WEIGHTED STANSFIELD ALGORITHM IN THREE DIMENSIONS

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ABSTRACT

In bearings-only geolocation, strategies that accurately combine both azimuth and elevation information for three-dimensional position estimation are required. This paper introduces a novel bearings-only geolocation algorithm, the Weighted Stansfield Algorithm in Three Dimensions (WS3D). This procedure uses a linearized approximation to the maximum likelihood cost for both the azimuth and elevation measurements, and uses both the bearing accuracies and estimates of the range to emitter in the zero-elevation geometric plane to minimize this cost in pair of least-squares procedures. Simulations of the new algorithm show that range and bearing accuracy weighting is important to obtain best performance. The algorithm also asymptotically outperforms a recently-derived instrumental variable (IV) algorithm for 3D geolocation designed to remove bias effects.

***Index Terms*—** antenna arrays, azimuth, direction-of-arrival estimation, maximum likelihood estimation.

1. INTRODUCTION

Bearings-only geolocation algorithms are important for numerous applications, including health care, emergency response, and defense [1]. Such algorithm employ angles of arrival (AOAs) to locate an emitter using triangulation. All bearing lines intersect at a single point if the AOAs are exact, but due to bearing noise, an estimation procedure is required for practical geolocation. Perhaps the most well-known bearings-only geolocation algorithm is due to R.G. Stansfield [2, 3], which uses a linearized least-squares procedure to find an emitter in a 2D geometric plane using azimuth angles alone.

There are a number of practical scenarios where a three-dimensional position estimate of an emitter is required. For example, in an emergency response situation involving a high-rise building, knowledge of what floor a particular occupant is on would speed up the search time. Because the Stansfield algorithm is formulated as a 2D estimation procedure on azimuth angle measurements $\widehat{\theta}_i$, extensions of this and other procedures to three dimensions that employ both azimuth and elevation angle measurements $\{\widehat{\theta}_i, \widehat{\phi}_i\}$ are required. There have been several recent works that extend 2D geolocation to 3D, including the pseudo-linear estimator, the weighted instrumental variable (WIV) estimator, and the orthogonal vector estimator [4, 5, 6]. The theoretical understanding of the performance of bearings-only geolocation algorithms is still in its infancy, both in terms of bias [7] and finite-sample performance [8].

While straightforward to implement, the Stansfield algorithm is known to be biased asymptotically [3]. In [5], an Instrumental Variable (IV) algorithm is introduced in an attempt to overcome this bias. This algorithm introduces an instrumental variable matrix into the 2D Stansfield least-squares formulation that uses estimated bearing directions that are calculated using a separate Stansfield algorithm, and thus its operation is more-complex than the original Stansfield algorithm. Moreover, while the IV algorithm is described for 3D geolocation, it is unclear how the IV algorithm addresses positional bias due to noisy elevation measurements.

In this paper, we derive a new three-dimensional geolocation method termed the Weighted Stansfield in Three Dimensions (WS3D) algorithm. This algorithm uses a unique approximation to the maximum likelihood (ML) cost on the combined azimuth and elevation measurements, and it employs knowledge of both the bearing variances and the range to the emitter to minimize this cost. Two interconnected least-squares procedures – one 2D, and one 1D – are employed, such that the algorithm is simple to implement. Numerical simulations of this procedure show that it outperforms both the pseudo-linear estimator and the IV estimator in 3D position estimation for small bearing errors, and that range weighting is important to obtain best performance. Moreover, in many cases, the root MSE of the WS3D algorithm is less than that of the IV estimator that was specifically-designed to reduce estimation bias in an attempt to reduce the overall position error.

2. BEARINGS-ONLY GEOLOCATION

We first introduce the bearings-only geolocation task and describe prior work. Fig. 1 shows a 3D view of the fixed emitter

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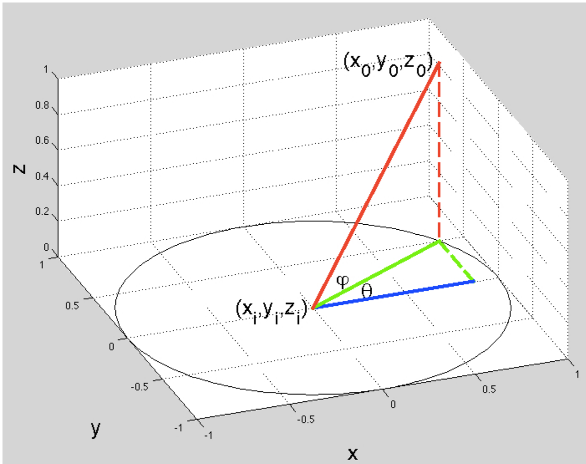

Fig. 1. Bearing geometry as a function of position.

position (x_0, y_0, z_0) and receiver position $\{x_i, y_i, z_i\}$ at time instant i. Suppose a sequence of azimuth $\widehat{\theta}_i$ and elevation $\widehat{\phi}_i$ measurements are taken at various receiver positions for different i. We assume a noisy model for these measurements, such that $\widehat{\theta}_i = \theta_i + \eta_i$ and $\widehat{\phi}_i = \phi_i + \zeta_i$, where θ_i and ϕ_i are the true values and η_i and ζ_i are i.i.d Gaussian zero mean random variables that are independent from bearing to bearing with a known variance δ_i^2 and ϵ_i^2, respectively. Thus,

$$\theta_i(x_0, y_0) = \arctan\left(\frac{y_0 - y_i}{x_0 - x_i}\right) \quad (1)$$

$$\phi_i(x_0, y_0, z_0) = \arctan\left(\frac{z_0 - z_i}{\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2}}\right) \quad (2)$$

The pseudo-linear estimator (PLE) is an extension of the 2D Stansfield algorithm to three dimensions proposed by Dogancay and Ibal [4]. In the first step of this procedure, the 2D Stansfield algorithm is used to estimate (x_0, y_0) in the zero-elevation geometric plane over n azimuth measurements $\widehat{\theta}_i$ at positions (x_i, y_i) via the approximate relation

$$\mathbf{A}_n \widehat{\mathbf{p}}_{xy}(n) \approx \mathbf{b}_n \quad (3)$$

$$\begin{bmatrix} \sin\widehat{\theta}_1 & -\cos\widehat{\theta}_1 \\ \vdots & \vdots \\ \sin\widehat{\theta}_n & -\cos\widehat{\theta}_n \end{bmatrix} \begin{bmatrix} \widehat{x}_0(n) \\ \widehat{y}_0(n) \end{bmatrix} \approx \begin{bmatrix} \sin\widehat{\theta}_1 x_1 - \cos\widehat{\theta}_1 y_1 \\ \vdots \\ \sin\widehat{\theta}_n x_n - \cos\widehat{\theta}_n y_n \end{bmatrix} \quad (4)$$

The estimated 2D position $\widehat{\mathbf{p}}_{xy}(n)$ is found as

$$\widehat{\mathbf{p}}_{xy}(n) = (\mathbf{A}_n^T \mathbf{A}_n)^{-1} \mathbf{A}_n^T \mathbf{b}_n \quad (5)$$

In the second step of the procedure, the estimate $\widehat{z}_0(n)$ is computed using the elevation angle measurements $\widehat{\phi}_i$ as

$$\widehat{z}_0(n) = \frac{1}{n}\sum_{i=1}^{n}\left(z_i + \sqrt{(\widehat{x}_0(n) - x_i)^2 + (\widehat{y}_0(n) - y_i)^2}\tan\widehat{\phi}_i\right). \quad (6)$$

The Stansfield algorithm is known to produce biased estimates of position due to bearing noise. To attempt to reduce this bias, the Weighted Instrumental Variable algorithm uses the instrumental variable matrix $\mathbf{G}_n$ defined as

$$\mathbf{G}_n = \begin{bmatrix} \sin\overline{\theta}_1 & -\cos\overline{\theta}_1 \\ \vdots & \vdots \\ \sin\overline{\theta}_n & -\cos\overline{\theta}_n \end{bmatrix}, \quad (7)$$

where $\overline{\theta}_i$ is found from a separate 2D Stansfield procedure via (5) and

$$\overline{\theta}_i = \arctan\left(\frac{\widehat{y}_0(n) - y_i}{\widehat{x}_0(n) - x_i}\right), \quad (8)$$

Then, a new 2D position estimate $\overline{\mathbf{p}}_{xy}(n) = [\overline{x}_i(n)\ \overline{y}_i(n)]^T$ is found as

$$\overline{\mathbf{p}}_{xy}(n) = (\mathbf{G}_n^T \overline{\mathbf{W}}_n^{-1} \mathbf{A}_n)^{-1} \mathbf{G}_n^T \overline{\mathbf{W}}_n^{-1} \mathbf{b}_n, \quad (9)$$

where $\overline{\mathbf{W}}_n$ is a diagonal matrix of squared range values $\widehat{r}_i(\widehat{x}_0(n), \widehat{y}_0(n)) = (\widehat{x}_0(n) - x_i)^2 + (\widehat{y}_0(n) - y_i)^2$. Finally, $\widehat{z}_i(n)$ is estimated using (6) with $(\overline{x}_0(n), \overline{y}_0(n))$ replacing $(\widehat{x}_0(n), \widehat{y}_0(n))$.

3. WEIGHTED STANSFIELD ALGORITHM IN THREE DIMENSIONS

Our proposed algorithm considers the maximum likelihood (ML) cost function for both the azimuth and elevation measurements, given by

$$J_n(x_0, y_0, z_0) = \sum_{i=1}^{n} \frac{1}{\delta_i^2}(\theta_i(x_0, y_0) - \widehat{\theta}_i)^2 + \frac{1}{\epsilon_i^2}(\phi_i(x_0, y_0, z_0) - \widehat{\phi}_i)^2 \quad (10)$$

Maximum likelihood estimation has a number of advantages, including statistical efficiency. We first approximate $J_n(x_0, y_0, z_0)$ as the sum of two cost functions

$$J_n(x_0, y_0, z_0) = J_n(x_0, y_0) + J_n(z_0|\widehat{x}_0, \widehat{y}_0) \quad (11)$$

where $J_n(z_0|\widehat{x}_0, \widehat{y}_0)$ is minimized using the solution $(\widehat{x}_0, \widehat{y}_0)$ obtained by minimization of $J_n(x_0, y_0)$. This approach is suboptimal relative to the ML cost and assumes that the elevation measurements ϕ_i have a weak effect on the ability to estimate (x_0, y_0).

Both $J_n(x_0, y_0)$ and $J_n(z_0|\widehat{x}_0, \widehat{y}_0)$ are nonlinear in the position variables x_0, y_0, and z_0. In an attempt to linearize this problem, we alter the cost functions to

$$\widehat{J}_n(x_0, y_0) = \sum_{i=1}^{n} \frac{1}{\delta_i^2} \sin^2(\theta_i(x_0, y_0) - \widehat{\theta}_i) \quad (12)$$

$$\widehat{J}_n(z_0|\widehat{x}_0, \widehat{y}_0) = \sum_{i=1}^{n} \frac{1}{\epsilon_i^2} \tan^2(\phi_i(\widehat{x}_0, \widehat{y}_0, z_0) - \widehat{\phi}_i)^2 \quad (13)$$

Both approximations are only valid when the angular estimation error is small, as then $\tan(\varphi) \approx \sin(\varphi) \approx \varphi$.

Minimization of $\widehat{J}_n(x_0, y_0)$ is straightforward and involves the same steps as used to derive the original 2D Stansfield algorithm, namely,

$$\sin(\theta_i(x_0, y_0) - \widehat{\theta}_i) = \frac{(y_0 - y_i)\cos\widehat{\theta}_i - (x_0 - x_i)\sin\widehat{\theta}_i}{\sqrt{(y_0 - y_i)^2 + (x_0 - x_i)^2}} \quad (14)$$

Let $r_i(x_0, y_0) = \sqrt{(y_0 - y_i)^2 + (x_0 - x_i)^2}$. Then, direct minimization of $\widehat{J}_n(x_0, y_0)$ yields the solution

$$\widehat{\mathbf{p}}_{xy}(n) = (\mathbf{A}_n^T \mathbf{W}_n^{-1} \mathbf{A}_n)^{-1} \mathbf{A}_n^T \mathbf{W}_n^{-1} \mathbf{b}_n \quad (15)$$

where $\mathbf{W}_n$ is a diagonal matrix whose ith diagonal entry is $r_i^2(\widehat{x}_0, \widehat{y}_0)\delta_i^2$. To evaluate $\mathbf{W}_n$, we use the position $\widehat{\mathbf{p}}_{xy}(n-1)$ estimated at block size $(n-1)$ to compute $\mathbf{W}_n$ for a block size of n. This algorithm assumes that the bearing variances δ_i^2 are known or can be estimated using side information. Thus, at time n, the solution to the 2D emitter position $(\widehat{x}_0(n), \widehat{y}_0(n))$ at measurement time n is found using (15), from which the values $r_i^2(\widehat{x}_0(n), \widehat{y}_0(n))$, $1 \leq i \leq n$ are calculated for use in both $\mathbf{W}_{n+1}$ and in the estimation of z_0.

We now consider the minimization of $J_n(z_0|\widehat{x}_0, \widehat{y}_0)$ in (13). To do so, note that

$$\begin{aligned}
&\tan(\phi_i(\widehat{x}_0, \widehat{y}_0, z_0) - \widehat{\phi}_i) \\
&= \frac{\tan(\phi_i(\widehat{x}_0, \widehat{y}_0, z_0)) - \tan(\widehat{\phi}_i)}{1 + \tan(\phi_i(\widehat{x}_0, \widehat{y}_0, z_0))\tan(\widehat{\phi}_i)} \quad (16) \\
&\approx \frac{\tan(\phi_i(\widehat{x}_0, \widehat{y}_0, z_0)) - \tan(\widehat{\phi}_i)}{1 + \tan^2(\widehat{\phi}_i)} \quad (17) \\
&= \cos^2(\widehat{\phi}_i)\left[\tan(\phi_i(\widehat{x}_0, \widehat{y}_0, z_0)) - \tan(\widehat{\phi}_i)\right]. \quad (18)
\end{aligned}$$

Furthermore,

$$\tan(\phi_i(\widehat{x}_0, \widehat{y}_0, z_0)) = \frac{z_0 - z_i}{\sqrt{(\widehat{x}_0 - x_i)^2 + (\widehat{y}_0 - y_i)^2}}. \quad (19)$$

Clearly, for small elevation bearing errors, the tangent of the elevation angle is approximately linear in z_0. Thus, the cost function in (13) is approximated by

$$\begin{aligned}
&\widehat{J}_n(z_0|\widehat{x}_0, \widehat{y}_0) \\
&= \sum_{i=1}^{n} \frac{\cos^4(\widehat{\phi}_i)}{\epsilon_i^2 r_i^2(\widehat{x}_0, \widehat{y}_0)}\left[z_0 - z_i - \tan(\widehat{\phi}_i) r_i(\widehat{x}_0, \widehat{y}_0)\right]^2 \quad (20)
\end{aligned}$$

Taking derivatives of $\widehat{J}_n(z_0|\widehat{x}_0, \widehat{y}_0)$ with respect to z_0, we obtain

$$\widehat{z}_0(n) = \frac{\sum_{i=1}^{n} d_i(n)\left[z_i + \tan\widehat{\phi}_i r_i(\widehat{x}_0(n), \widehat{y}_0(n))\right]}{\sum_{j=1}^{n} d_j(n)} \quad (21)$$

$$d_i(n) = \frac{\cos^4(\widehat{\phi}_i)}{\epsilon_i^2 r_i^2(\widehat{x}_0(n), \widehat{y}_0(n))} \quad (22)$$

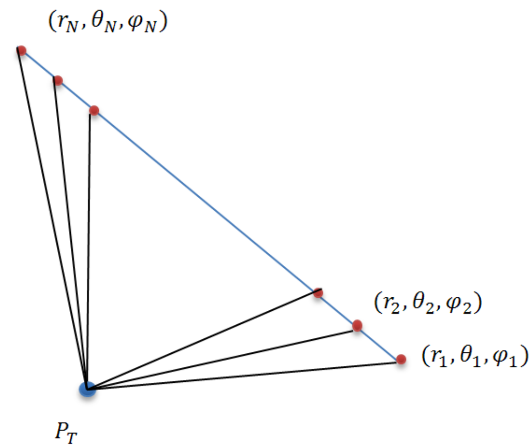

Fig. 2. Sensor array movement geometry in the simulation examples.

Taken together, Eq. (15), (21) and (22) describe the weighted Stansfield algorithm in three dimensions. It has three important differences as compared to both the pseudo-linear estimator and the IV estimator:

1. The WS3D algorithm is derived by starting with a maximum likelihood cost function for both the azimuth and elevation measurements.
2. The WS3D algorithm uses the bearing variances δ_i^2 and ϵ_i^2, when they are available, as weightings within the algorithm to improve estimation performance.
3. The WS3D algorithm uses range information as weightings in the estimation of both z_0 and the pair (x_0, y_0).

4. SIMULATION RESULTS

We now explore the proposed algorithm via simulations. A stationary emitter is sensed by a single moving antenna array with the following geometry: a starting distance of 1000 m and zero elevation, a straight line trajectory with a velocity of 15 m/s in the (x, y) plane and 2 m/s in the z direction, and an incident angle of 40° away from a direct path. Fig. 2 shows the corresponding geometry. Bearings $\widehat{\theta}_i$ and $\widehat{\phi}_i$ are recorded at a rate of 5 Hz along this trajectory. The pseudo-linear estimator, the WIV estimator, and the proposed WS3D estimator are all used to compute the root mean-squared error (RMSE) for these measurements. We use averages of 1000 Monte Carlo simulation runs to produce the plots shown.

In the first example, both the azimuth and elevation bearings have a constant bearing standard deviation of $\delta_i = \epsilon_i = 5°$. In this case, the WS3D does not require bearing accuracy knowledge to function, as both δ_i and ϵ_i can be set to any single positive value in the algorithm without changing its functional form for this case. Fig. 3 shows the root MSE for the three algorithms. As can be seen, the WIV algorithm has the best initial performance over the first 60 seconds. However, the long-term performance of the WS3D algorithm is the best of the three approaches, and both the pseudo-linear estimator

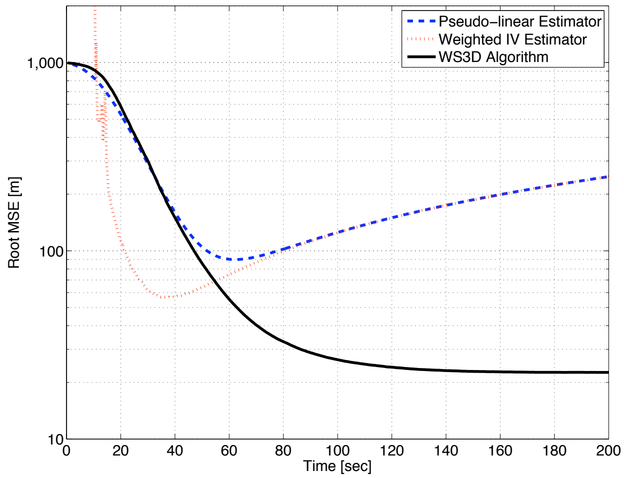

Fig. 3. Performances for a constant 5° bearing accuracy.

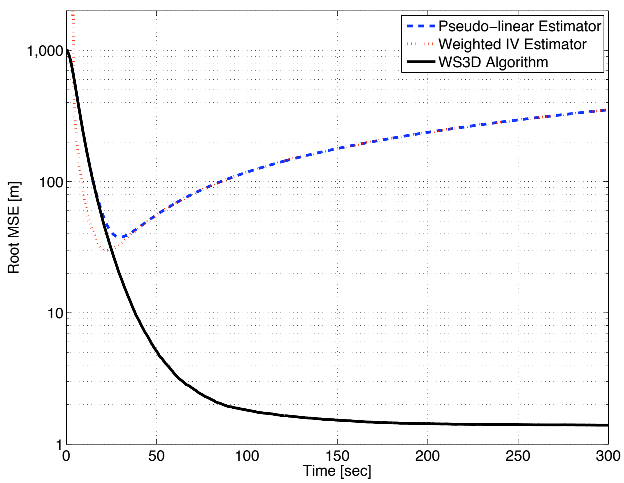

Fig. 4. Performances for a constant 1° bearing accuracy.

and WIV estimator show an increasing root MSE over time.

Fig. 4 shows the root MSE for the three algorithms in the case where more-accurate bearing measurements with $\delta_i = \epsilon_i = 1^\circ$ are available. The WS3D algorithm has a much better performance than the other two algorithms in this case after 25 seconds and achieves a much lower steady-state error. Again, both the pseudo-linear estimator and WIV estimator have an increasing root MSE over time. All three algorithms do not require knowledge of the bearing accuracy to function.

Fig. 5 shows the performance of the three algorithms in the case where 20% of the bearings have a 10° accuracy and 80% of the bearings have a 1° accuracy. The WS3D algorithm outperforms the other two algorithms at all time instants, and its performance is not significantly different from the previous case where all bearings have a 1° average error. This result shows the power of weighting by both estimated range and bearing accuracy, although knowledge of each bearing's accuracy is required to obtain this benefit.

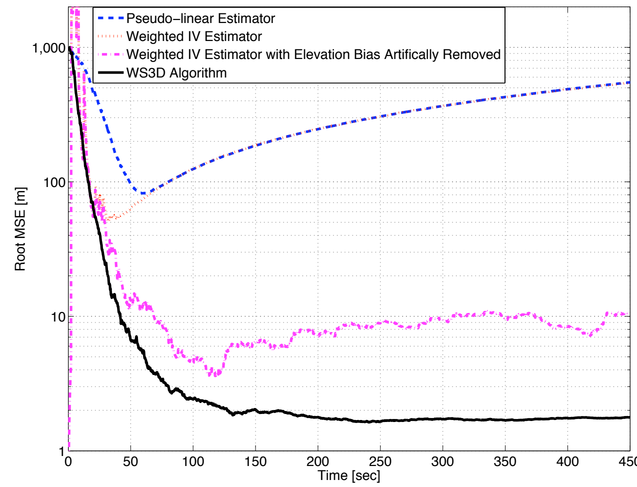

Fig. 5. Performances for varying bearing qualities; see text for explanation.

To better understand the issues surrounding estimation bias in existing algorithms, Fig. 5 also shows the performance of the WIV algorithm in this simulation scenario in which the bias associated with the z_0 estimate is artificially removed from the root MSE by subtracting out in quadrature its estimated value at each time instant. As can be seen, the remaining estimation error is still larger than that produced by the WS3D algorithm. This also shows that the existing algorithms do not accurately account for bias due to elevation measurements.

5. CONCLUSION

In this paper, we have proposed a novel extension of the well-known 2D Stansfield algorithm for three-dimensional geolocation tasks. The weighted Stansfield algorithm in three dimensions is a simple and robust procedure that provides better performance than existing approaches due to its maximum likelihood formulation. The WS3D algorithm uses weightings that depend on estimated 2D range and bearing quality. While estimation of 2D range is easily implementable within the algorithm, methods to estimate bearing quality are the subject of future research.

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