ANALYSIS OF THE CROSS-TARGET MEASUREMENT FUSION LIKELIHOOD FOR RSSI-BASED SENSORS

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ABSTRACT

In this paper an analysis is conducted regarding the likelihood function of an RSSI-based sensor measurement that is affected by a target of interest (TOI) and an interfering target source. The interferer's true location is unknown but is assumed to be Gaussian distributed with known parameters. This analysis is motivated by its potential application within a multi-agent distributed tracking system, where each agent is tasked with tracking a single TOI while treating others as sources of interference. By exchanging TOI information, each agent can use the results established here to effectively compensate for "out-of-scope" target interference by fusing this external information. An exact analytical form is established for the aforementioned likelihood and a Gaussian approximation is analytically developed. An application of these results is presented through an example scenario, with computer simulation results demonstrating performance.

Index Terms— RSSI-based target tracking, interference modeling, multiple particle filtering

1. INTRODUCTION

Received-Signal-Strength-Indicator (RSSI) sensors have attracted significant attention for use in target localization due to their low cost, relative ease of implementation, and pervasiveness in existing wireless sensor networks. Numerous target tracking algorithms that make specific use of RSSI sensors have been developed, including [1], [2], [3], and [4]. There is a strong interest in developing algorithms suitable for handling multiple targets, along with the ability to maintain tracking in the presence of interference. The specific nature of RSSI-based measurements (namely that each measurement is additively affected by all existing targets) allows these challenges to be more effectively addressed within a multi-agent distributed estimation framework. An algorithm following this concept was proposed in [5] and further developed in [6].

It is the aim of this paper to study a particular form of the measurement likelihood function with a standard RSSI model under the assumption that prior information is available regarding the target that is not directly being tracked. This prior information is assumed to follow a Gaussian distribution that is communicated by other agents within the environment. The approach followed is similar in concept to [7]. However, in that paper ultrasound sensors were considered and approximations to the likelihood were made based on maximum likelihood estimates of the interference locations. The main contributions of this paper are (a) an exact closed form solution for the problem considered, (b) analytical approximations facilitating practical computation, and (c) application of the presented results to a tracking scenario.

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2. PROBLEM FORMULATION

Let us assume that there exists a single TOI moving in a 2D plane that is described by $\mathbf{x}_t = [x_{t,1},\, x_{t,2},\, \dot{x}_{t,1},\, \dot{x}_{t,2}]^\top$ at time t, along with an interfering target, located at $\mathbf{l}_t$, at time t. An RSSI measurement, denoted by y_t is taken at time t, and is modeled as,

$$\begin{aligned} y_t &= y_{\mathbf{x}_t} + y_{\mathbf{l}_t} + v_t \\ &= \left(\frac{\Phi}{\|\mathbf{s}_t - \mathbf{x}_{t,1:2}\|^\alpha + \epsilon}\right) + \left(\frac{\Phi}{\|\mathbf{s}_t - \mathbf{l}_t\|^\alpha + \epsilon}\right) + v_t, \end{aligned} \tag{1}$$

where Φ represents the transmitted power, $\mathbf{s}_t = \left[s_{1,t},\, s_{2,t}\right]^\top$ is the position vector of the sensor at time t, α is the path-loss coefficient, ϵ is a saturation parameter, $y_{\mathbf{x}_t}$ and $y_{\mathbf{l}_t}$ are the respective measurement contributions from the TOI and the interfering target, and v_t represents the uncorrelated sensor noise. With the assumption that $\mathbf{l}_t \sim \mathcal{N}\left(\bar{\mathbf{l}}_t, \mathbf{Q}\right)$ where $\mathbf{Q} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$, we wish to determine an expression for the probability density $f_{y_{\mathbf{l}_t}}\left(y_{\mathbf{l}_t} \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right)$ that would facilitate computation of the likelihood function $f_{y_t}\left(y_t \mid \mathbf{x}_t, \bar{\mathbf{l}}_t, \mathbf{Q}\right)$. Introducing the auxiliary variable θ, one can express the coordinates of $\mathbf{l}_t$ as:

$$\begin{bmatrix} l_{t,1} \\ l_{t,2} \end{bmatrix} = \left(\frac{\Phi}{y_{\mathbf{l}_t}} - \epsilon\right)^{\frac{1}{\alpha}} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + \begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix}. \tag{2}$$

The measurement component $y_{\mathbf{l}_t}$ can then be rewritten as,

$$y_{\mathbf{l}_t} = \frac{\Phi}{\left(\mathbf{d}_t^\top \mathbf{d}_t\right)^{\frac{\alpha}{2}} + \epsilon}, \tag{3}$$

where $\mathbf{d}_t \sim \mathcal{N}\left(\bar{\mathbf{l}}_t - \mathbf{s}_t, \mathbf{Q}\right)$. A simple illustration of the model and defined notation is shown in Fig. 1.

3. EXACT SOLUTION

It is clear that the pair $(y_{\mathbf{l}_t}, \theta)$ maps uniquely to the random vector $\mathbf{l}_t$; thus, an expression for $f_{y_{\mathbf{l}_t},\theta}\left(y_{\mathbf{l}_t}, \theta \mid \bar{\mathbf{x}}_t, \mathbf{Q}\right)$ can be obtained by

$$f_{y_{\mathbf{l}_t},\theta}\left(y_{\mathbf{l}_t}, \theta \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right) = \left|J(\mathbf{l}_t)\right|\big|_{\mathbf{l}_t = \mathbf{l}_t^{-1}} f_{\mathbf{l}_t}\left(\mathbf{l}_t^{-1} \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right), \tag{4}$$

where $\mathbf{l}_t^{-1}$ denotes the value of $\mathbf{l}_t$ that corresponds to $(y_{\mathbf{l}_t}, \theta)$ as in (3) and $|J(\mathbf{l}_t)|$ is the absolute value of the determinant of the Jacobian matrix of the transformation from $(y_{\mathbf{l}_t}, \theta)$ to $\mathbf{l}_t$. Due to space

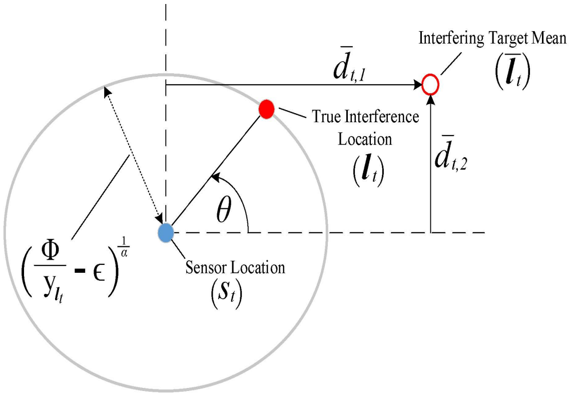

Fig. 1. RSSI interference target geometry.

limitation, we use the following definition for the remainder of the paper,

$$\delta_{y_{1_t}} = \frac{\Phi}{y_{1_t}} - \epsilon. \tag{5}$$

Note that $\delta_{y_{1_t}} \geq 0$, since y_{1_t} is supported on the interval $\left[0, \frac{\Phi}{\epsilon}\right]$. We can then write

$$\begin{aligned}
|J(\mathbf{l}_t)| &= \begin{vmatrix} \frac{\partial l_{t,1}}{\partial y_{1_t}} & \frac{\partial l_{t,1}}{\partial \theta} \\ \frac{\partial l_{t,2}}{\partial y_{1_t}} & \frac{\partial l_{t,2}}{\partial \theta} \end{vmatrix} \\
&= \begin{vmatrix} \delta_{y_{1_t}}{}^{\left(\frac{1}{\alpha}-1\right)} \left(\frac{-\Phi}{\alpha {y_{1_t}}^2}\right) \cos(\theta) & -\delta_{y_{1_t}}{}^{\frac{1}{\alpha}} \sin(\theta) \\ \delta_{y_{1_t}}{}^{\left(\frac{1}{\alpha}-1\right)} \left(\frac{-\Phi}{\alpha {y_{1_t}}^2}\right) \in (\theta) & \delta_{y_{1_t}}{}^{\frac{1}{\alpha}} \cos(\theta) \end{vmatrix} \\
&= \frac{\Phi}{\alpha {y_{1_t}}^2} \delta_{y_{1_t}}{}^{\left(\frac{2}{\alpha}-1\right)}.
\end{aligned}$$

Writing out the full form of the joint density for a bivariate Gaussian r.v., we can then express the density of the pair (y_{1_t}, θ) as

$$\begin{aligned}
& f_{y_{1_t},\theta}\left(y_{1_t}, \theta \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right) \\
&= \frac{\Phi}{2\pi\alpha\sigma_1\sigma_2 {y_{1_t}}^2 \sqrt{1-\rho^2}} \delta_{y_{1_t}}{}^{\left(\frac{2}{\alpha}-1\right)} \exp\left\{ \frac{-1}{2\left(1-\rho^2\right)} \right. \\
&\times \left[\frac{1}{\sigma_1^2} \left(\delta_{y_{1_t}}{}^{\frac{1}{\alpha}} \cos\theta - \bar{d}_{t,1} \right)^2 + \frac{1}{\sigma_2^2} \left(\delta_{y_{1_t}}{}^{\frac{1}{\alpha}} \sin\theta - \bar{d}_{t,2} \right)^2 \right. \\
&\left.\left. - \frac{2\rho}{\sigma_1\sigma_2} \left(\delta_{y_{1_t}}{}^{\frac{1}{\alpha}} \cos\theta - \bar{d}_{t,1} \right) \left(\delta_{y_{1_t}}{}^{\frac{1}{\alpha}} \sin\theta - \bar{d}_{t,2} \right) \right] \right\}.
\end{aligned} \tag{6}$$

In order to obtain our desired expression, $f_{y_{1_t}}\left(y_{1_t} \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right)$, we must marginalize the auxiliary variable θ by integrating from 0 to 2π. While there does not appear to be an exact simple form for this integral, it has been investigated at lengths under various contexts and numerous approaches have been identified as in [8], [9], and [10]. A particularly attractive solution appears in [11], which is of closed form and involves an infinite series of Bessel function products. The solution is for the case where individual components of the random vector have different variances, but are uncorrelated. A more general solution, allowing for nonzero correlation between components, can be obtained by noting that the argument of the exponential in (6) can be rewritten as

$$\begin{aligned}
& \mathcal{T}_1\left(y_{1_t}, \theta, \bar{\mathbf{l}}_t, \mathbf{Q}\right) \\
&= \frac{-1}{2\left(1-\rho^2\right)} \left(k_1 + \left(k_2 - \sqrt{k_5^2 + k_6^2} \right) \delta_{y_{1_t}}{}^{\frac{2}{\alpha}} \right. \\
&+ \left(\sqrt{k_3^2 + k_4^2} \right) \delta_{y_{1_t}}{}^{\frac{1}{\alpha}} \cos\left(\theta - \phi_1\right) \\
&\left. + 2 \left(\sqrt{k_5^2 + k_6^2} \right) \delta_{y_{1_t}}{}^{\frac{2}{\alpha}} \cos^2\left(\theta - \phi_2\right) \right),
\end{aligned} \tag{7}$$

where the different parameters are defined as

$$\begin{aligned}
k_1 &= \frac{\bar{d}_{t,1}^2}{\sigma_1^2} + \frac{\bar{d}_{t,2}^2}{\sigma_2^2} - \frac{2\rho\bar{d}_{t,1}\bar{d}_{t,2}}{\sigma_1\sigma_2} & k_2 &= \frac{1}{2}\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right) \\
k_3 &= 2\left(\frac{\rho\bar{d}_{t,2}}{\sigma_1\sigma_2} - \frac{\bar{d}_{t,1}}{\sigma_1^2}\right) & k_4 &= 2\left(\frac{\rho\bar{d}_{t,1}}{\sigma_1\sigma_2} - \frac{\bar{d}_{t,2}}{\sigma_2^2}\right) \\
k_5 &= \frac{1}{2}\left(\frac{1}{\sigma_1^2} - \frac{1}{\sigma_2^2}\right) & k_6 &= -\frac{\rho}{\sigma_1\sigma_2}. \\
\phi_1 &= \text{atan2}\left(\frac{k_4}{k_3}\right) & \phi_2 &= \frac{1}{2}\text{atan2}\left(\frac{k_6}{k_5}\right).
\end{aligned} \tag{8}$$

With the argument in this form, we can now make use of the result derived in [12],

$$\begin{aligned}
& \int_0^{2\pi} e^{in\theta} \exp\left[a\cos\left(\theta-\alpha\right) + 2b\cos^2\left(\theta-\beta\right)\right] d\theta \\
&= 2\pi e^b e^{in\alpha} \sum_{j=-\infty}^{\infty} e^{2ij(\alpha-\beta)} \mathrm{I}_{2j+n}\left(a\right) \mathrm{I}_j\left(b\right),
\end{aligned} \tag{9}$$

where $i = \sqrt{-1}$ and $\mathrm{I}_j(\cdot)$ is the j-th order Modified Bessel function of the first kind. Matching the corresponding terms in (9) with those in the integral we aim to solve, and noting that for j an integer, $\mathrm{I}_{-j}\left(x\right) = \mathrm{I}_j\left(x\right)$, and $\mathrm{I}_{2j}\left(-x\right) = \mathrm{I}_{2j}\left(x\right)$, we obtain the final form

$$\begin{aligned}
& f_{y_{1_t}}\left(y_{1_t} \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right) \\
&= \frac{\Phi\delta_{y_{1_t}}{}^{\left(\frac{2}{\alpha}-1\right)}}{\alpha\sigma_1\sigma_2\left(\sqrt{1-\rho^2}\right){y_{1_t}}^2} e^{-\frac{k_1 + k_2\delta_{y_{1_t}}{}^{\frac{2}{\alpha}}}{2\left(1-\rho^2\right)}} \\
&\times \left\{ \mathrm{I}_0\left(\frac{-\sqrt{k_5^2+k_6^2}}{2\left(1-\rho^2\right)} \delta_{y_{1_t}}{}^{\frac{2}{\alpha}} \right) \mathrm{I}_0\left(\frac{\sqrt{k_3^2+k_4^2}}{2\left(1-\rho^2\right)} \delta_{y_{1_t}}{}^{\frac{1}{\alpha}} \right) \right. \\
&+ 2\sum_{j=1}^{\infty} \left[\mathrm{I}_j\left(\frac{-\sqrt{k_5^2+k_6^2}}{2\left(1-\rho^2\right)} \delta_{y_{1_t}}{}^{\frac{2}{\alpha}} \right) \right. \\
&\left.\left. \times \mathrm{I}_{2j}\left(\frac{\sqrt{k_3^2+k_4^2}}{2\left(1-\rho^2\right)} \delta_{y_{1_t}}{}^{\frac{1}{\alpha}} \right) \cos\left(2j\left(\phi_1-\phi_2\right)\right) \right] \right\},
\end{aligned} \tag{10}$$

with $\delta_{y_{1_t}}$ defined in (5), and the rest of the parameters defined in (8).

The shape of $f_{y_{1_t}}\left(y_{1_t} \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right)$ varies dramatically depending on the specific values of the parameters. To demonstrate this and to provide a rough verification of accuracy (formal convergence proofs and accuracy bounds will be addressed elsewhere). a comparison between the truncated-sum approximation to (10), computed with 200 terms, and the empirical histogram (generated by drawing 500,000 samples of $\mathbf{l}_t$, each corresponding to a sample of $y_{\mathbf{l}_t}$) are plotted in Fig. 2 (solid red lines denote the histograms) for three distinct parameter sets, each denoted by S_i and given as

$$S_1 : \left\{ \bar{\mathbf{l}}_t = \begin{bmatrix} -0.2 \\ -0.2 \end{bmatrix}, \quad \begin{matrix} \sigma_1 = 0.5 \\ \sigma_2 = 0.9 \\ \rho = 0.9 \end{matrix} \right\},$$

$$S_2 : \left\{ \bar{\mathbf{l}}_t = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}, \quad \begin{matrix} \sigma_1 = 0.5 \\ \sigma_2 = 0.9 \\ \rho = 0.1 \end{matrix} \right\},$$

$$S_3 : \left\{ \bar{\mathbf{l}}_t = \begin{bmatrix} 1.0 \\ 0.0 \end{bmatrix}, \quad \begin{matrix} \sigma_1 = 2.0 \\ \sigma_2 = 0.9 \\ \rho = -0.5 \end{matrix} \right\}.$$

Note that in all the sets, the other parameters are set to $\phi = 10$, $\alpha = 1.6$, and $\epsilon = 0.8$.

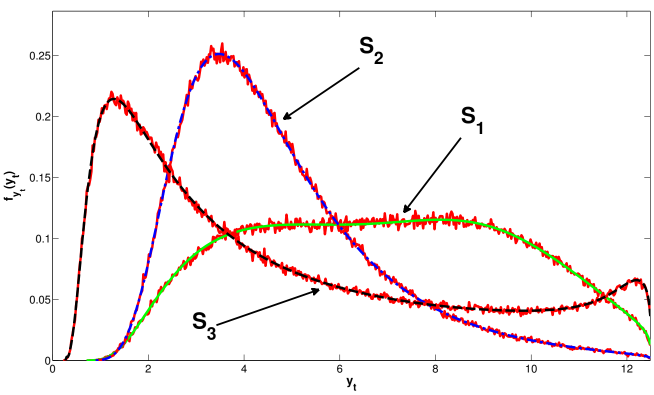

Fig. 2. Interference component density for various parameter sets.

4. SIMPLIFYING APPROXIMATIONS

A dramatic simplification of (10) can be obtained with the restriction that $\rho = 0$ and $\sigma = \sigma_1 = \sigma_2$, yielding $k_5 = k_6 = 0$. Noting that for $j \geq 0$, $\mathrm{I}_j(0) = 0$ and $\mathrm{I}_0(0) = 1$, the expression in (10) then reduces to

$$f_{y_{\mathbf{l}_t}}\left(y_{\mathbf{l}_t} \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right) = \frac{\Phi \delta_{y_{\mathbf{l}_t}}{}^{\left(\frac{2}{\alpha}-1\right)}}{\alpha\sigma^2 {y_{\mathbf{l}_t}}^2} e^{-\frac{1}{2\sigma^2}\left(\bar{d}_{t,1}^2 + \bar{d}_{t,2}^2 + \delta_{y_{\mathbf{l}_t}}{}^{\frac{2}{\alpha}}\right)} \times \mathrm{I}_0\left(\frac{\sqrt{\bar{d}_{t,1}^2 + \bar{d}_{t,2}^2}}{\sigma^2} \delta_{y_{\mathbf{l}_t}}{}^{\frac{1}{\alpha}}\right). \quad (11)$$

Assuming the target prior mean is located at some minimum distance away from the sensor, the constant factor $\frac{\|d\|}{\sigma^2}$, where $\|d\|$ is defined by

$$\|d\| = \sqrt{\bar{d}_{t,1}^2 + \bar{d}_{t,2}^2}$$

within the Bessel function argument in (11) will be large, allowing one to use the large argument approximation for modified Bessel functions, $\mathrm{I}_0(x) \approx \frac{e^x}{\sqrt{2\pi x}}$. As a result, (11) can be approximated as

$$f_{y_{\mathbf{l}_t}}\left(y_{\mathbf{l}_t} \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right) \approx \frac{\phi}{\alpha\sigma\sqrt{2\pi}} \delta_{y_{\mathbf{l}_t}}{}^{\frac{3-2\alpha}{2\alpha}} \frac{\exp\left[-\frac{1}{2\sigma^2}\left(\|d\| - \delta_{y_{\mathbf{l}_t}}{}^{\frac{1}{\alpha}}\right)^2\right]}{{y_{\mathbf{l}_t}}^2 \sqrt{\|d\|}}, \quad (12)$$

which is valid on the domain of $f_{y_{\mathbf{l}_t}}\left(y_{\mathbf{l}_t} \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right)$, $\left[0 \leq y_{\mathbf{l}_t} \leq \frac{\Phi}{\epsilon}\right]$. As long as the factor $\frac{\|d\|}{\sigma^2}$ is sufficiently large, it can be shown analytically that this is well approximated by a Gaussian (a similar approximation can in fact be established with general $\mathbf{C}$), i.e., $f_{y_{\mathbf{l}_t}}\left(y_{\mathbf{l}_t} \mid \bar{\mathbf{l}}_t, \mathbf{C}\right) \approx \mathcal{N}\left({y_{\mathbf{l}_t}}^*, \sigma^*\right)$ with,

$${y_{\mathbf{l}_t}}^* = \frac{\Phi}{\|d\|^\alpha + \epsilon} \qquad \sigma^* = \frac{\Phi\alpha\sigma\left(\bar{d}_{t,1}^2 + \bar{d}_{t,2}^2\right)^{\frac{\alpha-1}{2}}}{\left(\left(\bar{d}_{t,1}^2 + \bar{d}_{t,2}^2\right)^{\frac{\alpha}{2}} + \epsilon\right)^2}. \quad (13)$$

Before we proceed, we point out that if we have a general $\mathbf{C}$ for the target prior, replacing this with $\max(\sigma_1, \sigma_2)\,\mathbf{I}$ can be interpreted as intentionally ignoring some prior information in favor of a more conservative prior. The assumption that $\|d\| >> 0$ can be easily justified in typical tracking conditions.

5. APPLICATION

We now describe a basic scenario demonstrating how the aforementioned results can be applied. Before doing so, we reiterate that the results given in equations (10)-(13) are for the contribution $y_{\mathbf{l}_t}$ to y_t of the interfering target $\mathbf{l}_t$ only. It is straightforward to derive an expression for the measurement likelihood,

$$f_{y_t}\left(y_t \mid \mathbf{x}_t, \bar{\mathbf{l}}_t, \mathbf{Q}\right) = f_{y_{\mathbf{l}_t}}\left(y_t - y_{\mathbf{x}_t} \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right) * f_{v_t}\left(v_t\right), \quad (14)$$

where $*$ denotes the convolution operator and it is understood that $y_{\mathbf{x}_t}$ is not random here. When the Gaussian approximation given in (13) is used and assuming $v_t \sim \mathcal{N}(0, \sigma_v)$, this simplifies to $\mathcal{N}\left(y_t \mid y_{\mathbf{x}_t} + {y_{\mathbf{l}_t}}^*, \sqrt{(\sigma^\star)^2 + \sigma_v^2}\right)$. When a non-Gaussian form such as (11) must be used, one can always perform numerical integration to compute the convolution, which has been found to perform well when v_t is Gaussian.

Let us now consider a scenario with three mobile sensors (whose respective measurements are denoted $y_{t,i}$) that track and follow a single target obeying the dynamic model,

$$\mathbf{x}_t = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_{t-1} + \mathbf{w}_t,$$

$$w_t \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \sigma_w^2 \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 1 \end{bmatrix}\right).$$

At each point the sensors are positioned equiangularly about a circle of radius r centered at the predicted target location. Assume there also exists a single interfering target $\mathbf{l}_t$ with location at each time distributed as $\mathbf{l}_t \sim \mathcal{N}\left(\mathbf{s}_{t,1} + \mathbf{d}, \sigma^2\mathbf{I}\right)$, i.e., the interference location is randomly distributed at some constant offset $\mathbf{d}$ away from sensor 1 at each time t. Note that this scenario can be representative of a distributed processing scheme; the given information regarding $\mathbf{l}_t$ could have originated from a separate tracker tasked with estimating that

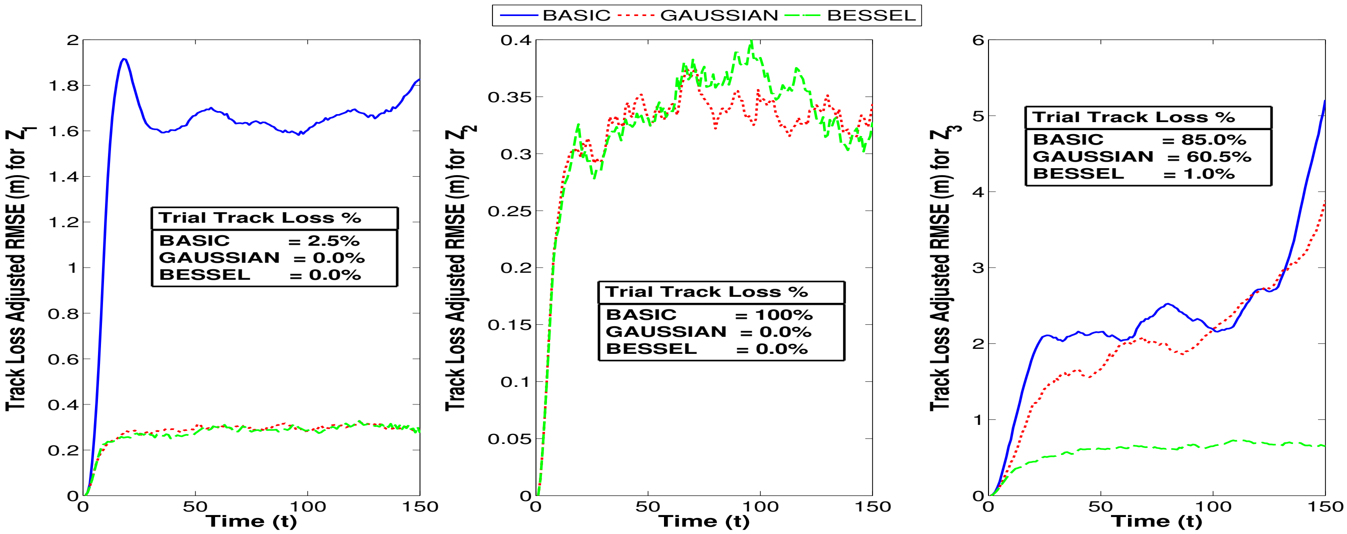

Fig. 3. Application performance comparison.

target (a true multi-target tracking scenario is not considered here due to space limitations, see [5] for more details). As in [13], a particle filter is used to fuse individual sensor measurements and track the target, operating in the same fashion as described within that paper, with the exception that the particle weights are now computed as

$$w_t^{(m)} \propto w_{t-1}^{(m)} \prod_{i=1}^{3} f_{y_{t,i}} \left(y_{t,i} \mid \mathbf{x}_t^{(m)}, \bar{\mathbf{l}}_t, \mathbf{Q} \right), \tag{15}$$

where $\mathbf{x}_t^{(m)}$ and $w_t^{(m)}$ respectively denote the mth particle and weight at time t. Note that even with exact $f_{y_{t,i}}(\cdot)$, this is still an approximation to the joint likelihood $f_{y_{t,1:3}}(y_{t,1:3} \mid \cdot)$ since the individual sensor measurements are no longer independent due to the interfering target.

This scenario is simulated for three distinct parameter sets, denoted by Z_i and given as

$$Z_1: \left\{ \mathbf{d} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad \sigma = .02 \right\}$$

$$Z_2: \left\{ \mathbf{d} = \begin{bmatrix} -0 \\ -3 \end{bmatrix} \quad \sigma = .2 \right\}$$

$$Z_3: \left\{ \mathbf{d} = \begin{bmatrix} -0 \\ -3 \end{bmatrix} \quad \sigma = 2 \right\}.$$

The estimation performance of the PF is compared for three different approaches: using (11) in (14) to compute (15) (labeled as BESSEL), using the Gaussian approximation in (13) for (14) (labeled as GAUSSIAN), and using the approach that was introduced in [5] whereby the simple approximation $\mathbf{l}_t = \bar{\mathbf{l}}_t$ is made to evaluate (14) (labeled as BASIC). The numerical convolution is performed by representing $f_{v_t}(v_t)$ discretely with 500 equally spaced points. The "track-loss adjusted" RMSE (only trials with error less than $3r$ are counted in the RMSE) over 200 trials of the target estimate position norm is plotted in Fig. 3. In all simulations the remaining model parameters were set to $\Phi = 10$, $\alpha = 2$, $\epsilon = 0$, $\sigma_v = 0.1$, $\sigma_w = 0.2$, and $r = 4$. We used $M = 200$ particles in each filter, resampling was performed at every time step, and initial particle sets were placed at the true target location.

Notice that in every case, the performance of the GAUSSIAN and BESSEL methods were vastly improved over BASIC; with Z_2, BASIC could not maintain tracking for any given trial. It is also apparent that BESSEL performed much better than GAUSSIAN for Z_3, which is to be expected since with $\frac{\|d\|}{\sigma^2} = \frac{3}{4}$ the assumption that $f_{y_{1_t}}\left(y_{1_t} \mid \bar{\mathbf{l}}_t, \mathbf{Q}\right)$ is Gaussian no longer holds.

6. CONCLUSION

In this paper an exact closed form was presented for the measurement likelihood function of an RSSI sensor following a standard model, whose measurements are affected not only by the target being tracked (TOI) but also by an additional "out-of-scope" target which acts as a source of interference. The underlying assumption is that Gaussian prior information regarding the interferer's location is available, which is communicated to the tracker by another cooperative agent. Exact analytical solutions were developed and it was shown that this likelihood function can be adequately approximated by a Gaussian distribution as long as specific conditions on the parameters are satisfied. A simple application was presented that can be directly interpreted as an approach to interference compensation, but which also has significantly deeper implications for a distributed multi-agent cooperative tracking system. The validity of the method's performance was confirmed via computer simulations and it was demonstrated that the proposed approaches are far superior to a method previously proposed, even when a Gaussian approximation to the likelihood function is no longer appropriate.

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