual linear MIMO model

$$\forall i, f: \mathbf{y}_{f,i} = \mathbf{H}_{f,i} \mathbf{s}_{f,i} + \mathbf{n}_{f,i}. \quad (3)$$

with the channel matrix $\mathbf{H}_{f,i} \in \mathbb{C}^{N_R \times N_T}$ and the zero-mean Gaussian noise $\mathbf{n}_{f,i} \in \mathbb{C}^{N_R}$ whose covariance matrix is $\mathbf{N}_{f,i} = \mathbb{E}[\mathbf{n}_{f,i} \mathbf{n}_{f,i}^H]$ and that is not correlated over time and frequency.

After reception and CP-removal, the discrete Fourier transform (DFT) is applied to the signals of the N_R receive antennas. We assume that an out-dated but otherwise perfect channel estimate $\hat{\mathbf{H}}_{f,i} = \mathbf{H}_{f, \lfloor i/N_P \rfloor N_P}$ is made available periodically every N_P -th time slot to the receiver via a pilot channel. Then, in principle, an array of N_F parallel MIMO detectors computes the LLRs for the individual bits of $c_{t,f,i}$ using the receive vector $\mathbf{y}_{f,i}$ with no exchange of information among the detectors or a-posteriori information from the channel decoder. The single-shot LLRs $\lambda(c_i)$ are then passed to the decoder. The signal flow of the receiver is depicted in Fig. 2.

3. MIMO SOFT DETECTION

Without feedback of a-posteriori information from the channel decoder, the optimal MIMO detector computes the LLR of the n -th bit of $c_{f,i}$ according to the MAP rule as

$$\lambda_{c_{f,i}}(n) = \ln \frac{\Pr(\mathbf{e}_n^T c_{f,i} = +1 | \hat{\mathbf{H}}_{f,i}, \mathbf{y}_{f,i})}{\Pr(\mathbf{e}_n^T c_{f,i} = -1 | \hat{\mathbf{H}}_{f,i}, \mathbf{y}_{f,i})}. \quad (4)$$

The probability that the i -th bit $\mathbf{e}_n^T c_{f,i}$ is $v \in \mathbb{B}$ can be written as $\sum_{\mathbf{m}(\mathbf{x}) \in \mathbb{M}^{N_T} | \mathbf{e}_n^T \mathbf{x} = v} \exp \left\{ -\mathbf{N}_{f,i}^{-1/2} (\mathbf{y}_{f,i} - \hat{\mathbf{H}}_{f,i} \mathbf{m}(\mathbf{x})) \right\}_2^2$ where the sums of exponentials can be approximated by their largest summands (max-log approximation, MLA), i.e.,

$$\lambda_{c_{f,i}}(n) \approx \tilde{\lambda}_{c_{f,i}}(n) = \zeta_{c_{f,i}}(n, -1) - \zeta_{c_{f,i}}(n, +1) \quad (5)$$

with search for the closest symbol vector ($v \in \mathbb{B}$)

$$\zeta_{c_{f,i}}(n, v) = \min_{\mathbf{m}(\mathbf{x}) \in \mathbb{M}^{N_T} | \mathbf{e}_n^T \mathbf{x} = v} \left\| \mathbf{N}_{f,i}^{-1/2} (\mathbf{y}_{f,i} - \hat{\mathbf{H}}_{f,i} \mathbf{m}(\mathbf{x})) \right\|_2^2. \quad (6)$$

Note that the computational complexities of (4) and (6) rise exponentially in N_T . For a high signal-to-noise ratio (SNR) the computational complexity of the MLA approach can be reduced by using a sphere decoder (see e.g., [16], [17]) for finding the closest symbol vector as shown in [18]. The same authors suggest a parallel

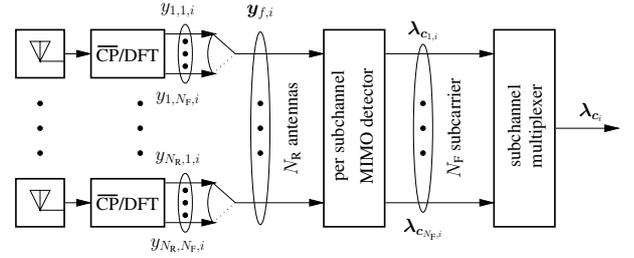


Fig. 2: Receiver signal flow from the receive antenna array to the log-likelihood ratios $\lambda(c_i)$ at time instance i .

approach for computing (6) without requiring a repeated tree search for each individual bit in [19], which mitigates the increase of computational costs w.r.t. using the sphere decoder for hard detection.

3.1. Linear MMSE Soft Detection

In linear detection, a filter \mathbf{G} is applied to the received signal $\mathbf{y}_{f,i}$, e.g., the MMSE filter $\mathbf{G}_{\text{MMSE},f,i} = \arg \min_{\mathbf{G}} \mathbb{E} [\| \mathbf{G} \mathbf{y}_{f,i} - \mathbf{s}_{f,i} \|^2]$, to obtain an estimate for $\mathbf{s}_{f,i}$. For the soft detection of the transmitted bit vectors, the LLR computation is based on symbol-by-symbol hypothesis pairs instead of vector-by-vector pairs as in MAP detection. With $\mathbf{g}_{\text{MMSE},t,f,i}^H = \mathbf{e}_t^T \mathbf{G}_{\text{MMSE},f,i}$, i.e., the MMSE filter to estimate the symbol $s_{t,f,i}$ sent from the t -th transmit antenna, and the corresponding channel vector $\hat{\mathbf{h}}_{t,f,i} = \hat{\mathbf{H}}_{f,i} \mathbf{e}_t$, the LLR of the n -th coded bit contained in the symbol $s_{t,f,i}$ can be found to be

$$\tilde{\lambda}_{c_{t,f,i}}(n) = \frac{1}{1 - \mathbf{g}_{\text{MMSE},t,f,i}^H \hat{\mathbf{h}}_{t,f,i}} \times \left(\min_{\mathbf{m}(\mathbf{x}) \in \mathbb{M} | \mathbf{e}_n^T \mathbf{x} = -1} \left| \mathbf{g}_{\text{MMSE},t,f,i}^H (\mathbf{y}_{f,i} - \hat{\mathbf{h}}_{t,f,i} \mathbf{m}(\mathbf{x})) \right|^2 - \min_{\mathbf{m}(\mathbf{x}) \in \mathbb{M} | \mathbf{e}_n^T \mathbf{x} = +1} \left| \mathbf{g}_{\text{MMSE},t,f,i}^H (\mathbf{y}_{f,i} - \hat{\mathbf{h}}_{t,f,i} \mathbf{m}(\mathbf{x})) \right|^2 \right). \quad (7)$$

Note that (7) is based on the assumption that the noise and interference are complex Gaussian with variance equal to the MMSE when estimating $s_{t,f,i}$. The major benefit of soft linear MMSE detection is that the computational complexity of (7) rises only linearly in N_T . For a detailed discussion of the soft MMSE detector see, e.g., [20].

3.2. MMSE-SIC Soft Detection

A straightforward extension of the linear receiver design is the SIC VBLAST detector, which approximates the LLRs by

$$\tilde{\lambda}_{\text{DF-C}\mu(t),f,i}(n) = \frac{1}{1 - \mathbf{g}_{\text{MMSE-SIC},\mu(t),f,i}^{(t),H} \hat{\mathbf{h}}_{\mu(t),f,i}} \times \left(\min_{\mathbf{m}(\mathbf{x}) \in \mathbb{M} | \mathbf{e}_n^T \mathbf{x} = -1} \left| \mathbf{g}_{\text{MMSE-SIC},\mu(t),f,i}^{(t),H} (\mathbf{y}_{f,i}^{(t)} - \hat{\mathbf{h}}_{\mu(t),f,i} \mathbf{m}(\mathbf{x})) \right|^2 - \min_{\mathbf{m}(\mathbf{x}) \in \mathbb{M} | \mathbf{e}_n^T \mathbf{x} = +1} \left| \mathbf{g}_{\text{MMSE-SIC},\mu(t),f,i}^{(t),H} (\mathbf{y}_{f,i}^{(t)} - \hat{\mathbf{h}}_{\mu(t),f,i} \mathbf{m}(\mathbf{x})) \right|^2 \right)$$

with $\mathbf{g}_{\text{MMSE-SIC},f,i} = \mathbf{e}_t^T \arg \min_{\mathbf{G}} \mathbb{E} [\| \mathbf{G} \mathbf{y}_{f,i}^{(t)} - \mathbf{s}_{f,i}^{(t)} \|^2]$. The LLR computation (8) is almost identical to that of the linear equalizer (7), with the exception that the detection is performed on the receive vector

$$\mathbf{y}_{f,i}^{(t)} = \begin{cases} \mathbf{y}_{f,i} & \text{if } t = 1 \\ \mathbf{y}_{f,i}^{(t-1)} - \hat{\mathbf{h}}_{\mu(t-1),f,i} \hat{s}_{\mu(t-1),f,i} & \text{otherwise.} \end{cases} \quad (9)$$

ℓ	X	$\approx \Pr(s_{\mu(n)} = \ell \hat{s}_{\mu(n)})$
$\hat{s}_{\mu(n)} + a$	E	$P_{\mathfrak{R}+}(1 - P_{\mathfrak{I}+} - P_{\mathfrak{I}-})$
$\hat{s}_{\mu(n)} - a$	D	$P_{\mathfrak{R}-}(1 - P_{\mathfrak{I}+} - P_{\mathfrak{I}-})$
$\hat{s}_{\mu(n)} + ja$	B	$P_{\mathfrak{I}+}(1 - P_{\mathfrak{R}+} - P_{\mathfrak{R}-})$
$\hat{s}_{\mu(n)} - ja$	G	$P_{\mathfrak{I}-}(1 - P_{\mathfrak{R}+} - P_{\mathfrak{R}-})$
$\hat{s}_{\mu(n)} + a + ja$	C	$P_{\mathfrak{R}+}P_{\mathfrak{I}+}$
$\hat{s}_{\mu(n)} - a + ja$	A	$P_{\mathfrak{R}-}P_{\mathfrak{I}+}$
$\hat{s}_{\mu(n)} + a - ja$	H	$P_{\mathfrak{R}+}P_{\mathfrak{I}-}$
$\hat{s}_{\mu(n)} - a - ja$	F	$P_{\mathfrak{R}-}P_{\mathfrak{I}-}$

Table 1: Detection error event probabilities

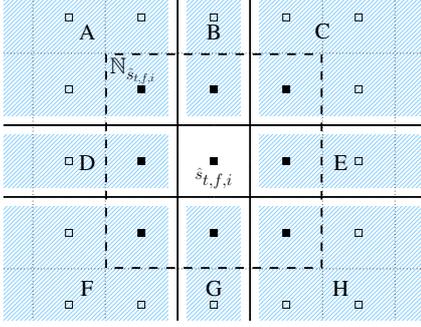


Fig. 3: Decision borders for the set $\mathbb{N}_{\hat{s}_{\mu(t),f,i}}$. Solid lines show the edges of the half-planes of the error probability measure.

The interference cancellation can be highlighted by redefining the transmitted symbol vector as

$$\mathbf{s}_{f,i}^{(t)} = \mathbf{s}_{f,i} - \sum_{j=1}^{t-1} \mathbf{e}_{\mu(j)} s_{\mu(j),f,i}. \quad (10)$$

For an efficient computation of the VBLAST-type detection order $\mu(t)$ and the MMSE-SIC filter $\mathbf{g}_{\text{MMSE-SIC},\mu(t),f,i}^{(t),H}$, see, e.g., [21]. In the interference cancellation process (9),

$$\hat{s}_{\mu(t),f,i} = \underset{s \in \mathbb{M}}{\operatorname{argmin}} |\mathbf{g}_{\text{MMSE-SIC},\mu(t),f,i}^{(t),H} (\mathbf{y}_{f,i}^{(t)} - \hat{\mathbf{h}}_{\mu(t),f,i} s)|$$

is the hard estimate of $s_{\mu(t),f,i}$. Although above derivations were based on the MMSE-SIC filter, with $\beta = \mathbf{g}_{\text{MMSE-SIC},\mu(t),f,i}^{(t),H} \hat{\mathbf{h}}_{\mu(t),f,i}$ we employed the unbiased MMSE-SIC filter

$$\mathbf{g}_{\text{unbiased-MMSE-SIC},\mu(t),f,i}^{(t),H} = \beta^{-1} \mathbf{g}_{\text{MMSE-SIC},\mu(t),f,i}^{(t),H} \quad (11)$$

which increases the mean square error and reduces the SNR. However, it also reduces the error probability (see [22]).

3.3. MMSE-SIC Soft Detection with Error Regularization

Our proposed technique is based upon taking into account that errors can occur in the interference cancellation (9). We therefore rewrite the MIMO model as

$$\mathbf{y}_{f,i}^{(t)} = \mathbf{H} \mathbf{s}_{f,i}^{(t)} + \mathbf{H} \mathbf{d}_{f,i}^{(t)} + \mathbf{n}_{f,i} \quad (12)$$

where $\mathbf{d}_{f,i}^{(t)} = \sum_{j=1}^t (s_{\mu(j),f,i} - \hat{s}_{\mu(j),f,i}) \mathbf{e}_{\mu(j)}$ is the detection error vector during the SIC detection. Note that $\mathbf{d}_{f,i}^{(t)}$ is zero where $\mathbf{s}_{f,i}^{(t)}$ is non-zero and vice versa. Since $\mathbf{s}_{f,i}^{(t)}$ is unknown at the receiver, $\mathbf{d}_{f,i}^{(t)}$ cannot be observed at the receiver either. However, only statistical

knowledge of $\mathbf{d}_{f,i}^{(t)}$ is required to regularize the error. With (12), the MMSE-SIC filter with error regularization (ER) reads as

$$\mathbf{G}_{\text{MMSE-SIC-ER},f,i}^{(t)} = \underset{\mathbf{G}}{\operatorname{argmin}} \mathbb{E} \left[\left\| \mathbf{G} \mathbf{y}_{f,i}^{(t)} - \mathbf{s}_{f,i}^{(t)} \right\|_2^2 \right] \quad (13)$$

$$= \left(\mathbf{H}^H \mathbf{N}_{f,i}^{-1} \mathbf{H} + (\mathbf{S}_{f,i}^{(t)} + \mathbf{D}_{f,i}^{(t)})^{-1} \right)^{-1} \mathbf{H}^H \mathbf{N}_{f,i}^{-1}$$

with $\mathbf{S}_{f,i}^{(t)} = \mathbb{E}[\mathbf{s}_{f,i}^{(t)} \mathbf{s}_{f,i}^{(t),H}]$ and $\mathbf{D}_{f,i}^{(t)} = \mathbb{E}[\mathbf{d}_{f,i}^{(t)} \mathbf{d}_{f,i}^{(t),H}]$. Note that we have neglected covariance matrices $\mathbb{E}[\mathbf{d}_{f,i}^{(t)} \mathbf{n}_{f,i}^{(t),H}]$ and $\mathbb{E}[\mathbf{d}_{f,i}^{(t)} \mathbf{s}_{f,i}^{(t),H}]$ in (13) for the sake of simplicity, which are non-zero when using MMSE filter on low SNR.

Since $\mathbf{d}_{f,i}^{(1)} = \mathbf{0}$, $\mathbf{g}_{\text{MMSE-SIC-ER},\mu(1),f,i}^{(1),H} = \mathbf{e}_{\mu(1)}^T \mathbf{G}_{\text{MMSE-SIC},f,i}^{(1)}$ can be computed as the standard unbiased MMSE filter (11). Then the first symbol can be detected and the interference can be cancelled [cf. (9)]. With the filter output $\tilde{s}_{\mu(1),f,i} = \beta^{-1} \mathbf{g}_{\text{MMSE-SIC-ER},\mu(1),f,i}^{(1),H} \mathbf{y}_{f,i}$ and the hard estimate $\hat{s}_{\mu(1),f,i}$, we can regularize the filter $\mathbf{e}_{\mu(2)}^T \mathbf{G}_{\text{MMSE-SIC-ER},f,i}^{(2)}$ [see (13)] taking into account a possible detection error by updating the elements of $\mathbf{D}_{f,i}^{(2)}$. Thus, we choose the $\mu(1)$ -th diagonal element of $\mathbf{D}_{f,i}^{(2)}$ according to

$$\mathbf{D}_{f,i}^{(2)} = \sum_{j=1}^{t-1} \mathbf{E}_{\mu(j)} \sum_{x \in \mathbb{M}} |x - \hat{s}_{\mu(j),f,i}|^2 p_{\mu(j),f,i}(x) \quad (14)$$

$$\approx \sum_{j=1}^{t-1} \mathbf{E}_{\mu(j)} \sum_{x \in \mathbb{M} \cap \mathbb{N}_{\hat{s}_{\mu(j),f,i}}} |x - \hat{s}_{\mu(j),f,i}|^2 p_{\mu(j),f,i}(x) \quad (15)$$

with $p_{\mu(j),f,i}(x) = \Pr(s_{\mu(j),f,i} = x | \tilde{s}_{\mu(j),f,i})$ and the matrix $\mathbf{E}_{\mu(j)} = \mathbf{e}_{\mu(j)} \mathbf{e}_{\mu(j)}^T$. We omit the off-diagonal entries of $\mathbf{D}_{f,i}^{(2)}$ since they seem to have a negligible impact on the regularization.

By treating the multimodal conditional Gaussian distribution $\Pr(\tilde{s}_{t,f,i} - s_{t,f,i} | s_{t,f,i})$ as unimodal with the equivalent (unbiased) variance $\sigma_{t,f,i}^2 = \beta/(1 - \beta)$, we can approximate the conditional probability $p_{\mu(j),f,i}(x)$ under the application of the Bayes rule with the Q-function $Q(x) = \int_x^\infty (1/\sqrt{2\pi}) \exp(-u^2/2) du$ for each $x \in \mathbb{M}$. With $\mathbf{D}_{f,i}^{(2)}$ given, $\mathbf{G}_{\text{MMSE-SIC-ER}}^{(2)}$ can be computed. With the following detected symbols, the procedure is repeated until all symbols have been detected. The LLRs can be computed after the detection of each symbol analogously to (8).

While the number of summands in (14) rises with the QAM modulation order, we can avoid this complexity by using the lower bound (15) where it is assumed that $s_{\mu(t),f,i} \in \mathbb{M} \cap \mathbb{N}_{\hat{s}_{\mu(t),f,i}}$ with

$$\mathbb{N}_{\hat{s}_{\mu(t),f,i}} = \mathbb{N}_{\bar{s}} = \{\bar{s} + a, \bar{s} - a, \bar{s} + a - aj, \bar{s} - a + aj, \bar{s} + aj, \bar{s} - aj, \bar{s} + a - aj, \bar{s} - a - aj\}.$$

Note that $\mathbb{M} \cap \mathbb{N}_{\hat{s}_{\mu(t),f,i}}$ contains (at most) eight QAM constellation points which are closest to the hard detected symbol $\hat{s}_{\mu(t),f,i}$ in the QAM alphabet (see Fig. 3), where $a = \sqrt{6/(|\mathbb{M}| - 1)}$ is the spacing between the elements of \mathbb{M} . The approximation (15) is especially attractive for higher order modulation alphabets (e.g., $|\mathbb{M}| \in \{256, 1024, \dots\}$), since the number of Q-function evaluations required is independent from the modulation order. Then, for any modulation order, only four probabilities have to be computed, viz.,

$$P_{\mathfrak{R}+} = Q(\sqrt{2} \sigma_{t,f,i}^{-1} (\Re(\hat{s}_{\mu(t)} - \tilde{s}_{\mu(t)}) + a/2)) \quad (16)$$

$$P_{\mathfrak{R}-} = Q(\sqrt{2} \sigma_{t,f,i}^{-1} (a/2 - \Re(\hat{s}_{\mu(t)} + \tilde{s}_{\mu(t)}))), \quad (17)$$

$$P_{\mathfrak{I}+} = Q(\sqrt{2} \sigma_{t,f,i}^{-1} (\Im(\hat{s}_{\mu(t)} - \tilde{s}_{\mu(t)}) + a/2)), \quad (18)$$

$$P_{\mathfrak{I}-} = Q(\sqrt{2} \sigma_{t,f,i}^{-1} (a/2 - \Im(\hat{s}_{\mu(t)} + \tilde{s}_{\mu(t)}))). \quad (19)$$

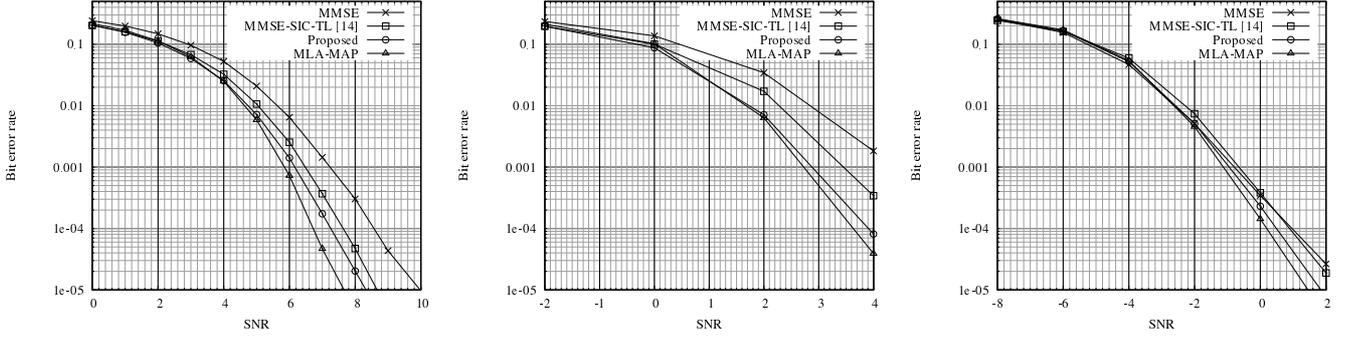


Fig. 4: Bit error rates for a given SNR with 16-QAM modulation for a MIMO system with 3×3 (left) and 6×6 (center) antennas at 5Hz Doppler fading, and a 4×4 system with QPSK modulation at 200 Hz Doppler fading (right).

Since the estimation error $s_{\mu(t),f,i} - \tilde{s}_{\mu(t),f,i}$ is assumed to be circularly symmetric complex Gaussian, the probability $p_{\mu(t),f,i}(x) = \Pr(s_{\mu(t),f,i} = x | \tilde{s}_{\mu(t),f,i})$ can be approximated based on $P_{\Re+}$, $P_{\Re-}$, $P_{\Im+}$, and $P_{\Im-}$ as given in Table 1 by a set of intersecting half-planes in the plane of complex numbers. The edges of the respective half planes define the decision borders for the hard detection of $\hat{s}_{\mu(t),f,i}$ that are shown in Fig. 3. Each half-plane defines a probability event for a detection error resulting from a deviation $\tilde{s}_{\mu(t),f,i} - s_{\mu(t),f,i}$ along the real and/or the imaginary axis. Note that only the (at most) eight neighbors of $s_{\mu(t),f,i}$ are taken into account in (15). For high SNR, however, this approximation has negligible effect on the performance, as a detection error with $|\hat{s}_{\mu(t),f,i} - s_{\mu(t),f,i}| > \sqrt{2}a$ is unlikely.

Since the error covariance matrix is updated during the SIC, the filter (13) must be adapted to the remaining interference before each of the N_T symbol detections. In order to reduce the complexity of the matrix inversion involved in (13), we can use the Sherman-Morrison formula to update the linear filter $\mathbf{G}_{\text{MMSE-SIC-ER},f,i}^{(t)}$. By defining $\mathbf{Q}_{f,i}^{(t)} = (\mathbf{H}^H \mathbf{N}_{f,i}^{-1} \mathbf{H} + (\mathbf{S}_{f,i}^{(t)} + \mathbf{D}_{f,i}^{(t)})^{-1})^{-1}$, we use

$$\mathbf{Q}^{(t+1)} = \mathbf{Q}^{(t)} - \frac{\mathbf{Q}^{(t)} \mathbf{e}_{\mu(t)} \mathbf{e}_{\mu(t)}^T \mathbf{Q}^{(t),H}}{\frac{1}{(\sigma_{\mu(t)}^2 - [\mathbf{D}^{(t)}]_{\mu(t)})} - [\mathbf{Q}^{(t)}]_{\mu(t)}}. \quad (20)$$

successively after every symbol detection, where the complexity of the rank-1 update step is $\mathcal{O}(N_T^2)$. The MMSE optimal detection and cancellation order [9, 21] is then successively identified by the largest diagonal element of $\mathbf{Q}^{(t)}$, i.e.,

$$\mu(t) = \underset{t \in \{1, \dots, N_T\} \setminus \{\mu(1), \dots, \mu(t-1)\}}{\operatorname{argmin}} [\mathbf{Q}^{(t)}]_t \quad (21)$$

for $t \in \{1, \dots, N_T\}$. The computation of the LLRs is then performed according to (8). Similar to the linear MMSE filter and the MMSE-SIC without regularization, the resulting MMSE-SIC-ER soft detection algorithm has a computational complexity of $\mathcal{O}(N_T^3)$.

4. SIMULATION RESULTS

To evaluate the performance of the proposed algorithm, we have simulated the MIMO system with i.i.d. Rayleigh fading channels at different Doppler frequencies. We have used the LTE turbo code [23] to generate the transmit vectors \mathbf{c}_i with a coding block length of 6144 bits. We have assumed $N_F = 72$ OFDM sub-carriers with 15 kHz spacing in the same transmission band, a 1/12 time slot of cyclic prefix as guard interval, and channel estimations every $N_P = 7$ time slots. For the time domain tap channel we have used the ITU Extended Terrestrial Urban (ETU) channel with 9 taps and a maximum

delay spread of $5 \mu\text{s}$ [24]. At the receiver side, we have used the standard iterative BCJR decoder [25] to decode the LLRs given by the MIMO detectors. For the sake of comparison, we have fixed the number of decoding iterations to 6. For our proposed method, we have approximated the error covariance matrix using the half-planes method as described in Subsection 3.3. The simulation results are shown in Fig. 4 for $N_T = N_R \in \{3, 6\}$ with 16-QAM for a slowly time-varying channel and for $N_T = N_R = 4$ with QPSK in a fast fading environment. The table lookup method from [14] is denoted as MMSE-SIC-TL in the plots.

The improvement of the bit error rate through error regularization is already visible for as few as $N_T = 3$ transmitted symbols. In the 3×3 system, the standard MMSE filter performs with a loss of -2.0 dB at a bit error rate of 10^{-4} in comparison to the MLA-MAP detector. The MMSE-SIC-TL regularizes the error partially and provides a 1.0 dB gain to the linear MMSE filter. Our proposed method is as close as 0.4 dB to the MLA-MAP detector and outperforms the MMSE by 1.4 dB and the MMSE-SIC-TL by 0.4 dB. As expected, the relative performance gap of the algorithms decreases slightly when the number of antennas is increased to $N_T = N_R = 6$. Since the number of symbol cancellations increases with the number of antennas, the regularization of early detection errors becomes more prevalent. As a result, we see that the bit error rate of our proposed detector comes closer to the MLA-MAP bit error rate, while the relative performance of the MMSE-SIC-TL method decreases.

While SIC approaches are known to perform poor for imperfect CSI at the receiver, we can see in the 4×4 MIMO system at 200 Hz Doppler frequency that our proposed method is robust to channel estimation errors. For the low SNR regime, the MMSE filter achieves nearly the same performance as the MLA-MAP and the proposed error regularization method, while the MMSE-SIC-TL method performs slightly worse. For higher SNR, we observe that the performance of the MMSE detection degrades, while the error regularization methods show a constant performance gap to MLA-MAP detection. Also in this case, our proposed detector outperforms the MMSE-SIC-TL by 0.4 dB in terms of SNR.

5. CONCLUSION

We have shown in this paper, that the MMSE-SIC principle can be applied for the detection of coded signals in iterative receivers by preventing the propagation of the detection errors by a regularization with a significant performance gain compared to linear MMSE filtering. The resulting algorithm has a fixed runtime, is robust to imperfect CSI at the receiver and is especially applicable to high-order turbo-coded QAM systems with three or more antennas.

6. REFERENCES

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