ENVIRONMENTALLY SENSITIVE PARTICLE FILTER TRACKING IN MULTISTATIC AUV NETWORKS WITH PORT-STARBOARD AMBIGUITY

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ABSTRACT

This paper presents a Bayesian multi-sensor tracking strategy for a network of autonomous underwater vehicles (AUVs) for the purpose of anti-submarine warfare (ASW). A bistatic configuration and the corresponding acoustic model for the bistatic signal-to-noise ratio (SNR) is used. The Bayesian posterior distribution of the target state based on all available information from sensors and on the acoustic model is reconstructed via particle filtering methods, taking into account the port-starboard ambiguity typical of horizontal line arrays. The posterior distribution is the optimal estimation procedure, the only approximation derives from the particle representation. The effectiveness of the proposed algorithm is demonstrated on a real data set collected by the NATO Centre for Maritime Research and Experimentation (CMRE) during the NATO Proud Manta 2012 exercise (ExPOMA12).

***Index Terms*—** Data fusion, Antisubmarine warfare, multistatic active sonar, target tracking, particle filtering, port-starboard ambiguity, underwater wireless sensor networks, autonomous underwater vehicles.

1. MOTIVATION AND RELATED WORKS

Submarine detection, localization, and tracking in shallow water continues to be a problem of interest. While passive systems suffer from the low radiated noise levels of modern submarines, active systems are plagued by reverberation and clutter. A multistatic system [1–4] offers several advantages over a single source-receiver pair in either a monostatic or bistatic configuration. For example, it offers aspect diversity in the target echoes which, because of the strongly aspect dependent target strength, increases the chance of catching a specular target echo, or "glint." This depends on the usually unknown, or poorly known, target heading and the placement of the source(s) and receivers. One fundamental advantage of multistatics is that, through the simultaneous deployment of multiple sensors, the probability of detecting a "glint" echo is increased.

Traditional ASW assets generally use large aperture towed or flank arrays, but these platforms are expensive to operate. Distributed mobile and stationary sensors, such as sonobuoy fields and AUVs, have recently been suggested to replace or supplement conventional assets. Compared to standard assets these small, low-power, mobile devices have limited onboard processing and communication capabilities. Due to their low hardware/software needs and the complexity of the shallow water environment, individual sensors can only perform simple onboard computation and communicate over a short range at low data rates. When deployed in large numbers across a surveillance region, these primitive sensors can form an intelligent network achieving high performance. An overview of underwater wireless sensor networks can be found in [5].

Horizontal line arrays are typically used as receivers. These are cylindrically symmetric and therefore cannot discriminate left from right, or port from starboard. Such an ambiguity complicates the detection and tracking algorithms and may severely degrade performance. Several approaches have been proposed to overcome these difficulties, including multiline arrays, e.g. twin arrays [6] and triplet arrays [7]. However the use of multiline arrays requires the use of a higher number of hydrophones to achieve the same directivity of a single line array (e.g. double for the twin array). This reduces the aperture size for a fixed number of hydrophones, which for an AUV with limited complexity is generally small.

In [8, 9] a Bayesian tracking approach is proposed to track the target state (position and velocity) in presence of port–starboard ambiguous data. In this paper the Bayesian filtering approach is extended in order to take into account environmental acoustic effects. A model for the probability of detection (P_D) for each of the source/receiver pairs as a function of target position is given. The probability of detection can vary dramatically over the surveillance region, and is in general a function of the receiver array parameters, source parameters, and environmental parameters as such as bottom scattering strength, water column and bottom sediment density and sound speed, and bottom and surface roughness and reflection loss. Numerical and/or closed-form acoustic propagation models [10–12] are then used to calculate the predicted SNR and P_D. The proposed tracking technique takes advantage of the ability of the acoustic model to predict the detection probability of the target in any position in the surveillance region.

The approach presented here has been applied on real-world data collected during ExPOMA12, conducted by the CMRE (formerly known as SACLANTCEN and NURC). The NATO Research Vessel (NRV) *Alliance* and the CMRE's underwater networks with the multistatic sonar system were used, and results from these data are reported in the following sections.

2. PROBLEM FORMALIZATION

A network of N_s sensors (or vehicles) is considered. A single target is assumed to sail across the surveillance region $\mathcal{S}$, and the objective of the sensor network is to estimate its kinematic state at each time scan k.

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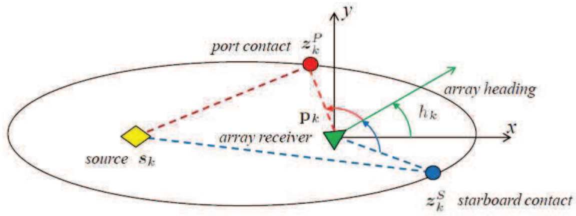

Fig. 1. Sketch of the port-starboard ambiguity in the bistatic geometry.

2.1. Target dynamic model

The target dynamic, defined in Cartesian coordinates, is expressed in terms of a Markovian process [13], the target motion state vector is $\boldsymbol{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^T$, where the two positions are x_k, y_k, and $\dot{x}_k, \dot{y}_k$ are the corresponding velocities. Given the typical motion of such targets, a nearly constant velocity model [13] can be adopted

$$\boldsymbol{x}_k = \boldsymbol{F}_k \boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}_k \boldsymbol{v}_k, \tag{1}$$

where $\boldsymbol{F}_k$ is the state transition matrix, $\boldsymbol{\Gamma}_k \boldsymbol{v}_k$ takes into account the target acceleration or unmodeled dynamics. The term $\boldsymbol{v}_k$ is typically assumed to be Gaussian with zero-mean and covariance matrix $\boldsymbol{Q}$.

2.2. Target measurement model with PS ambiguity

In this subsection the model for the target-originated measurements is introduced for a generic sensor. Consider the geometry given in Fig. 1. Let $\boldsymbol{s}_k = [s_k^x, s_k^y]^T$ denote the source position at time scan k, while the sensor array position and his heading angle are indicated by $\mathbf{p}_k = [p_k^x, p_k^y]^T$ and h_k, respectively. The sensor measures the bistatic range b_k from source to target to receiver and the bearing angle relative to the array heading θ_k. The non-ambiguous measurement is given by

$$\boldsymbol{z}_k = \begin{bmatrix} b_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} \|\boldsymbol{x}_k^p - \mathbf{p}_k\| + \|\boldsymbol{x}_k^p - \boldsymbol{s}_k\| & + & w_k^b \\ \tan^{-1}\left(\frac{y_k - p_k^y}{x_k - p_k^x}\right) - h_k & + & w_k^\theta \end{bmatrix},$$

$$\boldsymbol{w}_k = \begin{bmatrix} w_k^b \\ w_k^\theta \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix} \right),$$

where w_k^b and w_k^θ are the additive noise to the range and bearing. The port-starboard ambiguous contacts have the same bistatic range measurement but different bearing angles, one θ_k^P from receiver to the target on the port side, and another θ_k^S from receiver to target on the starboard side. Then $\boldsymbol{z}_k^P = \left[b_k, \theta_k^P\right]^T$ and $\boldsymbol{z}_k^S = \left[b_k, \theta_k^S\right]^T$ are given by

$$\begin{cases} \theta_k^P = \theta_k,\ \theta_k^S = -\theta_k^P, & \text{if } \tan^{-1}\left(\dfrac{y_k - p_k^y}{x_k - p_k^x}\right) \geq h_k, \\ \theta_k^S = \theta_k,\ \theta_k^P = -\theta_k^S, & \text{if } \tan^{-1}\left(\dfrac{y_k - p_k^y}{x_k - p_k^x}\right) < h_k, \end{cases}$$

note that it always holds that $\theta_k^P = -\theta_k^S \in [0, \pi]$, with $\theta_k \in [-\pi, \pi]$.

2.3. Measurement model in presence of missed detections and clutter

At each time scan k a set of data is observed, whose cardinality is the number of detections. The target is considered a point and is observed at the s^{th} sensor with a detection probability $P_{D,s}(\mathbf{x})$ which depends on the target state $\mathbf{x}$. All the other echoes are clutter, independent of the target's state, the number of which is typically modeled as a Poisson random variable with rate λ, see also [13–17].

The data set $Z_{s,k}$ of the whole measurement set for the s^{th} sensor at time k is defined as

$$Z_{s,k} = \left\{\boldsymbol{z}_{i,s,k}^P\right\}_{i=1}^{m_{s,k}}, \tag{2}$$

where $m_{s,k}$ is the number of measurements. Only the port contacts have been considered, since they form a *sufficient statistic*, because of the deterministic dependence of the starboard contacts on the port contacts. The aggregate in time of the data up to time step k is indicated as $Z_{1:k} = Z_1, Z_2, \ldots, Z_k$, where $Z_k = \{Z_{s,k}\}_{s=1}^{N_s}$.

3. OPTIMAL BAYESIAN INFERENCE WITH PS AMBIGUITY

In the Bayesian approach to dynamic state estimation, the goal is to construct the posterior probability density distribution (pdf) of the state based on all available information, including the set of received measurements. Since this pdf embodies all available statistical information, it contains the complete solution to the estimation problem, and the optimal (with respect to any criterion) estimate of the state may be obtained from the posterior.

The posterior of the target's state in Eq. (1) is given by the Bayes' rule

$$\mathcal{P}\left(\boldsymbol{x}_k \,|Z_{1:k}\right) = \frac{\mathcal{L}_k\left(Z_k \,|\boldsymbol{x}_k\right)\mathcal{P}\left(\boldsymbol{x}_k \,|Z_{1:k-1}\right)}{\mathcal{P}\left(Z_k \,|Z_{1:k-1}\right)}, \tag{3}$$

where the prior at time k is given by

$$\mathcal{P}\left(\boldsymbol{x}_k \,|Z_{1:k-1}\right) = \int \mathcal{P}\left(\boldsymbol{x}_k \,|\boldsymbol{x}_{k-1}\right)\mathcal{P}\left(\boldsymbol{x}_{k-1} \,|Z_{1:k-1}\right) d\boldsymbol{x}_{k-1}$$

and $\mathcal{P}\left(\boldsymbol{x}_k \,|\boldsymbol{x}_{k-1}\right)$ is ruled by the dynamic model in Eq. (1). Given that the sensors are conditionally independent on target position, the likelihood $\mathcal{L}_k\left(Z_k \,|\boldsymbol{x}_k\right)$ can be factorized

$$\mathcal{L}_k\left(Z_k \,|\boldsymbol{x}_k\right) = \prod_{s=1}^{N_s} \mathcal{L}_{s,k}\left(Z_{s,k} \,|\boldsymbol{x}_k\right), \tag{4}$$

where $\mathcal{L}_{s,k}\left(Z_{s,k} \,|\boldsymbol{x}_k\right)$ is the likelihood of the s^{th} sensor at time k. The likelihood, derived in [9], is given by

$$\begin{aligned} \mathcal{L}_{s,k}\left(Z \,|\boldsymbol{x}\right) =& \frac{\mu_c(m-1;\lambda)P_{D,s}(\boldsymbol{x})}{mV^{m-1}} \sum_{i=1}^{m} f_{s,k}^{PS}\left(\boldsymbol{z}_i^P \,|\boldsymbol{x}\right) \\ &+ \frac{\mu_c(m;\lambda)(1 - P_{D,s}(\boldsymbol{x}))}{V^m}, \end{aligned} \tag{5}$$

where $\boldsymbol{x}$ is the target state, $Z = \left\{\boldsymbol{z}_i^P\right\}_{i=1}^m$ is the set of data of a given sensor at time k, V is the area of the region, $\mu_c(m;\lambda)$ is the distribution of the number of false alarms. The target originated likelihood $f_{s,k}^{PS}\left(\boldsymbol{z}_i^P \,|\boldsymbol{x}\right)$ is given by [9]

$$f_{s,k}^{PS}\left(\boldsymbol{z}^P \,|\boldsymbol{x}\right) = \begin{cases} f_{s,k}\left(\boldsymbol{z}^P \,|\boldsymbol{x}\right), & \text{if } \boldsymbol{x} \text{ on the port,} \\ f_{s,k}\left(g_k^{-1}\left(\boldsymbol{z}^P\right) |\boldsymbol{x}\right), & \text{if } \boldsymbol{x} \text{ on the starb.} \end{cases}$$

where $f_{s,k}\left(\boldsymbol{z} \,|\boldsymbol{x}\right)$ is the non-ambiguous conditional pdf of the target originated data, $g_k(\boldsymbol{z}) = [b, -\theta]^T$, $\boldsymbol{z} = [b, \theta]^T$, and b is the bistatic range. Note that all the variables in Z in Eq. (5) are expressed in Cartesian coordinates, and hence the distribution of the exact target-originated measurement $f_{s,k}(\boldsymbol{z}\,|\boldsymbol{x})$ can be derived using the Fundamental Theorem of transformation of random variables [18], see details in [9]. An approximation of the likelihood function is provided in [2], in which the small-error assumption is made allowing the use of the first-order linearization expression.

Note that the detection probability $P_{D,s}(\boldsymbol{x})$ in Eq. (5) is dependent on the target location, and is given in the following section.

4. ACOUSTIC MODEL FOR BISTATIC SNR AND DETECTION PROBABILITY

The acoustic model of SNR used here is based on the work of Harrison [10–12, 19, 20]. For ease of notation, here we skip the dependency on the time and sensor index. A closed-form solution exists when the bathymetry is range dependent but the sound speed is isovelocity. The bottom reflection coefficient R as a function of grazing angle θ is approximated as $R = \exp(-\alpha\,\theta)$, where α is a constant. Assuming the data are reverberation-limited, the bistatic signal to reverberation ratio (SRR) is given by

$$\frac{I_T^S(\phi,t)}{I_R(\phi,t)} = \frac{4\pi\alpha^2 S_T H_{eff,s} H_{eff,r}}{H_s^2 H_r^2 (1-\exp(-A_s r_s))(1-\exp(-A_r r_r))} \cdot \frac{(c\,t - L\cos\phi)^2}{\mu\delta\phi\sqrt{[(c\,t)^2 + L^2 - 2L\,c\,t\,\cos\phi]\,[(ct)^2 - L^2]}}$$

where ϕ is the angle at the receiver between the source and the point of interest $\boldsymbol{x}^p$, t is the time, $r_s = \|\boldsymbol{x}^p - \boldsymbol{s}\|$, $r_r = \|\boldsymbol{x}^p - \mathbf{p}\|$ are the ranges to the source $\boldsymbol{s}$ and receiver $\mathbf{p}$ from $\boldsymbol{x}^p$, $L = \|\mathbf{p} - \boldsymbol{s}\|$ is the distance between the source and receiver, and c is the range independent sound speed. The $\delta\phi$ term is the beamwidth of the array, which is in general a function of the steering angle, and the terms A_s and A_r are defined as

$$A_s = \frac{\alpha\,\theta_c^2\,H_{c,s}^2\,H_{eff,s}}{2H_{scat}^2\,H_s^2}, \quad A_r = \frac{\alpha\,\theta_c^2\,H_{c,r}^2\,H_{eff,r}}{2H_{scat}^2\,H_r^2}, \tag{6}$$

where θ_c is the critical angle of the water-sediment interface. The terms

$$H_{eff,s}(r) = (H_s^2\,H_{scat}^2/r)\int_0^r \frac{\mathrm{d}r'}{H^3(r')}, \tag{7}$$

$$H_{eff,r}(r) = (H_r^2\,H_{scat}^2/r)\int_0^r \frac{\mathrm{d}r'}{H^3(r')}, \tag{8}$$

are the "effective" water depths for the source and receiver, and $H_{c,s}$ and $H_{c,r}$ are the "critical" depths for the source and receiver, meaning the depth where the steepest rays are incident above the critical angle. The $H_{s,r}$ are the water depths at the source and receiver locations, and H_{scat} is the water depth at the point of interest. The function $H(r)$ is the water depth profile along a given path. In this work, we assume the bottom scattering follows Lambert's law [11, 19].

The target strength S_T is assumed to be constant in this work, but can be a function of target aspect angle in more complex models. If the sound speed is range- and/or depth-dependent, a numerical model such as ARTEMIS or BELLHOP can be used. The SNR can be converted to the probabilities of detection and false alarm assuming the envelope of the acoustic data is Rayleigh distributed. The P_D corresponding to a desired P_{FA} for a signal with $\mathrm{SNR}_{dB}(\mathbf{x})$ (which takes into account the reverberation and the ambient noise) depending on the target state $\mathbf{x}$

$$P_D(\mathbf{x}) = \exp\left(\frac{\log(P_{FA})}{10^{\mathrm{SNR_{dB}}(\mathbf{x})/10}}\right) \tag{9}$$

5. EXPERIMENTAL RESULT USING EXPOMA12

The ExPOMA12 exercise was held in the Mediterranean Sea off the coast of Sicily, Italy during February-March 2012. The setup of the experiment is given in Fig. 2, where we depict the location of the source *DEMUS* (yellow diamond), position of AUVs, *Harpo* (blue circle) and *Groucho* (green circle) with related headings (arrow), and the trajectory of the target (black dashed line). An echo-repeater (ER) towed by the NRV is used in the experiment as a reproducible and controllable target.

The DEMUS is located at $(12.3\,\mathrm{km}, 23.2\,\mathrm{km})$. The target sails from the location $(16.5\,\mathrm{km}, 16.9\,\mathrm{km})$ to $(17.2\,\mathrm{km}, 9.8\,\mathrm{km})$ and then goes to $(15.8\,\mathrm{km}, 11.3\,\mathrm{km})$. The AUVs sail south-east of the source position and the target trajectory. The duration of the experiment is approximately 2 hours. Further details about the experiment are available in [9].

A key element of this procedure is that the SNR, and consequently the detection probability, at each AUV depends on the geometry of the bistatic system and on the environment, e.g. multipath time spread, reverberation, bathymetry, sound speed, etc. The detection probability is calculated according to the equations in Sec. 4 using a target strength of 15 dB.

The detections of each AUV are combined to estimate the target state distribution, defined in Sec. 3, by using a multi-sensor particle filtering strategy. Additional details related to the particle filter implementation can be found in [9].

In Fig. 2, we report the behaviour of the particle filter using detection probabilities that are not constant over the surveillance region. Figure 2 (right) shows the so-called blanking region between the AUV and the source and also the degradation of the detection probability (< 0.3) near the edges of the surveillance region. However, note that the target is mostly moving in the region where the detection probability is large ($\approx 0.7 - 0.9$).

Using the proposed procedure we are able to correctly estimate the target trajectory and to reject ghost and false contacts.

6. CONCLUSIONS

A method of data fusion and Bayesian target tracking has been presented for a network of multiple AUVs. The port-starboard ambiguity problem, present on horizontal line arrays, is addressed. We take into account the hypothesized probability of detection as a function of target position. The latter is itself a function of the bistatic geometry and environmental parameters of the acoustic waveguide, taking into account all available environmental information. The methodology is optimal provided there is exactly one target in the surveillance region. Future work will extend this method to allow the possibility of zero or multiple targets, as well as use the Bayesian posterior distribution to influence AUV navigation decisions [21].

7. ACKNOLEDGEMENTS

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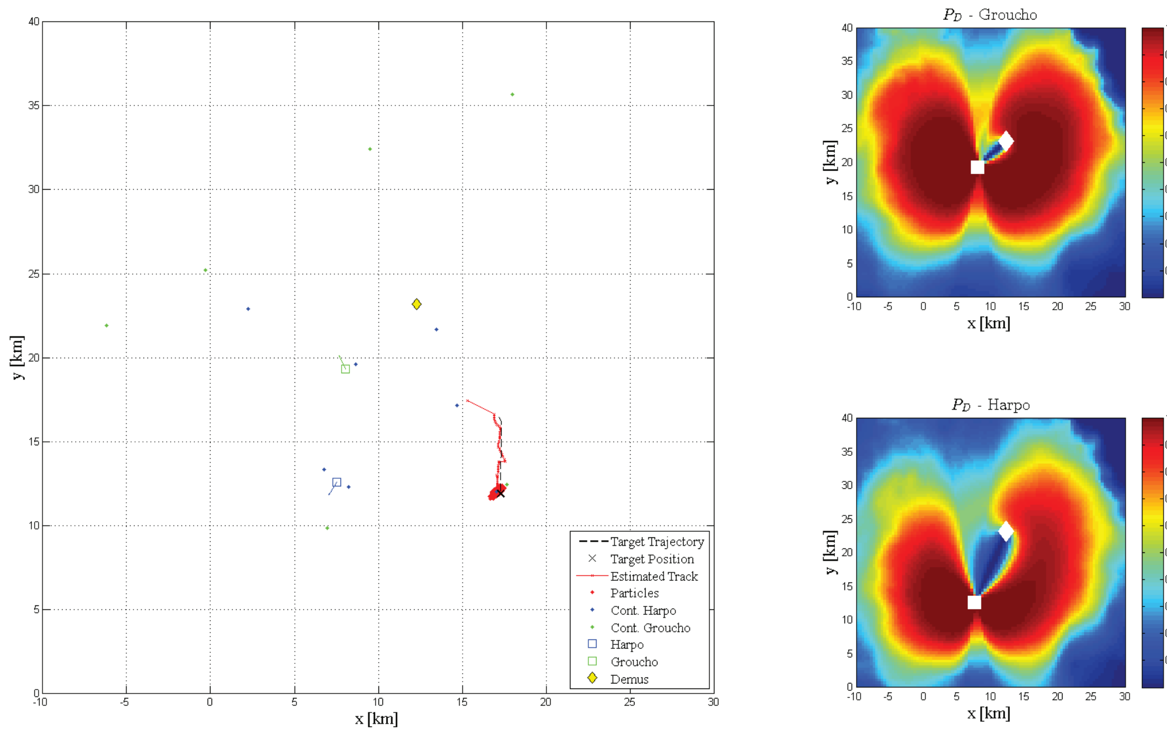
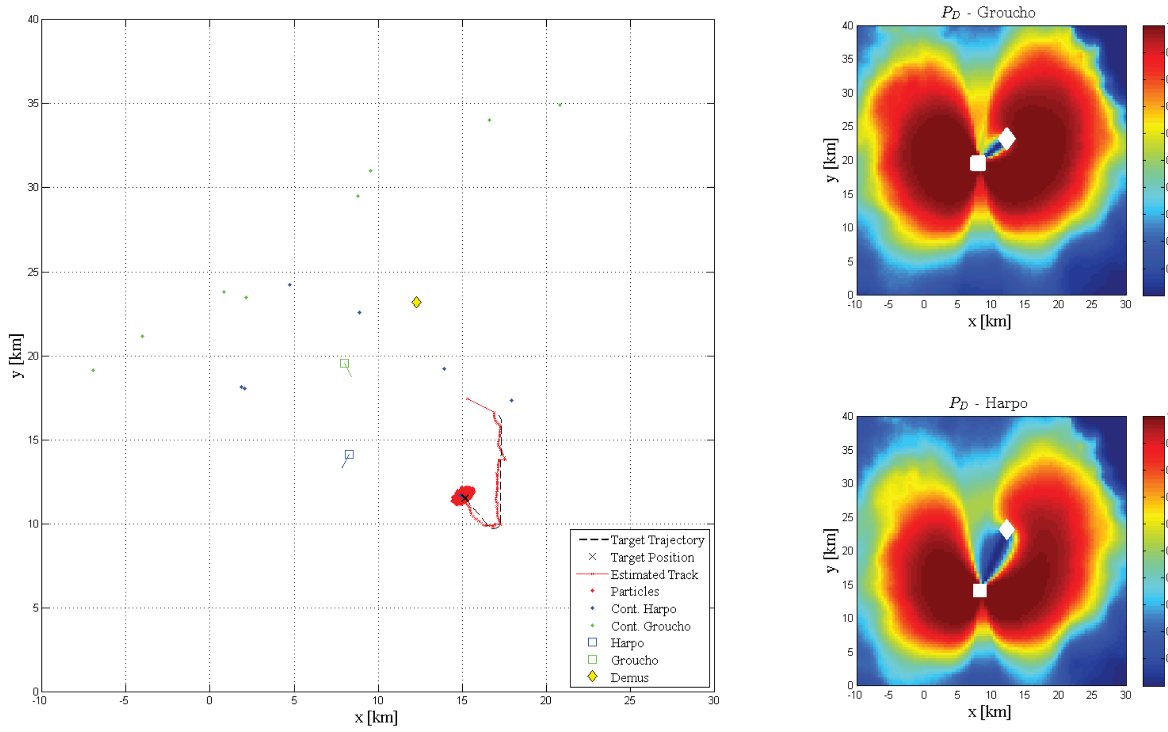


(a) k=50



(b) k=100

Fig. 2. Sea trials ExPOMA12. Behaviour of the PF algorithm, using the ExPOMA12 data set, with the detection probability at Harpo and Groucho given by the acoustic model. Time scan $k = 50$ and $k = 100$. Right-side detection probability map for Harpo and Groucho at time scan k, Groucho and Harpo positions (white square '□'), source position (white diamond '♢'). Left-side we report the target trajectory (dashed black line), current target position ('x'), estimated track at time k (red line '-x'), Harpo contacts at time k (blue dots), Groucho contacts at time k (green dots), Harpo position at time k (blue square '□'), Groucho position at time k (green square '□'), source position (yellow diamond '♢').

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