OPTIMAL INFORMATION ORDERING IN SEQUENTIAL DETECTION PROBLEMS WITH COGNITIVE BIASES

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Abstract—In this paper sequential detection problems are treated in the context of cognitive biases. We present a general bias model and we design a generalized sequential probability ratio test (GSPRT) to mitigate the bias impact following a composite hypothesis testing approach. We also derive an optimal ordering of the incoming observations for fast detection defined in terms of the average sample number (ASN) of observations. We verify through numerical analysis that the designed detector fulfills the time and accuracy requirements. Results show that its performance emulates that of a Bayesian detector optimized for fast sequential detection in absence of biases.

Index — Cognitive biases, Ordering, Mitigation, GSPRT, Bayesian testing

I. INTRODUCTION

Bayesian inference stands as a model for drawing conclusions based on observing data in the sense that it maximizes the odds of the detected hypothesis from a probabilistic point of view. When it comes to sequential hypothesis testing then the newly coming observations serve to update the level of confidence for decision making on the valid hypothesis. The more observations are made, the less the probability of detecting the wrong hypothesis is. However, when fast detection is desired a trade-off arises between the level of confidence for choosing a hypothesis and the timing constraint. Capturing the optimal balance between the two objectives has gained notable attention in the literature. Yet less attention was paid to perform that taking into account the cognitive biases a human might have when building knowledge of observed data and drawing out conclusions.

The sequential probability ratio test is the optimal binary sequential hypothesis test when the observed data is i.i.d [1]. At every incoming observation a metric is updated and evaluated with respect to fixed thresholds to either decide on some hypothesis or take a new observation. The thresholds are explicit functions of the prescribed probabilities of errors. Optimality of SPRT is extended in [2] to correlated and nonhomogeneous processes and statisical problems with nuisance parameters. Still, for non-stationary independent observations it is proved in [3] that time-varying thresholds are optimal over fixed thresholds though there is no standard way to define this variation. For example, in [4] the thresholds are constructed so that the probability of a false alarm and a miss are upper bounded at every iteration of the sequential test, and then the ASN is minimized by arranging the incoming data in the non-decreasing order of the Kullback Leibler (KL) divergence. The idea of presenting significant information at the start of the sequential test is also utilized but in a different context in [5]. Data is preprocessed using short/medium fast Fourier transforms (FFTs) and the FFT outputs are sorted so that the samples in which the signal energy is mostly concentrated enter the sequential test first.

Accommodating human cognitive biases in Bayesian inference

has been considered in a different realm than sequential hypothesis testing. Cognitive biases refer to the well documented tendency of humans to deviate from good judgment. Such biases have been confirmed by reproducible research and result from a number of factors including information that humans are exposed to prior to making a given decision. They can lead to systematic deviations from unbiased and rational decisionmaking. In [6] they try to produce an optimal time-accuracy trade-off as a function of the number of iterations needed to estimate a random quantity. A cost function is defined for both the estimation error and the number of observations and the bias is quantified by the time cost and error cost at the stage of yielding the estimate. The bias is thus attributed to the rationality in utilizing the finite resources of the brain when tackling an inference problem and trying to optimize the utility function that involves the time requirement. In [7] and [8] the starting point bias is treated in a willingness to pay (WTP) estimation setup. Successive bid values are introduced by the interviewer and the anchoring effect is modeled as the impact of the initial bid value on the posterior WTP. The yeasaying bias is also addressed in [8] where the data distribution is adjusted by an additive half-normal random variable that compensates for the interviewee's willingness to accept higher bid values.

In this paper we treat the sequential hypothesis testing problem when cognitive biases affect the decision making while keeping the ASN of observations and probability of error at a minimum. In Section II we motivate the problem through a case study and formalize its statement. In Section III we design a GSPRT detector to meet the problem requirements. The performance of the detector is evaluated through numerical analysis in Section IV. Section V concludes the paper.

II. PROBLEM STATEMENT

In this section, we will study the problem of interviewing a candidate for a given position as an example of sequential detection problems with cognitive biases. An interviewee is asked a series of questions and the grade evaluation for each answer is modeled as a Gaussian random variable of mean representing the average knowledge of the interviewee about the subject and variance characteristic of the fluctuation of the interviewee's ability to express that knowledge. We will assume that only two outcomes are possible. Either the interviewee fails to meet expectations. The questions are of different importance so they are weighted differently. We can always deduct the average grade for a particular question from the "observed" grade given any hypothesis, and thus the observations can be reformed as follows:

$$H_0: Y_n = W_n, \forall n \ge 1$$

$$H_1: Y_n = m_n + W_n, \forall n \ge 1$$
(1)

where $W_n \sim N(0, \sigma^2)$ are iid and m_n is the difference in the means of the n^{th} evaluation under the two hypotheses.

The interviewer has to decide which hypothesis is true. This is a sequential detection problem. With l_n being the log-likelihood ratio for observation Y_n , and L_n being the cumulative log-likelihood ratio up to observation Y_n , then

$$L_n = \log\left(\prod_{i=1}^n \frac{f_i(Y_i|H_1)}{f_i(Y_i|H_0)}\right) = \sum_{i=1}^n \log\left(\frac{f_i(Y_i|H_1)}{f_i(Y_i|H_0)}\right) = \sum_{i=1}^n l_i$$
(2)

and $l_i = \frac{2m_i Y_i - m_i^2}{2\sigma^2}$. L_n is compared to thresholds a_n and b_n . We assume here that a_n and b_n are absolute thresholds. These thresholds will be modified by the cognitive biases of the interviewer. We will model the cognitive bias modification by a data dependent correction factor Δ_n . We will develop a model for Δ_n below. As in regular sequential detection problems, when $L_n > b_n H_1$ is assumed true, the interviewee is qualified and the test terminates. When $L_n < a_n H_0$ is assumed true, the test terminates and the interviewee is disqualified. When $a_n \leq L_n \leq b_n$ is true an additional question should be directed to the interviewee.

Given a prescribed probability of false alarm P_f and probability of a miss P_m , it is required to design a_n and b_n so that $P_f^{(n)} = P(a_1 < L_1 < b_1, ..., a_{n-1} < L_{n-1} < b_{n-1}, L_n \ge b_n |H_0) \le P_f$ and $P_m^{(n)} = P(a_1 < L_1 < b_1, ..., a_{n-1} < L_{n-1} < b_{n-1}, L_n \le b_n |H_1) \le P_m$ at stage n. This should accommodate the fact that the interviewer may be biased by the last n-1 answers of the interviewee. In addition, questions should be ordered for the fastest detection of the correct hypothesis.

III. SEQUENTIAL TEST DESIGN

In this section we design a general sequential probability ratio test to detect the correct hypothesis in the problem statement under the constraints of time and accuracy and in the presence of data-dependent bias. The test design is split into three stages: the computation of the thresholds, the validation of the convergence test conditions and the ordering of the observations.

A. Computation of the GSPRT Thresholds

Assume that the interviewer is subject to some bias, then given the same distribution of observations Y_n , the interviewer perturbs thresholds a_n and b_n and then decides on what hypothesis is true or whether to ask another question by comparing L_n to the new thresholds. Since the interviewer's tendency to qualify the interviewee is consistent with the tendency not to reject the interviewee and vice-versa, then a_n and b_n are perturbed by the same quantity Δ_n . Denote by $g(L_i|H_0)$ and $g(L_i|H_1)$ the distributions of L_n under hypotheses H_0 and H_1 respectively. Then we have the following:

$$P_f^{(n)} \le p_f(n) = \int_{b_n - \Delta_n}^{\infty} g(L_i|H_0) dL_i \tag{3}$$

$$P_m^{(n)} \le p_m(n) = \int_{-\infty}^{a_n - \Delta_n} g(L_i | H_1) dL_i$$
(4)

We note that any variation in the distribution of L_n can be captured by assigning to Δ_n an adequate random distribution. Therefore we can assume that $g(L_n|H_0)$ and $g(L_n|H_1)$ are given by the respective distributions $N\left(-\frac{\sum_{i=1}^n m_i^2}{2\sigma^2}, \frac{\sum_{i=1}^n m_i^2}{\sigma^2}\right)$ and $N\left(\frac{\sum_{i=1}^n m_i^2}{2\sigma^2}, \frac{\sum_{i=1}^n m_i^2}{\sigma^2}\right)$ in the presence of a bias. Plugging the corresponding expressions in (3) and (4) we get

$$p_f(n) = \frac{\sigma}{\sqrt{2\pi\sum_{i=1}^n m_i^2}} \int_{b_n}^{\infty} e^{-\frac{\left(\frac{x-\Delta_n + \sum_{i=1}^n m_i^2}{2\sigma^2}\right)^2}{2\times \sum_{i=1}^n m_i^2}} dx \quad (5)$$

$$p_m(n) = \frac{\sigma}{\sqrt{2\pi\sum_{i=1}^n m_i^2}} \int_{-\infty}^{a_n} e^{-\frac{\left(x - \Delta_n - \frac{\sum_{i=1}^n m_i^2}{2\sigma^2}\right)^2}{2 \times \frac{\sum_{i=1}^n m_i^2}{\sigma^2}}} dx \quad (6)$$

Letting Δ_n denote the cumulative data-dependent bias at observation *n*, we suggest the following bias model:

$$\Delta_n = \sum_{i=1}^{n-1} \alpha_{in} \left(l_i - \frac{m_i^2}{2\sigma^2} + \frac{m_i t h_i}{\sigma^2} \right) = \sum_{i=1}^{n-1} \frac{\alpha_{in} m_i (Y_i + t h_i - m_i)}{\sigma^2}$$
(7)

This model is fully characterized by α_{in} and th_i , $1 \leq i < n$. In particular, Δ_n is chosen as a linear combination of the Y_i s in the α_{in} terms, thus mimicking the anchoring bias modeling in [7] and [8]. On the other hand, the th_i terms model the shift in the interviewer's attitude and thus generalize the yeasaying model in [8]. To see this, we first assume without loss of generality that $\alpha_{in} \geq 0$. Notice that when the equality holds the bias is absent. The evaluation of the i^{th} answer positively impacts the evaluation of the n^{th} answer when the former exceeds the threshold mark $m_i - th_i$. A fair value of th_i is zero. Positive th_i improves the evaluation in later observations while negative th_i establishes a negative future attitude from the interviewer. The larger α_{in} is then the more the bias impact stands out. Therefore th_i is a shifting factor while α_{in} is a scaling factor.

Denote by μ_{L_n} the mean of L_n in presence of Δ_n , then

$$H_{0}: \mu_{L_{n}} \sim N\left(\sum_{i=1}^{n-1} \frac{\alpha_{in}m_{i}(th_{i}-m_{i})}{\sigma^{2}} - \frac{\sum_{i=1}^{n}m_{i}^{2}}{2\sigma^{2}}, \sum_{i=1}^{n-1} \frac{\alpha_{in}^{2}m_{i}^{2}}{\sigma^{2}}\right)$$
$$H_{1}: \mu_{L_{n}} \sim N\left(\sum_{i=1}^{n-1} \frac{\alpha_{in}m_{i}th_{i}}{\sigma^{2}} + \frac{\sum_{i=1}^{n}m_{i}^{2}}{2\sigma^{2}}, \sum_{i=1}^{n-1} \frac{\alpha_{in}^{2}m_{i}^{2}}{\sigma^{2}}\right)$$
(8)

The transformed problem defined by importing Δ_n into the mean of L_n falls under composite hypothesis testing [9]. Given that μ_{L_n} has a prior distribution, we proceed by replacing it in the distribution of L_n by its Bayesian least square (BLS) estimate.

Consider a linear combination of the Gaussian random variables μ_{L_n} and L_n . It is a Gaussian random variable and therefore μ_{L_n} and L_n are jointly Gaussian. We derive the BLS estimate of μ_{L_n} based on observing L_n and consequently $Y_i \forall 1 \leq i \leq n$. For the jointly Gaussian case, the BLS estimate is a linear least square (LLS) estimate.

Deducting from μ_{L_n} and L_n their respective means we get

$$H_{0}: \mu_{L_{n}} - \bar{\mu}_{L_{n}} = \sum_{i=1}^{n-1} \frac{\alpha_{in} m_{i} Y_{i}}{\sigma^{2}}$$

$$H_{1}: \mu_{L_{n}} - \bar{\mu}_{L_{n}} = \sum_{i=1}^{n-1} \frac{\alpha_{in} m_{i} (Y_{i} - m_{i})}{\sigma^{2}}$$
(9)

and

$$H_{0}: L_{n} - \bar{L}_{n} = \sum_{i=1}^{n} \frac{m_{i}Y_{i}}{\sigma^{2}}$$

$$H_{1}: L_{n} - \bar{L}_{n} = \sum_{i=1}^{n} \frac{m_{i}(Y_{i} - m_{i})}{\sigma^{2}}$$
(10)

Under both hypothesis the covariance terms are

$$\Lambda_{\mu_{L_n},L_n} = E\left[(\mu_{L_n} - \bar{\mu}_{L_n})(L_n - \bar{L}_n)\right] = \sum_{i=1}^{n-1} \frac{\alpha_{in} m_i^2}{\sigma^2}$$
(11)

and

$$\Lambda_{L_n} = E\left[(L_n - \bar{L}_n)^2\right] = \sum_{i=1}^n \frac{m_i^2}{\sigma^2}$$
(12)

The BLS estimate $\hat{\mu}_{L_n}(Y) = \bar{\mu}_{L_n} + \Lambda_{\mu_{L_n},L_n} \Lambda_{L_n}^{-1}(Y - \mu_Y)$ of μ_{L_n} becomes

$$H_{0}: \hat{\mu}_{L_{n}}(Y) = \sum_{i=1}^{n-1} \frac{\alpha_{in}m_{i}(th_{i} - m_{i})}{\sigma^{2}} - \frac{\sum_{i=1}^{n}m_{i}^{2}}{2\sigma^{2}} \\ + \frac{\sum_{i=1}^{n-1}\alpha_{in}m_{i}^{2}}{\sum_{i=1}^{n}m_{i}^{2}} \times \frac{\sum_{i=1}^{n}m_{i}Y_{i}}{\sigma^{2}} \\ H_{1}: \hat{\mu}_{L_{n}}(Y) = \sum_{i=1}^{n-1} \frac{\alpha_{in}m_{i}th_{i}}{\sigma^{2}} + \frac{\sum_{i=1}^{n}m_{i}^{2}}{2\sigma^{2}} \\ + \frac{\sum_{i=1}^{n-1}\alpha_{in}m_{i}^{2}}{\sum_{i=1}^{n}m_{i}^{2}} \times \frac{\sum_{i=1}^{n}m_{i}(Y_{i} - m_{i})}{\sigma^{2}}$$
(13)

Replace the means of the integrated pdfs in (5) and (6) by their estimates in (13). Following the same approach as in [4] we set $p_f(n)$ and $p_m(n)$ to the prescribed probabilities P_f and P_m and solve for the thresholds at the boundary of the integration. We obtain

$$b_{n} = max \left(\frac{Q^{-1}(P_{f})\sqrt{\sum_{i=1}^{n}m_{i}^{2}}}{\sigma} - \frac{\sum_{i=1}^{n}m_{i}^{2}}{2\sigma^{2}} + \frac{\sum_{i=1}^{n-1}\alpha_{in}m_{i}^{2}}{\sum_{i=1}^{n}m_{i}^{2}} \times \frac{\sum_{i=1}^{n}m_{i}Y_{i}}{\sigma^{2}} + \sum_{i=1}^{n-1}\frac{\alpha_{in}m_{i}(th_{i}-m_{i})}{\sigma^{2}}, 0 \right)$$
(14)

and

$$a_{n} = min \left(\frac{Q^{-1}(1-P_{m})\sqrt{\sum_{i=1}^{n}m_{i}^{2}}}{\sigma} + \frac{\sum_{i=1}^{n}m_{i}^{2}}{2\sigma^{2}} + \frac{\sum_{i=1}^{n-1}\alpha_{in}m_{i}^{2}}{\sum_{i=1}^{n}m_{i}^{2}} \times \frac{\sum_{i=1}^{n}m_{i}(Y_{i}-m_{i})}{\sigma^{2}} + \sum_{i=1}^{n-1}\frac{\alpha_{in}m_{i}th_{i}}{\sigma^{2}}, 0 \right)$$
(15)

where Q is the standard Q-function. The comparison to zero allows us to make sure that $a_n < 0 < b_n$ so that the sequential test is valid. From (5) and (6), $p_f(n)$ and $p_m(n)$ are upper bounds of $P_f^{(n)}$ and $P_m^{(n)}$ and therefore at any stage of observations the probabilities of a false alarm and a miss are always below the prescribed values for P_f and P_m respectively.

B. Conditions for Convergence

In order to have a finite number of observations before the test terminates, we need $\tilde{P}_n = P(a_n < L_n < b_n | H_1) \rightarrow 0$ as n tends to ∞ . Using the above expressions of a_n and b_n we have

$$\tilde{P}_{n} = P\left(\frac{a_{n} - \hat{\mu}_{L_{n}|H_{1}}}{\operatorname{var}(L_{n})} < \frac{L_{n} - \hat{\mu}_{L_{n}|H_{1}}}{\operatorname{var}(L_{n})} < \frac{b_{n} - \hat{\mu}_{L_{n}|H_{1}}}{\operatorname{var}(L_{n})}|H_{1}\right)$$

$$= 1 - P_{m} - Q\left(\frac{Q^{-1}(P_{f}) + \hat{\mu}_{L_{n}|H_{0}} - \hat{\mu}_{L_{n}|H_{1}}}{\operatorname{var}(L_{n})}\right)$$
(16)

Setting \tilde{P}_n to zero we obtain

$$Q^{-1}(P_f) - Q^{-1}(1 - P_m) - \frac{\sqrt{\sum_{i=1}^n m_i^2}}{\sigma} = \epsilon$$
(17)

where $\epsilon > 0$. This is only valid for an energy sequence of observations: $lim_{n\to\infty}\sum_{i=1}^n m_i^2 < \infty$. Interestingly, by (17) convergence conditions are independent of any bias term.

C. Ordering of the Observations

We desire an ordering of observations for a fast hypothesis detection within prescribed error bounds. Note the following:

$$E[\hat{\mu}_{L_n|H1} - \hat{\mu}_{L_n|H_0}] = \frac{\sum_{i=1}^n m_i^2}{\sigma^2} + \frac{\sum_{i=1}^{n-1} \alpha_{in} m_i^2}{\sigma^2} \quad (18)$$

The expected mean difference between the 2 hypotheses thus depends on α_{in} but not th_i . Inspecting (7) α_{in} affects the variance of Y_i while th_i affects its mean. Since all observations have the same variance then the bias will not vary the order of KL divergence and from [4] fastest detection is ensured by sorting the observations in the decreasing order of their means.

IV. RESULTS AND ANALYSIS

In this section we validate the various design stages of the GSPRT detector. The sequence m_i is an exponential decay with ratio r = 0.96 and $m_1 = 1$. $P_f = P_m = 10^{-3}$ and $\sigma^2 = 0.4$ in order to satisfy (17).

We choose two bias schemes. For both schemes $\alpha_{in} > 0, 1 \le i \le n-1$ so that the bias, if it exists, increases with the better performance of the interviewee. Moreover, we choose $\sum_{i=1}^{n-1} \alpha_{in} < 1$ so that by (18) the bias contributes to the evaluation of the n^{th} answer by less than what the n^{th} answer itself does. In the first scheme, referred to as the *first impression* scheme, $\alpha_{in} = \frac{6}{\pi^2} \frac{1}{i^2} \beta$. The intuition is that the first answers are weighted with higher α -values and thus the more fit is the first set of answers, the higher the evaluation the interviewee receives throughout the interview. In the second scheme, referred to as the *short-term memory* scheme, $\alpha_{in} = \frac{6}{\pi^2} \frac{1}{(n-i)^2} \beta$. In this case it is mainly the

interviewee's last previous set of answers that could bias the evaluation of the current answer since it is associated with high α -weights. For both schemes, β is a scaling factor that increases with the bias, $0 \le \beta < 1$ and $\frac{6}{\pi^2}$ is a normalization factor.

We test the detector under H_1 . Since the potential error under this hypothesis is a miss, we let $th_i = -\frac{m_i}{2} < 0$ so that from (7) the interviewer is picky in the evaluation.



Fig. 1. Sample mean thresholds for the GSPRT in presence of bias under H_1 and H_0 .

In figure 1 thresholds a_n and b_n are presented for the *short-term memory* scheme. Since they are data-dependent, we only show their expected values for both H_1 and H_0 . The solid lines are for H_1 and the dashed lines are for H_0 . Since under H_1 the observations have a higher mean, the solid lines lie above the dashed lines. At the test start, less information is available and the thresholds bulge out. By (17) a_n and b_n converge to their limits in a finite time and are independent of the valid hypothesis. The asymmetry of the thresholds for both H_1 and H_0 is an indicator of a present bias. For $\alpha_{in} > 0$, a_n and b_n decrease when th_i decreases to counteract the negative attitude of the interviewer and vice-versa.



Fig. 2. Probability of a miss for different biases and bias treatments.

In figure 2 we check the validity of setting the mean of L_n to its BLS estimate for correct hypothesis testing in the presence of a bias. Results are presented for the two bias schemes. Observations are sequenced in the decreasing order of their means and an upper estimate $\lceil P_m^n \rceil$ of P_m^n is

obtained by running the GSPRT over the input samples for 10000 iterations. Note that an exact evaluation of P_m^n through simulations is tedious since the test terminates at an arbitrary stage n. For each scheme we evaluate (14) and (15) for two cases: once we plug for the α_i terms their values characteristic of each bias, and once we plug zeros. In the latter case we aim at checking how the detector performs when the bias is neglected. We then repeat the tests for different values of β . Noticeably, no matter how β grows below unity, $\lceil P_m^n \rceil$ remains close to the prescribed value $P_m = 10^{-3}$ for both schemes. This is not true when the bias is not treated and $\lceil P_m^n \rceil$ grows monotonically. The error growth for the shortterm memory scheme overwhelms that of the first impression scheme where the α -weights are higher for the first incoming data samples. Their high means facilitate the detection of H_1 and the corresponding error growth remains limited.



Fig. 3. Average sample number for the GSPRT for different orderings of the observations.

The optimal ordering of the observations is suggested in figure 3. Define the *p*-reverse ordering as the arrangement of the observations in decreasing order of their means followed by reversing the first p samples. The ASN is plotted against different *p*-reverse orderings. The ASN increases with p for the no-bias scheme and the two bias schemes and thus the 1-reverse ordering is optimal. The first impression scheme represents a scenario where the bias serves to terminate the GSPRT faster and with the correct conclusion under H_1 when the optimal ordering is adopted. This is because the higher means are weighted more in the bias. The short-term memory scheme represents a scenario where the bias serves to terminate the GSPRT slower under H_1 when the optimal ordering is adopted. This is because the low-mean observations are more emphasized in the bias. The reduced gap in ASN in the presence and absence of bias points out the optimality of the bias treatment given that all α_{in} and th_i terms are known at the design stage of the detector.

V. CONCLUSION

In this paper we presented the design of a sequential hypothesis testing problem where cognitive biases are involved in the decision process. The problem was transformed into composite hypothesis testing where a generalized model was employed to to capture the impact of the biases on the decision thresholds. Sorting the observations in the decreasing order of their means was suggested for fast detection, and numerical analysis validated the test design for both accuracy and speed.

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