NOVEL CLOSED-FORM AUXILIARY VARIABLES BASED ALGORITHMS FOR SENSOR NODE LOCALIZATION USING AOA

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ABSTRACT

Node localization is a key issue for wireless sensor networks (WSNs). The triangulation method and the maximum likelihood (ML) estimator are usually adopted for angle of arrival (AOA) based node localization in WSNs. However, the localization accuracy of the triangulation is low, and the ML estimator requires a good initialization close to the true location to avoid the divergence problem. In this paper, we develop two efficient closed-form AOA based localization algorithms derived from effective auxiliary variables based method. First, we formulate the node localization problem as a linear least squares problem using auxiliary variables. Based on its closed-form solution, a new auxiliary variables based pseudo-linear estimator (AVPLE) is developed. Then, we further propose an auxiliary variables based total least square (AVTLS) estimator to improve the localization accuracy. In addition, we investigate the impact of the orientation of the unknown node on estimation performance of the new algorithms. Simulation results demonstrate that the new algorithms achieve much higher localization accuracy than the triangulation method and also avoid local minima and divergence problem in ML estimator. Moreover, the AVTLS estimator has higher localization accuracy than the AVPLE, and its localization accuracy remains robust when the orientation angle of the unknown node varies from 0 to 180 degrees.

Index Terms— Angle of arrival; node localization; auxiliary variables based pseudo-linear estimator; auxiliary variables based total least square; closed-form solution

1. INTRODUCTION

Location information of sensor nodes in WSNs is essential since it is the basis for many applications, including, for example, target localization and tracking [1–3]. To obtain the location information of each node, one possible method is to equip each node with a GPS receiver. However, with severe constraints on cost and energy [4], only limited number of sensor nodes can be equipped with GPS receivers. These nodes are called beacons as their accurate locations are known by the GPS receivers. Other nodes without GPS receivers are called unknown nodes, whose locations have to be estimated.

In this paper, we focus on the AOA technique [5] for node localization in WSNs. One common method to obtain AOA measurements is to use an antenna array on each sensor node [6]. Different from target localization, AOA based node self-localization needs to estimate both the location and orientation of each unknown node. The orientation in the paper refers to the reference direction against which the AOAs are measured.

Over the past few years, several algorithms have been developed for AOA based node localization in WSNs, such as triangulation [5], [6], maximum likelihood (ML) estimator [7]. However, the localization accuracy of the triangulation method is low because of the error accumulation in the process of obtaining the center and radius of the circumscribed circle, and the ML method either requires a reasonable initialization close to the true solution or may suffer from local minima and even divergence problems [8], [9].

In this paper, we develop an effective auxiliary variables based method [10] to obtain the closed-form solution of AOA based node localization problem. It improves the localization accuracy compared with the triangulation method and avoids the problem of ML estimator. First, we develop an auxiliary variables based pseudo-linear estimator (AVPLE) algorithm, which employs auxiliary variables to formulate the self-localization problem as a linear least squares problem.

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Furthermore, we develop an auxiliary variables based total least square (AVTLS) estimator to improve the localization accuracy. In addition, we investigate the impact of the orientation of the unknown node on estimation performance of the new algorithms.

2. PROBLEM STATEMENT OF NODE LOCALIZATION

Let us assume that for an unknown node, there are N beacons within its sensing range, as shown in Fig. 1. Each beacon equips with a GPS receiver and knows its accurate location, denoted by $\boldsymbol{q}_j = [a_j, b_j]^T, j = 1, 2, \ldots, N$, but does not know its orientation. That is to say, we do not use the orientation information of the beacon in the estimation. The unknown node does not know its location or orientation, denoted by $\boldsymbol{p} = [x, y, \theta]^T$, is to be estimated in this paper.

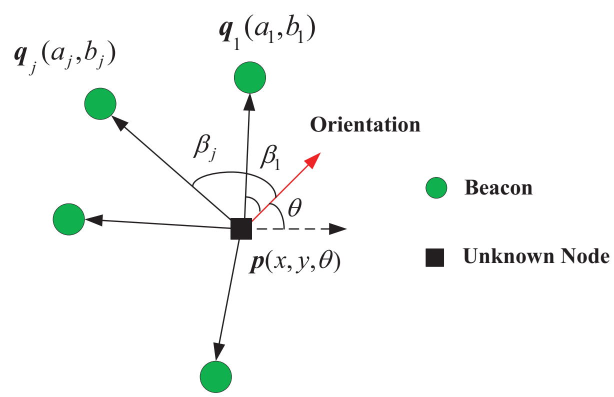

Fig. 1. Illustration of AOA based node localization in WSNs.

The beacons are signal emitters and the unknown node is a signal receiver. Hence, the AOAs of the beacons can be measured by the unknown node with respect to its own orientation. Let $\tilde{\beta}_j$ denote the measured AOA by the unknown node with respect to its own orientation θ from the jth beacon. The AOA measurements are given as follows:

$$\tilde{\beta}_j = \beta_j + \Delta\beta_j, \;\; \Delta\beta_j \sim \mathcal{N}(0, \sigma_j^2), \tag{1}$$

where β_j is the true AOA measurement from the jth beacon, $\Delta\beta_j \sim \mathcal{N}(0, \sigma_j^2)$ is the measurement noise following a Gaussian distribution with mean zero and variance of σ_j^2.

In this paper, our objective is to estimate the location and orientation of the unknown node based on the known locations of beacons and their AOAs measured by the unknown node.

3. AUXILIARY VARIABLE BASED NODE LOCALIZATION IN CLOSED-FORM

3.1. Auxiliary Variable based Pseudo-linear Estimator (AVPLE)

As shown in Fig. 1, the relationship between the unknown node and its neighbor beacons is:

$$\tan(\theta + \beta_j) = \frac{b_j - y}{a_j - x}, \; j = 1, 2, \ldots, N, \tag{2}$$

where (a_j, b_j) is the location of the jth beacon, (x, y) and θ denote the location and orientation of the unknown node, respectively.

After mathematical derivations [11], we could have

$$\begin{aligned} &\sin\beta_j u_1 - \cos\beta_j u_2 - (a_j \cos\beta_j + b_j \sin\beta_j) u_3 \\ &= a_j \sin\beta_j - b_j \cos\beta_j, \end{aligned} \tag{3}$$

where auxiliary variables

$$\begin{aligned} u_1 &= x + y\tan\theta, \\ u_2 &= y - x\tan\theta, \\ u_3 &= \tan\theta. \end{aligned} \tag{4}$$

Since the true AOA, β_j, cannot be obtained in the outdoor environment, we replace it with the measured AOA with noise, $\tilde{\beta}_j$, to formulate the linear least squares problem. Thus, (3) could be approximately expressed as

$$\boldsymbol{AU} \approx \boldsymbol{b}, \tag{5}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \sin\tilde{\beta}_1 & -\cos\tilde{\beta}_1 & -a_1\cos\tilde{\beta}_1 - b_1\sin\tilde{\beta}_1 \\ \vdots & \vdots & \vdots \\ \sin\tilde{\beta}_N & -\cos\tilde{\beta}_N & -a_N\cos\tilde{\beta}_N - b_N\sin\tilde{\beta}_N \end{bmatrix}, \tag{6a}$$

$$\boldsymbol{b} = \begin{bmatrix} a_1\sin\tilde{\beta}_1 - b_1\cos\tilde{\beta}_1 \\ \vdots \\ a_N\sin\tilde{\beta}_N - b_N\cos\tilde{\beta}_N \end{bmatrix}, \tag{6b}$$

$$\boldsymbol{U} = [u_1, u_2, u_3]^T. \tag{6c}$$

A least squares criteria can be used to solve (5). We adopt the PLE to get a closed-form solution. The estimated value of the auxiliary variables $\boldsymbol{U}$, denoted by $\hat{\boldsymbol{U}}_{avple}$, is

$$\hat{\boldsymbol{U}}_{avple} = \underset{\boldsymbol{U}}{\text{argmin}} ||\boldsymbol{AU} - \boldsymbol{b}||_2^2. \tag{7}$$

Based on (7) and (4), the location and orientation of the unknown node are given by

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{\theta} \end{bmatrix} = \begin{bmatrix} \frac{\hat{u}_1 - \hat{u}_3\hat{u}_2}{1+\hat{u}_3^2} \\ \frac{\hat{u}_2 + \hat{u}_1\hat{u}_3}{1+\hat{u}_3^2} \\ \arctan\hat{u}_3 \end{bmatrix}. \tag{8}$$

Remark 1 *Note that the tangent becomes unstable when the orientation of the unknown node is near $\pi/2$ (or $-\pi/2$). Thus, it may result in high estimation error of the auxiliary variables based PLE (AVPLE) method when the orientation is near $\pm\pi/2$.*

3.2. Auxiliary Variables based Total Least Squares (AVTLS)

In fact, when the noise in $\boldsymbol{A}$ is zero, and the noise is just confined to $\boldsymbol{b}$, the LS estimator can get the optimal solutions [12]. However, the measured AOAs in $\boldsymbol{A}$ exist noise in this paper, and thus the LS estimator, $\boldsymbol{X}_{ls} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b}$, is no longer optimal and it will suffer from bias and increased covariance owing to the accumulation of noise errors in $\boldsymbol{A}^T\boldsymbol{A}$ [13]. Therefore, for the node localization problem in (2) and (5), we develop an auxiliary variables based total least squares (AVTLS) estimator to improve the localization accuracy of the unknown node.

The AVTLS estimator aims to offset the noise present in $\boldsymbol{A}$ and $\boldsymbol{b}$ by perturbing $\boldsymbol{A}$ and $\boldsymbol{b}$, meanwhile minimizing the sum of squares of the norms of the perturbations, which is concern with solving the the following constrained minimization problem [12], [14]

$$[\hat{\boldsymbol{\Delta}}, \hat{\boldsymbol{\Gamma}}] = \underset{\boldsymbol{b}+\boldsymbol{\Gamma}\in \text{Range }(\boldsymbol{A}+\boldsymbol{\Delta})}{\arg\min} \|[\boldsymbol{\Delta}, \boldsymbol{\Gamma}]\|_F, \tag{9}$$

and $\|.\|_F$ represents the Frobenius norm.

Thus the AVTLS solution $\hat{\boldsymbol{U}}_{avtls}$ meets

$$(\boldsymbol{A} + \hat{\boldsymbol{\Delta}})\hat{\boldsymbol{U}}_{avtls} = \boldsymbol{b} + \hat{\boldsymbol{\Gamma}}, \tag{10}$$

where $\hat{\boldsymbol{\Delta}}$ and $\hat{\boldsymbol{\Gamma}}$ are the minimal perturbations as shown in (9). The AVTLS solution can be obtained from a singular value decomposition (SVD) of the augmented $N \times 4$ matrix $[\boldsymbol{A}, \boldsymbol{b}]$:

$$[\boldsymbol{A}, \boldsymbol{b}] = \boldsymbol{W}\boldsymbol{\Lambda}\boldsymbol{V}^T = \sum_{i=1}^{4} \lambda_i \boldsymbol{w}_i \boldsymbol{v}_i^T, \tag{11}$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq 0$ are the singular values, and $\boldsymbol{W} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3, \boldsymbol{w}_4]$ and $\boldsymbol{V} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4]$ are unitary matrix. Using the similar method in [12], [14], we can get the AVTLS solution of our problem as follows

$$\hat{\boldsymbol{U}}_{avtls} = -\frac{1}{v_{44}} \begin{bmatrix} v_{14} \\ v_{24} \\ v_{34} \end{bmatrix}, \tag{12}$$

where $\boldsymbol{v}_4 = [v_{14}, v_{24}, v_{34}, v_{44}]^T$ is the fourth column of matrix $\boldsymbol{V}$. Our solution is based on the assumption that the smallest singular value is unique. Further, when the smallest singular value is repeated at least two times, we can refer to literature such as [13] and [15] to obtain the solutions for this very rare occasions.

4. ERROR ANALYSIS FOR AVPLE AND AVTLS

4.1. Error Analysis for the AVPLE

The estimation bias of the auxiliary variables for the AVPLE in (7) is given by

$$\Delta\boldsymbol{U}_{avple} = -E\left\{(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{\phi}\right\}. \tag{13}$$

where

$$\boldsymbol{\phi} = \begin{bmatrix} d_1 \sin\Delta\beta_1 \\ \vdots \\ d_N \sin\Delta\beta_N \end{bmatrix}. \tag{14}$$

See [11] for mathematical details.

Then, based on Slutsky's theorem [16], for large N the AVPLE bias mentioned above can be approximated by

$$\Delta\boldsymbol{U}_{avple} \approx -E\left\{\frac{\boldsymbol{A}^T\boldsymbol{A}}{N}\right\}^{-1} E\left\{\frac{\boldsymbol{A}^T\boldsymbol{\phi}}{N}\right\}. \tag{15}$$

As $N \to \infty$, (15) becomes an equality.

4.2. Error Analysis for the AVTLS

Based on (10), the AVTLS solution can be given by

$$\hat{\boldsymbol{U}}_{avtls} = \left[(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})\right]^{-1}(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T(\boldsymbol{b}+\hat{\boldsymbol{\Gamma}}), \tag{16}$$

where $\hat{\boldsymbol{\Delta}}$ and $\hat{\boldsymbol{\Gamma}}$ are defined by [15]:

$$[\hat{\boldsymbol{\Delta}}, \hat{\boldsymbol{\Gamma}}] = -\lambda_4 \boldsymbol{w}_4 \boldsymbol{v}_4^T. \tag{17}$$

Similar to [14], we could have

$$(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T(\boldsymbol{A}+\hat{\boldsymbol{\Delta}}) = (\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T\boldsymbol{A}, \tag{18a}$$

$$\left[(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})\right]^{-1}(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T\hat{\boldsymbol{\Gamma}} = 0. \tag{18b}$$

Substituting (18) into (16), we have

$$\hat{\boldsymbol{U}}_{avtls} = \left[(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T\boldsymbol{A}\right]^{-1}(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T\boldsymbol{b}. \tag{19}$$

Note that [11],

$$\boldsymbol{b} = \boldsymbol{A}\boldsymbol{U} - \boldsymbol{\phi}, \tag{20}$$

where $\boldsymbol{U}$ is the true values of auxiliary variables. Thus, (19) can be expressed by

$$\hat{\boldsymbol{U}}_{avtls} = \boldsymbol{U} - \left[(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T\boldsymbol{A}\right]^{-1}(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T\boldsymbol{\phi}. \tag{21}$$

Further, the expectation is given by

$$E\{\hat{\boldsymbol{U}}_{avtls} - \boldsymbol{U}\} = -E\left\{\left[(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T\boldsymbol{A}\right]^{-1}(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T\boldsymbol{\phi}\right\}. \tag{22}$$

Using Slutsky's theorem [16], for sufficiently large N the AVTLS bias approximately equals to

$$E\{\hat{\boldsymbol{U}}_{avtls} - \boldsymbol{U}\} \approx -E\left\{\frac{(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T\boldsymbol{A}}{N}\right\}^{-1} E\left\{\frac{(\boldsymbol{A}+\hat{\boldsymbol{\Delta}})^T\boldsymbol{\phi}}{N}\right\}. \tag{23}$$

5. PERFORMANCE EVALUATION

Matlab simulations are conducted to evaluate the performance of our new algorithms. First, we investigate the impact of the orientation of the unknown node on estimation performance of the new algorithms. Then, the estimation errors of new algorithms, the triangulation method and the ML estimator are compared.

In this paper, we assume that all the AOA measurements are subject to independent identically distributed (i.i.d.) Gaussian white noise with the noise variance σ^2. Root mean square error (RMSE) is used to evaluate the estimation accuracy, denoted as $RMSE = \sqrt{\sum_{m=1}^{M} \|\hat{\boldsymbol{p}} - \boldsymbol{p}\|^2 / M}$, where $\hat{\boldsymbol{p}}$ is the estimated location, $\boldsymbol{p}$ is the true location of the unknown node and M is the number of the simulation runs. Simulation results are obtained with $M = 2,000$ Monte Carlo simulation runs.

5.1. Impact of the Orientation on Estimation Performance

We first investigate the impact of the orientation on estimation performance of the proposed algorithms. We randomly place eight beacons and one unknown node in the $100m \times 100m$ region. Fig. 2 shows the localization accuracy of the proposed algorithms as the orientation θ changes from 0 to 180 degrees. It can be seen that estimation error of the AVPLE is lower than that of the triangulation, except when θ is near 90 degrees (i.e., $\pi/2$), while the estimation error of the AVTLS remains stable when θ varies from 0 to 180 degrees. Therefore, we can conclude that the AVTLS estimator is more robust than the AVPLE. In addition, the AVTLS estimation error could be significantly reduced by means of coordinate translations using the method in [14].

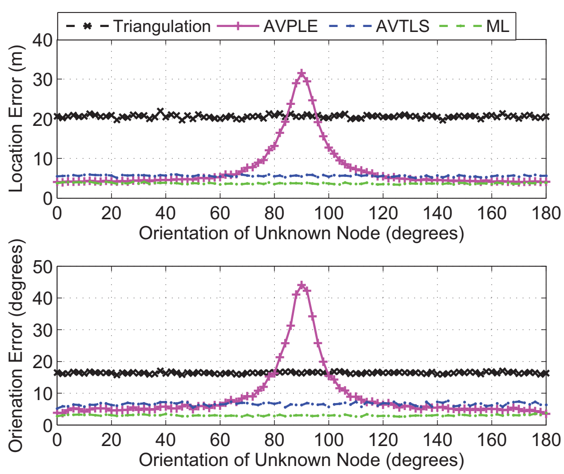

Fig. 2. Impact of the orientation angle on estimation performance of our proposed algorithms ($N = 8, \sigma = 4$).

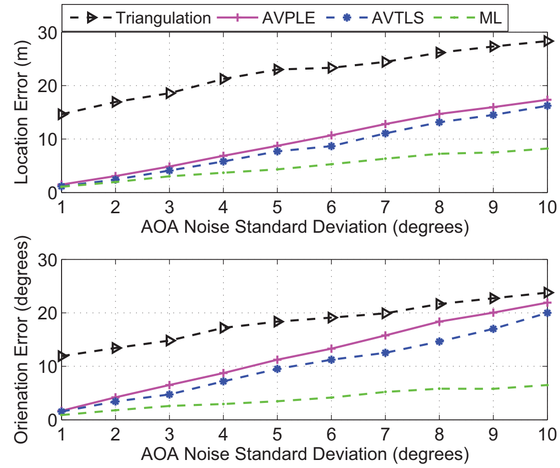

Fig. 3. Estimation errors of AVPLE and AVTLS in comparison with the triangulation and the ML estimator ($N = 8$).

5.2. Algorithm Comparisons

We consider a scenario with eight beacons and one unknown node randomly deployed in the $100m \times 100m$ region. The ML estimator is implemented by the Nelder-Mead simplex algorithm as in [17].

Fig. 3 illustrates the comparisons of estimation error (i.e, RMSE) for the triangulation method, the AVPLE, the AVTLS and the ML estimator initialized to the AVTLS. It can be seen that the AVPLE and the AVTLS have lower estimation error than the triangulation, which verifies the effectiveness and feasibility of our new algorithms. Moreover, the AVTLS has higher localization accuracy than the AVPLE, while the estimation error of the ML is lower than other three algorithms. However, the ML estimator is sensitive to initializations and may suffer from local convergence problem [11].

6. CONCLUSION AND FUTURE WORK

In this paper, we presented two novel auxiliary variables based closed-form algorithms for the efficient AOA based node localization. Compared with the triangulation, our new auxiliary variables based algorithms significantly improve the localization accuracy. And unlike the ML estimator, the new algorithms do not suffer from local convergence problem. Moreover, we investigated the impact the orientation on the estimation performance of the proposed algorithms. The results show that the estimation error of the AVPLE is large when the orientation θ is near 90 degrees, while the estimation error of the AVTLS remains robust as θ varies from 0 to 180 degrees. To further improve the localization accuracy, the constrained least squares [18] may be helpful to estimate the unknown node.

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