REACHING ASYMPTOTIC EFFICIENT PERFORMANCE FOR SQUARED PROCESSING OF RANGE AND RANGE DIFFERENCE LOCALIZATIONS IN THE PRESENCE OF SENSOR POSITION ERRORS

Shanjie Chen and K. C. Ho

ECE Department, University of Missouri, Columbia, MO 65211, USA.
scm89@mail.missouri.edu; hod@missouri.edu

ABSTRACT

Computationally efficient source location solutions from TOAs or TDOAs require the squaring of the measurements before optimization. The squaring operation changes the characteristics of the measurements and causes degradation in the localization accuracy, unless proper weightings are applied when forming the cost function for minimization. The previously developed weighting values are useful for measurements with accurate sensor positions only. This paper extends the study and derives the weightings when the sensor position uncertainties are present such as in sensor networks. The resultant cost functions for TOA and TDOA positionings are analyzed and the performance accuracy is shown to attain the CRLB asymptotically under Gaussian noise. Simulations validate the performance of the new cost functions and the theoretical investigations.

Index Terms— Source localization, TOA, TDOA, sensor position errors, efficient solution

1. INTRODUCTION

Source localization is a fundamental application in GPS, radar, sonar, mobile communication and sensor networks [1–16]. Most common localization approaches are ranged-based through the use of time of arrival (TOA), time difference of arrival (TDOA) or the received signal strength (RSS) from the source to a number of sensors at known positions. In practice, the sensor positions are not known exactly, such as in a sensor network in which the node positions are estimated by anchors. Sensor position errors degrade the localization performance considerably [17, 18] and their statistics should be taken into consideration in order to reach better performance [19, 20]. For example, [19] addressed the sensor position uncertainty by jointly estimating the source and sensor positions through an iterative implementation of the maximum likelihood estimator (MLE). [20] handled the situation by joint estimation as well using semi-definite programming.

Obtaining the source location from the range-based measurements is not an easy task, since the measurement equation is nonlinear with respect to the unknowns. Solving the source position directly from the measurements, such as by the MLE, requires an iterative solution whose performance depends highly on initializations. When the sensor positions have uncertainties, the sensor positions become nuisance variables and they need to be solved jointly with the source positions. The number of unknowns becomes large and it presents challenges to an iterative solution in reaching the global minimum and in maintaining the computational efficiency. This motivates the development of algebraic closed-form solutions for the localization problem and it has been an active research area.

Most of the closed-form or exact localization solutions in literature are based on squaring the range-based measurements. In particular, [21] has established an exact and computationally efficient solution for TOA localization by applying the generalized trust region subproblem (GTRS) [22] technique on the squared range measurements. The squaring operation, however, changes the characteristics of the measurements and the resulting solution is suboptimum [23]. Indeed, under some unfavorable geometries, the location error ratio of the squared measurement solution to that of the MLE is unbounded [24]. Fortunately, [25] has shown that by introducing suitable weightings to the squared measurements when forming the cost function for minimization, we are able to recover the optimum performance accuracy in reaching the Cramér-Rao Lower Bound (CRLB). The weightings do not alter the structure for obtaining computationally efficient solution and the previously developed solution methods with squared measurements remain applicable. The work in [25] only considers measurement noise only and is applicable when the sensor positions are accurate. The effect of sensor position errors on the localization accuracy is not negligible even if the number of sensors is large [18] This paper extends [25] and derives the new weightings for the squared TOA and TDOA measurements when sensor position errors are present.

The benefits for this extension are threefold. First, a typical algorithm such as the MLE needs to jointly estimate the source and sensor positions, while the proposed new cost function estimates the source position only. Obviously, the dimension reduction in optimization can significantly reduce the computational complexity. Second, the new cost functions have the same structure as those in [21], thereby existing algebraic or exact solutions to the squared measurements can still be used without requiring new optimization method. Third, the solutions of the new cost functions achieve asymptotically the CRLB performance under Gaussian noise as we will show in the theoretical analysis and simulations.

The rest of the paper is organized as follows. Section 2 introduces the scenarios for TOA and TDOA localizations and provides the CRLBs of the source location estimate. Section 3 proposes the new cost function for TOA positioning and analyzes its solution accuracy. Section 4 is for the new cost function of TDOA positioning and its analysis. Section 5 gives the simulation results to verify the performance of the proposed new cost functions and support the theoretical studies. Section 6 concludes the paper. We shall use TOA and range, as well as TDOA and range difference interchangeably because they are differed by a scaling factor.

2. LOCALIZATION SCENARIO AND CRLB

2.1. Localization scenario

Let us begin the source localization problem in 3D by having M sensors to collect the range-based measurements from a source as shown in Fig. 1. The source position to be estimated is repre-

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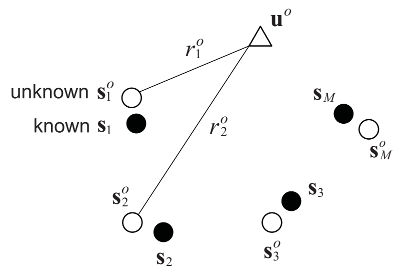

Fig. 1. Localization scenario. Open circles are the true sensor positions and closed circles are the available sensor positions.

sented by $\mathbf{u}^o = [x^o\, y^o\, z^o]^T$. The sensor positions during acquisition are $\mathbf{s}_i^o = [x_i^o\, y_i^o\, z_i^o]^T$, $i = 1, 2, \ldots, M$. They are not known to a location estimator and we only have the erroneous positions $\mathbf{s}_i = \mathbf{s}_i^o + \mathbf{n}_{si}$, where $\mathbf{n}_{si}$ is the position error of sensor i. We collect the sensor positions in a vector as $\mathbf{s} = \mathbf{s}^o + \mathbf{n}_s$, where $\mathbf{s}^o = \left[\mathbf{s}_1^{oT}\, \mathbf{s}_2^{oT}\, \cdots\, \mathbf{s}_M^{oT}\right]^T$ and $\mathbf{n}_s = \left[\mathbf{n}_{s1}^T\, \mathbf{n}_{s2}^T\, \cdots\, \mathbf{n}_{sM}^T\right]^T$. In this study, we consider $\mathbf{n}_s$ is zero-mean Gaussian with known covariance $\mathbf{Q}_s$. The localization problem has unknown parameters $[\mathbf{u}^{oT}\ \mathbf{s}^{oT}]^T$.

In this paper, we assume line-of-sight (LOS) propagation and sufficient SNR such that the acquired TOAs and TDOAs can well be modelled by Gaussian distribution with covariance matrix governed by their CRLBs. The localization accuracy is indirectly related to the received waveforms, signal and noise bandwidths, SNR and observation time through the covariance matrices of the TOAs and TDOAs [10, 26–29].

For range (TOA) localization, the measurement vector is $\mathbf{r} = \mathbf{r}^o + \mathbf{n}_r$ where $\mathbf{r}^o = [r_1^o\, r_2^o\, \cdots\, r_M^o]^T$, $r_i^o = \|\mathbf{u}^o - \mathbf{s}_i^o\|$ and $\mathbf{n}_r = [n_1\, n_2\, \cdots\, n_M]^T$ is the measurement noise vector that is zero-mean Gaussian with covariance matrix $\mathbf{Q}$.

For range difference (TDOA) localization, the measurement vector is $\mathbf{r}_d = \mathbf{r}_d^o + \mathbf{n}_d$ where $\mathbf{r}_d^o = [r_{21}^o\, r_{31}^o\, \cdots\, r_{M1}^o]^T$ and $r_{i1}^o = r_i^o - r_1^o, i = 2, 3, \ldots, M$. The noise vector $\mathbf{n}_d = [n_{21}\, n_{31}\, \cdots\, n_{M1}]^T$ is zero-mean Gaussian with covariance matrix $\mathbf{Q}_d$. In both range and range difference cases, the measurement noise and sensor position noise are assumed independent for ease of illustration. The collection of the measurement and sensor position noise is denoted by $\mathbf{n}$, which is either $[\mathbf{n}_r^T, \mathbf{n}_s^T]^T$ or $[\mathbf{n}_d^T, \mathbf{n}_s^T]^T$.

2.2. CRLB

In the asymptotic region in which the estimation bias is small compared to variance, the localization performance can be characterized by the CRLB. Utilizing the CRLB analysis in [17] and taking further simplification, the CRLB for a source position estimate in range localization is

$$\mathrm{CRLB}(\mathbf{u}^o) = \left[\boldsymbol{\Gamma}(\mathbf{Q} + \mathbf{A}^T\mathbf{Q}_s\mathbf{A})^{-1}\boldsymbol{\Gamma}^T\right]^{-1}, \tag{1}$$

where $\boldsymbol{\Gamma} = [\boldsymbol{\rho}_1\, \boldsymbol{\rho}_2\, \cdots\, \boldsymbol{\rho}_M]$, $\mathbf{A} = -\mathrm{blkdiag}\{\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \ldots, \boldsymbol{\rho}_M\}$, and blkdiag$\{\star\}$ is the block diagonal matrix notation. The vector $\boldsymbol{\rho}_i$ is $\boldsymbol{\rho}_i = (\mathbf{u}^o - \mathbf{s}_i^o)/\|\mathbf{u}^o - \mathbf{s}_i^o\|$, which is a unit vector pointing from sensor i to the source.

For range difference localization, we have

$$\mathrm{CRLB}(\mathbf{u}^o) = \left[\boldsymbol{\Gamma}_d(\mathbf{Q}_d + \mathbf{A}_d^T\mathbf{Q}_s\mathbf{A}_d)^{-1}\boldsymbol{\Gamma}_d^T\right]^{-1} \tag{2}$$

where $\boldsymbol{\Gamma}_d = [\boldsymbol{\rho}_{21}\, \boldsymbol{\rho}_{31}\, \cdots\, \boldsymbol{\rho}_{M1}]$, $\boldsymbol{\rho}_{i1} = \boldsymbol{\rho}_i - \boldsymbol{\rho}_1$,

$$\mathbf{A}_d = \begin{bmatrix} \boldsymbol{\rho}_1\mathbf{1}^T \\ -\mathrm{blkdiag}\{\boldsymbol{\rho}_2, \boldsymbol{\rho}_3, \ldots, \boldsymbol{\rho}_M\} \end{bmatrix},$$

and $\mathbf{1}$ is a length (M-1) vector of unity.

3. SQUARED RANGE COST FUNCTION AND ANALYSIS

3.1. New SR-WLS cost function

We shall define the squared range weighted least-squares (SR-WLS) cost function to obtain the source location estimate as

$$f_{\text{SR-WLS}}(\mathbf{u}) = \sum_{i,j=1}^{M} \tilde{w}_{ij}(r_i^2 - \|\mathbf{u} - \mathbf{s}_i\|^2)(r_j^2 - \|\mathbf{u} - \mathbf{s}_j\|^2). \tag{3}$$

Note that the unknown of the cost function is $\mathbf{u}$ only and $\mathbf{s}$ is kept as the noisy sensor positions. The objective is to find the weights $\tilde{w}_{ij}$ to improve as much accuracy as possible since the cost function is constructed with the noisy sensor positions.

The residual squared range error at the true source location is

$$\begin{aligned} e_i &= r_i^2 - \|\mathbf{u}^o - \mathbf{s}_i\|^2 = (r_i^o + n_i)^2 - \|\mathbf{u}^o - \mathbf{s}_i^o - \mathbf{n}_{si}\|^2 \\ &\simeq 2r_i^o n_i + 2(\mathbf{u}^o - \mathbf{s}_i^o)^T\mathbf{n}_{si} = 2r_i^o\left(n_i + \boldsymbol{\rho}_i^T\mathbf{n}_{si}\right). \end{aligned} \tag{4}$$

The approximation comes from ignoring the second order noise terms and we have used $\boldsymbol{\rho}_i$ to represent $(\mathbf{u}^o - \mathbf{s}_i^o)/r_i^o$. According to the WLS estimation theory [30], the weights should be the elements of $\mathbf{W} = \mathbf{C}^{-1}$, where $\mathbf{C}$ is the correlation matrix whose $(i, j) - th$ element is $E[e_i e_j]$. Defining $\mathbf{B}$ as $\mathbf{B} = 2\mathrm{diag}\{r_1^o, r_2^o, \ldots, r_M^o\}$,

$$\mathbf{W} = [\mathbf{B}(\mathbf{Q} + \mathbf{A}^T\mathbf{Q}_s\mathbf{A})\mathbf{B}]^{-1} \tag{5}$$

where $\mathbf{A}$ is defined below (1). $\mathbf{W}$ is not known since it depends on the true range values and the true source and sensor positions. Let us construct the noisy version of $\mathbf{B}$ from the measurements as $\tilde{\mathbf{B}} = 2\mathrm{diag}\{r_1, r_2, \ldots, r_M\}$ and that of $\mathbf{A}$ as $\tilde{\mathbf{A}}$ by replacing $\boldsymbol{\rho}_i$ as $\tilde{\boldsymbol{\rho}}_i = (\tilde{\mathbf{u}} - \mathbf{s}_i)/r_i$, where $\tilde{\mathbf{u}}$ is some estimate of $\mathbf{u}^o$. We choose the weights $\tilde{w}_{ij}$ in (3) as the elements of

$$\widetilde{\mathbf{W}} = [\tilde{\mathbf{B}}(\mathbf{Q} + \tilde{\mathbf{A}}^T\mathbf{Q}_s\tilde{\mathbf{A}})\tilde{\mathbf{B}}]^{-1}. \tag{6}$$

We shall show from the first order analysis that although we use the noisy measurement values to form the weights, the minimum of the cost function (3) is able to reach the CRLB accuracy in the asymptotic region.

3.2. MSE analysis

We shall evaluate the mean-square error (MSE) matrix $\mathbf{M}$ of the minimum solution of (3). Using the first order analysis, the MSE matrix of the solution from a smooth cost function $f(\mathbf{u})$ is [25]:

$$\mathbf{M} = \overline{\mathbf{F}}''^{-1} E\left[\mathbf{f}'\mathbf{f}'^T\right]\overline{\mathbf{F}}''^{-1} \tag{7}$$

where $\mathbf{f}' = \partial f(\mathbf{u})/\partial\mathbf{u}|_{\mathbf{u}=\mathbf{u}^o}$, and $\overline{\mathbf{F}}''$ is the component of $\mathbf{F}'' = \partial^2 f(\mathbf{u})/\partial\mathbf{u}\partial\mathbf{u}^T\big|_{\mathbf{u}=\mathbf{u}^o}$ by setting the noise to zero.

We shall use little-o notation $o(\star)$ to represent the noise component in the following analysis, where $a(x) \in o(b(x))$ means that $\lim_{x\to 0} a(x)/b(x) = 0$. It is reasonable to assume the source location $\tilde{\mathbf{u}}$ used in forming $\widetilde{\mathbf{W}}$ is different from the true source location by random noise. Hence the weights in (3) can be expressed as $\tilde{w}_{ij} = w_{ij} + o(1)$, where w_{ij} is the $(i, j) - th$ element of (5).

The first derivative from (3) is,

$$\frac{\partial f_{\text{SR-WLS}}(\mathbf{u})}{\partial\mathbf{u}} = -4\sum_{i,j=1}^{M} \tilde{w}_{ij}\left(r_i^2 - \|\mathbf{u} - \mathbf{s}_i\|^2\right)(\mathbf{u} - \mathbf{s}_j). \tag{8}$$

Expressing $\tilde{w}_{ij}$ in terms of w_{ij} and substituting $r_i = r_i^o + n_i$ and $\mathbf{s}_i = \mathbf{s}_i^o + \mathbf{n}_{si}$, we arrive at after some algebraic manipulations,

$$\begin{aligned}\mathbf{f}'_{\text{SR-WLS}} &= -8\sum_{i,j=1}^{M} r_i^o w_{ij} r_j^o (n_i + \boldsymbol{\rho}_i^T \mathbf{n}_{si})\boldsymbol{\rho}_j + o(\|\mathbf{n}\|)\,\mathbf{1} \\ &= -2\boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\mathbf{B}(\mathbf{n} - \mathbf{A}^T\mathbf{n}_s) + o(\|\mathbf{n}\|)\,\mathbf{1}\,, \qquad (9)\end{aligned}$$

where $\mathbf{1}$ represents a 3×1 vector of unity. Hence using (5),

$$\begin{aligned}E[\mathbf{f}'_{\text{SR-WLS}}\mathbf{f}'^T_{\text{SR-WLS}}] &\simeq 4\boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\mathbf{B}(\mathbf{Q} + \mathbf{A}^T\mathbf{Q}_s\mathbf{A})\mathbf{B}\mathbf{W}\mathbf{B}\boldsymbol{\Gamma}^T \\ &= 4\boldsymbol{\Gamma}(\mathbf{Q} + \mathbf{A}^T\mathbf{Q}_s\mathbf{A})^{-1}\boldsymbol{\Gamma}^T\,. \qquad (10)\end{aligned}$$

For the second derivative,

$$\begin{aligned}\frac{\partial^2 f_{\text{SR-WLS}}(\mathbf{u})}{\partial\mathbf{u}\partial\mathbf{u}^T} = -4\sum_{i,j=1}^{M}\tilde{w}_{ij}[-2(\mathbf{u}-\mathbf{s}_i)(\mathbf{u}-\mathbf{s}_j)^T \\ +(r_i^2 - \|\mathbf{u}-\mathbf{s}_i\|^2)\mathbf{I}] \qquad (11)\end{aligned}$$

where $\mathbf{I}$ is an identity matrix. Again, expressing the noisy qualities in terms of the true values yields

$$\begin{aligned}\mathbf{F}''_{\text{SR-WLS}} &= 8\sum_{i,j=1}^{M} r_i^o w_{ij} r_j^o \boldsymbol{\rho}_i\boldsymbol{\rho}_j^T + o(1)\mathbf{1}\mathbf{1}^T \\ &= 2\boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\mathbf{B}\boldsymbol{\Gamma}^T + o(1)\mathbf{1}\mathbf{1}^T. \qquad (12)\end{aligned}$$

Its constant component excluding noise is, after using (5),

$$\overline{\mathbf{F}}''_{\text{SR-WLS}} = 2\boldsymbol{\Gamma}(\mathbf{Q} + \mathbf{A}^T\mathbf{Q}_s\mathbf{A})^{-1}\boldsymbol{\Gamma}^T. \qquad (13)$$

Utilizing the MSE formula (7) gives immediately

$$\mathbf{M}_{\text{SR-WLS}} = \left[\boldsymbol{\Gamma}(\mathbf{Q} + \mathbf{A}^T\mathbf{Q}_s\mathbf{A})^{-1}\boldsymbol{\Gamma}^T\right]^{-1}, \qquad (14)$$

which is exactly the CRLB in (1) for range localization in the presence of sensor position errors. Thus the solution of the new SR-WLS cost function is asymptotically efficient.

4. SQUARED RANGE DIFFERENCE COST FUNCTION AND ANALYSIS

4.1. New SRD-WLS cost function

Let $p_i = (r_{i1} + \|\mathbf{u} - \mathbf{s}_1\|)^2 - \|\mathbf{u} - \mathbf{s}_i\|^2$. The new squared range difference weighted least-squares (SRD-WLS) cost function is

$$f_{\text{SRD-WLS}}(\mathbf{u}) = \sum_{i,j=2}^{M} \tilde{w}_{dij} p_i p_j, \qquad (15)$$

where the unknown is considered to be $\mathbf{u}$ only and $\tilde{w}_{dij}$ is the weights to be found to improve performance.

Since $\|\mathbf{u}^o - \mathbf{s}_1\| \simeq r_1^o - \boldsymbol{\rho}_1^T\mathbf{n}_{s1}$ by the Taylor series expansion, the residual error at the true source location is

$$\begin{aligned}&(r_{i1} + \|\mathbf{u}^o - \mathbf{s}_1\|)^2 - \|\mathbf{u}^o - \mathbf{s}_i\|^2 \\ \simeq\ & 2r_i^o(n_{i1} - \boldsymbol{\rho}_1^T\mathbf{n}_{s1} + \boldsymbol{\rho}_i^T\mathbf{n}_{si}) \qquad (16)\end{aligned}$$

after putting $r_{i1} = r_{i1}^o + n_{i1}$ and $\mathbf{s}_i = \mathbf{s}_i^o + \mathbf{n}_{si}$. Using the same argument as for the range case, the ideal weightings are the elements of

$$\mathbf{W}_d = [\mathbf{B}_d(\mathbf{Q}_d + \mathbf{A}_d^T\mathbf{Q}_s\mathbf{A}_d)\mathbf{B}_d]^{-1} \qquad (17)$$

where $\mathbf{B}_d = 2\text{diag}\{r_2^o, r_3^o, \ldots, r_M^o\}$ and $\mathbf{A}_d$ is defined below (2). Let $\tilde{r}_1 = \|\tilde{\mathbf{u}} - \mathbf{s}_1\|$ and $\tilde{\mathbf{u}}$ is an initial source location estimate. Also, let $\tilde{r}_i = r_{i1} + \tilde{r}_1$, $i = 2, 3, \ldots, M$ and $\tilde{\boldsymbol{\rho}}_i = (\tilde{\mathbf{u}} - \mathbf{s}_i)/\tilde{r}_i$. We shall define $\tilde{\mathbf{B}}_d = 2\text{diag}\{\tilde{r}_2, \tilde{r}_3, \ldots, \tilde{r}_M\}$ and $\tilde{\mathbf{A}}_d$ as $\mathbf{A}_d$ by replacing $\boldsymbol{\rho}_i$ with $\tilde{\boldsymbol{\rho}}_i$. The weights $\tilde{w}_{dij}$ in (15) are the elements of

$$\widetilde{\mathbf{W}}_d = [\tilde{\mathbf{B}}_d(\mathbf{Q}_d + \tilde{\mathbf{A}}_d^T\mathbf{Q}_s\tilde{\mathbf{A}}_d)\tilde{\mathbf{B}}_d]^{-1}\,. \qquad (18)$$

4.2. MSE analysis

Following the same procedure as in the range localization case, we have for the first derivative,

$$\mathbf{f}'_{\text{SRD-WLS}} = -2\boldsymbol{\Gamma}_d\mathbf{B}_d\mathbf{W}_d\mathbf{B}_d(\mathbf{n}_d - \mathbf{A}_d^T\mathbf{n}_s) + o(\|\mathbf{n}\|)\,\mathbf{1} \qquad (19)$$

and hence after using (17)

$$E[\mathbf{f}'_{\text{SRD-WLS}}\mathbf{f}'^T_{\text{SRD-WLS}}] \simeq 4\boldsymbol{\Gamma}_d(\mathbf{Q}_d + \mathbf{A}_d^T\mathbf{Q}_s\mathbf{A}_d)^{-1}\boldsymbol{\Gamma}_d^T. \qquad (20)$$

For the second derivative,

$$\overline{\mathbf{F}}''_{\text{SRD-WLS}} = 2\boldsymbol{\Gamma}_d(\mathbf{Q}_d + \mathbf{A}_d^T\mathbf{Q}_s\mathbf{A}_d)^{-1}\boldsymbol{\Gamma}_d^T. \qquad (21)$$

Putting them into (7) yields

$$\mathbf{M}_{\text{SRD-WLS}} = \left[\boldsymbol{\Gamma}_d(\mathbf{Q}_d + \mathbf{A}_d^T\mathbf{Q}_s\mathbf{A}_d)^{-1}\boldsymbol{\Gamma}_d^T\right]^{-1}. \qquad (22)$$

$\mathbf{M}_{\text{SRD-WLS}}$ is the CRLB expression (2) for range difference localization under sensor position errors. As a result, the solution of the new SRD-WLS cost function is also asymptotically efficient.

Generating the new weights $\tilde{w}_{ij}$ or $\tilde{w}_{dij}$ requires a coarse estimate of the source location $\tilde{\mathbf{u}}$. It can be easily generated by using a localization algorithm, e.g. from [25], by pretending the sensor position errors are absent.

5. SIMULATIONS

In this section, we shall validate the asymptotic efficient performance of the SR-WLS and SRD-WLS cost functions that address sensor position errors, using one specific geometry and 200 random geometries. The specific localization geometry is taken from [17], where the true locations of the sensors are shown in Table 1 and the source is at $\mathbf{u}^o = [700, 650, 550]^T$. The sensor position covariance is $\mathbf{Q}_s = \sigma_s^2\text{diag}\{1, 2, 10, 40, 20, 3\} \otimes \mathbf{I}_3$, $\mathbf{I}_3$ is an identity matrix of size 3 and $\otimes$ is the Kronecker product. For the random geometries, we use $M = 10$ sensors. The sensors and the source are placed with independent, identically distributed (IID) uniform distribution in each coordinate within a cube of length 1000. To avoid degenerate geometry that yields poor performance, we maintain a minimum distance of 25 between the source and a sensor. The sensor position covariance is $\mathbf{Q}_s = \sigma_s^2\text{diag}\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_M^2\} \otimes \mathbf{I}_3$, where σ_i's are created randomly with IID uniform distributions and are normalized so that $\sum_1^M \sigma_i^2 = 1$. A new $\mathbf{Q}_s$ is used for each random geometry.

Table 1: The true positions of sensors

sensor no. i	1	2	3	4	5	6
x_i^o	300	400	300	350	-100	200
y_i^o	100	150	500	200	-100	-300
z_i^o	150	100	200	100	-100	-200

The GTRS solution [22] is used to solve for the SR-WLS cost function and the exact solution in [21] for the SRD-WLS cost function. For reference purpose, we also provide the results of the MLE that jointly estimates the source and sensor positions. The MLE is implemented by the Gauss-Newton method, where the initialization of each coordinate of the source is the true value added with independent zero-mean Gaussian white noise with variance equal to two times the CRLB, and the initializations of sensor positions are the erroneous sensor positions. We stop the iteration once the parameter change in the current step is larger than that in the previous step.

The range measurement covariance matrix is $\mathbf{Q} = \sigma^2\mathbf{I}$, and that of the range difference measurement is $\mathbf{Q}_d = \sigma^2(\mathbf{I} + \mathbf{1}\mathbf{1}^T)/2$

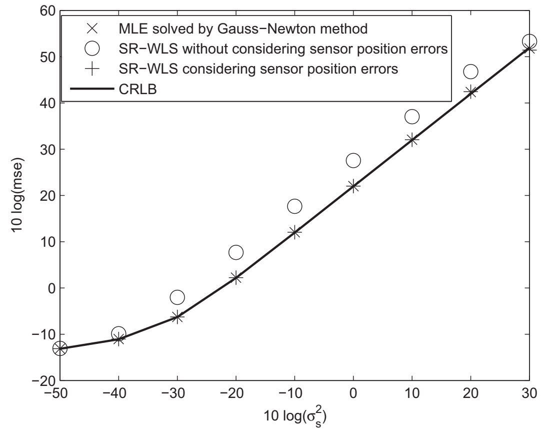

Fig. 2. Range (TOA) localization performance of SR-WLS considering sensor position errors, under the specific geometry in Table 1.

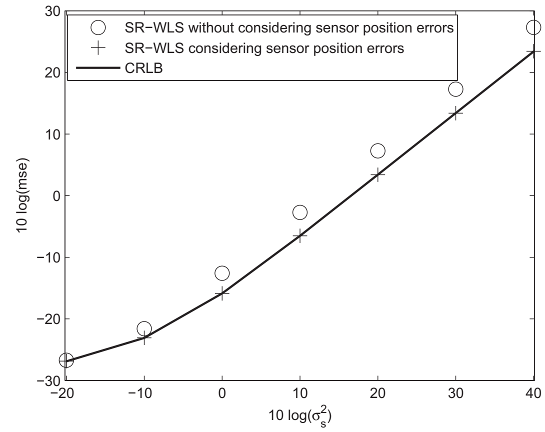

Fig. 3. Range (TOA) localization performance of SR-WLS considering sensor position errors, under the 200 random geometries.

[2], where we fix the noise level at $\sigma^2 = 10^{-3}$. The performance index is $\mathrm{mse} = \sum_{l=1}^{L} ||\mathbf{u}^{(l)} - \mathbf{u}^o||^2/L$, where $\mathbf{u}^{(l)}$ is the estimate at ensemble l and $L = 2000$ is the number of ensemble runs.

Fig. 2 shows the results for the specific geometry in range localization. The GTRS solution of the new cost function performs close to the MLE and attains the CRLB accuracy. It provides about 5.5 dB improvement over the previous cost function [25] that does not take the sensor position errors into account when σ_s^2 becomes significant. For the random geometry results shown in Fig. 3, the observations are consistent and the improvement is about 4 dB.

For range difference localization, the results for the specific geometry are depicted in Fig. 4. The new cost function yields the CRLB accuracy and matches the MLE performance. We would like to point out that the MLE experiences the thresholding effect at around $\sigma_s^2 = 10^{-0.2}$, which is caused by the sensitivity of initialization and by the large number of unknowns to be found. On the other hand, the solution from the new SRD-WLS cost function is relatively stable and provides about 4 dB improvement over previous SRD-WLS that ignores the sensor position errors. The observations are similar for the random geometry results shown in Fig. 5, and the new cost function has about 2.5 dB improvement.

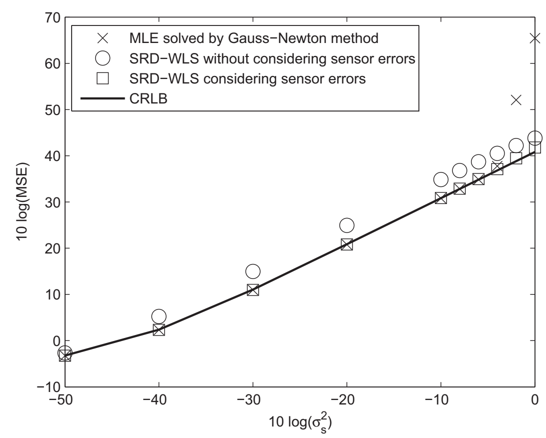

Fig. 4. Range difference (TDOA) localization performance of SRD-WLS considering sensor position errors, under the specific geometry in Table 1.

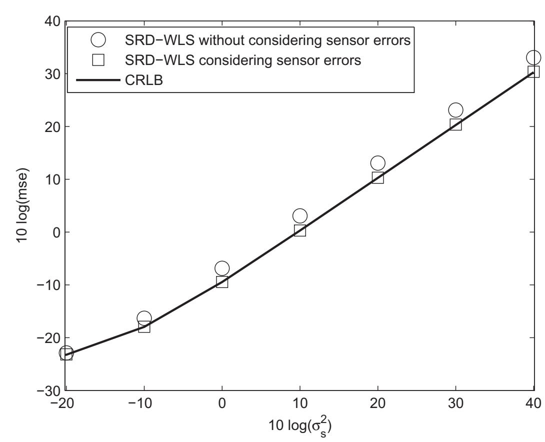

Fig. 5. Range difference (TDOA) localization performance of SRD-WLS considering sensor position errors, under the 200 random geometries.

6. CONCLUSIONS

Proper weightings must be used in the squared range and squared range difference cost functions to compensate for the effect of squaring. The weights derived in the previous work [25] is not adequate when the sensor positions contain errors. In this paper, we generalize the study and develop the new weights that take the sensor position errors into account to improve performance. We show from the first order analysis under Gaussian noise that although the weights are constructed from the measurements that are noisy, the performance of the source location solution closely follows the CRLB asymptotically. The new cost functions enable the application and development of algebraic solutions, and improve the computational efficiency relative to the iterative MLE.

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