# THE ELITIST PARTICLE FILTER BASED ON EVOLUTIONARY STRATEGIES AS NOVEL APPROACH FOR NONLINEAR ACOUSTIC ECHO CANCELLATION

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# ABSTRACT

In this article, we introduce a novel approach for nonlinear acoustic echo cancellation based on a combination of particle filtering and evolutionary strategies. The nonlinear echo path is modeled as a state vector with non-Gaussian probability distribution and the relation to the observed signals and near-end interferences are captured by nonlinear functions. To estimate the probability distribution of the state vector and the model parameters, we apply the numerical sampling method of particle filtering, where each set of particles represents different realizations of the nonlinear echo path. While the classical particle-filter approach is unsuitable for system identification with large search spaces, we introduce a modified particle filter to select elitist particles based on long-term fitness measures and to create new particles based on the approximated probability distribution of the state vector. The validity of the novel approach is experimentally verified with real recordings for a nonlinear echo path stemming from a commercial smartphone.

*Index Terms*— Echo cancellation, nonlinear AEC, system identification, particle filter, evolutionary strategies

# 1. INTRODUCTION

The problem of acoustic echo cancellation (AEC) for applications like teleconferencing and hands-free communication systems has been investigated for several decades and is still an active research topic. While linear AEC has reached a mature state as vital part of today's communication devices, nonlinear distortions created by amplifiers and transducers in miniaturized loudspeakers limit the practical performance of linear echo path models [1]. Although nonlinear residual echo suppression (NL-RES) can be applied to improve the performance of linear echo cancelers [2, 3], the unavoidable near-end distortions of NL-RES do not alleviate the need for nonlinear acoustic echo cancellation (NL-AEC).

A variety of concepts for NL-AEC have been proposed like the cascade of nonlinear signal transformation and linear filtering [4, 5]. The distortions which are part of the physical system can be modeled as adaptive [1, 6] or static [7] nonlinear functions. The second class of nonlinear approaches is based on the Volterra filters (VFs) which can be thought of as a Taylor-series expansion of a system with memory [8–10]. Various simplified realizations have been proposed to facilitate real-time implementations with a low order for the nonlinearity [11–13]. As third category, artificial neural networks (ANNs) are used to address the nonlinear characteristics with

a non-convex optimization problem [14–16]. Combined with the introduction of functional links as integration of linearly independent functions, the functional link artificial neural network (FLANN) and functional link adaptive filter (FLAF) have been proposed in [17] and [18], respectively. Finally, Kernel methods are based on a data mapping which facilitates the analysis of nonlinear methods by linear operators in a high-dimensional feature space [19–21].

In contrast to prior work, we introduce a novel approach for NL-AEC which is based on a sequential Monte Carlo method. The nonlinear relation between loudspeaker and microphone signal is modeled as part of a dynamical system whose state variables are not restricted to be normally distributed random variables. To approximate the statistics of the nonlinear echo path and to estimate the model parameters, we apply the classical particle filtering, where each particle represents one realization of the nonlinear echo path [22, 23]. In order to overcome conceptual disadvantages of the classical particle-filter approach, we introduce a new algorithm denoted as the elitist particle filter based on evolutionary strategies (EPFES). The fundamental idea is a selection of elitist particles by means of long-term fitness measures for the approximation of the statistics of the nonlinear echo path and the estimation of the model parameters. Furthermore, we introduce a sampling procedure which facilitates the dynamic identification of nonlinear characteristics. The validity of the proposed approach is experimentally verified with various smartphone recordings.

This paper is organized as follows: In Section 2, we briefly introduce NL-AEC based on modeling the nonlinear distortions by a memoryless preprocessor. This is subsequently addressed from a Bayesian network perspective in Section 3, where the classical particle filter is presented and its conceptional limitations described. In Section 4, we introduce the EPFES and discuss one possible realization for the application of NL-AEC. The experimental evaluation in Section 5 verifies the proposed algorithm with real smartphone recordings. Finally, conclusions are drawn in Section 6.

## 2. NL-AEC USING A MEMORYLESS PREPROCESSOR

The system model for NL-AEC is shown in Fig. 1. The acoustic path at time n between loudspeaker and microphone is partly modeled by the linear finite impulse response (FIR) filter

$$\mathbf{h}_{n} = [h_{0,n}, h_{2,n}, ..., h_{M-1,n}]^{T}$$
(1)

with coefficients  $h_{\kappa,n}$  which are dependent on the time instant, where  $\kappa = 0, ..., M - 1$ . Interfering components are captured by the additive variable  $v_n$  and nonlinearities of the echo path (as typically resulting from the loudspeaker and the corresponding amplifier) are modeled by a nonlinear function applied to the input [1].

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Fig. 1. NL-AEC with memoryless preprocessor

As a consequence, the observation equation models the microphone sample at time n,  $d_n$ , as

$$d_n = \mathbf{h}_n^T \cdot y(\mathbf{x}_n, \mathbf{a}_n) + v_n, \qquad (2)$$

where the input signal vector  $\mathbf{x}_n$  is defined as

$$\mathbf{x}_{n} = [x_{n}, x_{n-1}, ..., x_{n-M+1}]^{T}$$
(3)

with time-domain samples  $x_n$ . Furthermore, the time-dependent length-P vector

$$\mathbf{a}_{n} = [a_{0,n}, a_{1,n}, \dots, a_{P-1,n}]^{T}$$
(4)

parameterizes the nonlinear function  $y(\cdot)$ . The random variable  $v_n$  is assumed to be statistically independent from the input signal vector. To estimate the relevant system parameters  $\hat{\mathbf{a}}_n$  and  $\hat{\mathbf{h}}_n$ , we make use of the error  $e_n$  between the observation  $d_n$  and the estimated microphone signal  $\hat{d}_n$ .

# 3. NL-AEC USING CLASSICAL PARTICLE FILTERING

Assume the relevant information of a nonlinear system to be captured by the state vector

$$\mathbf{z}_{n} = [z_{0,n}, z_{2,n}, ..., z_{R-1,n}]^{T} = [\mathbf{h}_{n}^{T}, \mathbf{a}_{n}^{T}]^{T}$$
(5)

with coefficients  $z_{\nu,n}$  and  $\nu = 0, ..., R-1$ , where R = M + P. This unobservable or latent vector is dependent on the time instant n = 1, ..., N and its temporal evolution described by the system model

$$\mathbf{z}_n = f\left(\mathbf{z}_{n-1}, \mathbf{w}_n\right),\tag{6}$$

where  $f(\cdot)$  represents the so-called nonlinear progress [24]. The uncertainty of the state vector is denoted as  $\mathbf{w}_n$  and equally structured as  $\mathbf{z}_n$  in (5) with coefficients  $w_{\nu,n}$ . The relationship between the state vector  $\mathbf{z}_n$  and the observation  $d_n$  is described as

$$d_n = g\left(\mathbf{x}_n, \mathbf{z}_n\right) + v_n,\tag{7}$$

where  $g(\cdot)$  represents a nonlinear function which also depends on the input signal vector  $\mathbf{x}_n$ . With respect to (2), the state vector  $\mathbf{z}_n$  intends to model the echo path including the nonlinear transformation of the input signal vector, as well as the acoustic wave propagation from the loudspeaker to the microphone, so that

$$g(\mathbf{x}_n, \mathbf{z}_n) = \mathbf{h}_n^T \cdot y(\mathbf{x}_n, \mathbf{a}_n).$$
(8)

The uncertainty of the observation  $d_n$  is modeled by the additive variable  $v_n$ . From a Bayesian network perspective, this corresponds to the graphical model shown in Fig. 2. The directed links express statistical dependencies between the nodes and observed variables, such as  $d_n$ , are marked by shaded circles. The estimate of the state



Fig. 2. Bayesian network for NL-AEC

vector  $\hat{\mathbf{z}}_n$  is derived as an minimum mean square error (MMSE) estimate

$$\hat{\mathbf{z}}_n = \operatorname*{argmin}_{\tilde{\mathbf{z}}_n} \mathcal{E}\{||\tilde{\mathbf{z}}_n - \mathbf{z}_n||_2^2\},\tag{9}$$

where  $||\cdot||_2$  is the euclidean norm and  $\mathcal{E}\{\cdot\}$  the expectation operator. The minimization of (9) with respect to  $\hat{\mathbf{z}}_n$  yields the mean vector of the posterior probability density function (PDF)  $p(\mathbf{z}_n|d_{1:n})$  as estimate for the state vector

$$\hat{\mathbf{z}}_n = \mathcal{E}\{\mathbf{z}_n | d_{1:n}\},\tag{10}$$

where  $d_{1:n} = d_1, ..., d_n$ . In the case of linear relations between the variables in (6) and (7), and for a linear estimator for  $\hat{\mathbf{z}}_n$ , the MMSE estimate of (9) leads to the Kalman filter equations. This estimation is optimal in the Bayesian sense for normally distributed random variables. Due to the nonlinear structures of (6) and (7), the non-Gaussian PDF of the state vector  $\mathbf{z}_n$  precludes a closed-form solution for the Bayesian estimate of  $\hat{\mathbf{z}}_n$  in (10), so that we employ the particle filter to approximate the posterior PDF

$$p(\mathbf{z}_n|d_{1:n}) = \frac{p(d_n|\mathbf{z}_n)p(\mathbf{z}_n|d_{1:n-1})}{\int p(d_n|\mathbf{z}_n)p(\mathbf{z}_n|d_{1:n-1})\mathrm{d}\mathbf{z}_n}$$
(11)

by a discrete distribution [25, 26]

$$p(\mathbf{z}_n|d_{1:n}) \approx \sum_{l=1}^{L} \frac{p(d_n|\mathbf{z}_n^{(l)})\delta(\mathbf{z}_n - \mathbf{z}_n^{(l)})}{\int p(d_n|\mathbf{z}_n^{(l)})\delta(\mathbf{z}_n - \mathbf{z}_n^{(l)})d\mathbf{z}_n}$$
(12)

where  $\delta(\cdot)$  is the Dirac delta distribution and l = 1, ..., L. Based on (12), the set of L particles  $\mathbf{z}_n^{(l)}$  is characterized by the weights  $\omega_n^{(l)}$ 

$$p(\mathbf{z}_{n}|d_{1:n}) \approx \sum_{l=1}^{L} \omega_{n}^{(l)} \delta(\mathbf{z}_{n} - \mathbf{z}_{n}^{(l)}) \quad \text{with} \quad \omega_{n}^{(l)} = \frac{p(d_{n}|\mathbf{z}_{n}^{(l)})}{\sum_{l=1}^{L} p(d_{n}|\mathbf{z}_{n}^{(l)})},$$
(13)

which describe the likelihoods that the observation is obtained by the corresponding particle. These likelihoods are used as measures for the probability of the samples to be drawn from the true PDF [22]. Finally, the estimate for the state vector

$$\hat{\mathbf{z}}_n = \mathcal{E}\{\mathbf{z}_n | d_{1:n}\} \approx \sum_{l=1}^L \omega_n^{(l)} \mathbf{z}_n^{(l)}$$
(14)

is given as the estimated mean vector of the approximated posterior PDF. This fundamental concept is illustrated in Fig. 3. As starting point, a fixed number of L samples are drawn from the initial PDF  $p(\mathbf{z}_0)$  and evaluated based on (13). With this set of particles and



Fig. 3. Concept of the classical particle filter

corresponding weights, the estimate  $\hat{z}_1$  and the discrete distribution  $p(\mathbf{z}_1|d_1)$  are determined based on (14) and (12), respectively. This PDF is the basis for the sampling procedure of the following time step. As shown by the circular structure in Fig. 3, the particle filter is characterized by a finite number of state vectors which are determined at the initialization step and are subsequently evaluated in terms of their likelihood producing the observations  $d_n$  [23]. This results in the problem of degeneracy which implies that many particles have negligible weights after a few iteration steps. To address this issue, resampling methods have been introduced performing socalled sample impoverishment: The particles with very low weights are replaced, so that the set of particles is represented by only a few members with high weights. To cope with these conceptional limitations, a variety of sophisticated resampling methods have been proposed [22, 27, 28]. However, these modifications of the classical particle filter are still associated with a high computational load and a low performance for large search spaces [22, 29].

#### 4. THE EPFES FOR NL-AEC

In this section, we introduce the elitist particle filter based on evolutionary strategies (EPFES) as a numerical sampling method derived from classical particle filtering and discuss one possible realization of the EPFES for the task of NL-AEC.

#### 4.1. Properties and realization of the EPFES

As first modification with respect to the classical particle filter, we approximate the discrete distribution  $p(\mathbf{z}_n|d_{1:n})$  with a continuous PDF  $\hat{p}(\mathbf{z}_n|d_{1:n})$ , as proposed for the Gaussian case in [30]. This leads to the advantage of addressing degeneracy and sample impoverishment without introducing resampling methods.

Furthermore, the instantaneous adaptation of the estimated state vector  $\hat{\mathbf{z}}_n$  solves the local optimization problem regardless of the generalization of the instantaneous solution. Therefore, we employ a recursive calculation of the particle weights

$$\omega_{n}^{(l)} = \lambda \,\omega_{n-1}^{(l)} + (1-\lambda) \frac{p\left(d_{n} | \mathbf{z}_{n}^{(l)}\right)}{\sum_{l=1}^{L} p\left(d_{n} | \mathbf{z}_{n}^{(l)}\right)},\tag{15}$$

where  $\lambda$  is the so-called forgetting factor.

The major conceptual change with respect to classical particle filtering is based on evolutionary strategies (ES) and associated with the so-called "natural selection" [31]: At time instant n, we consider the set of L samples  $\mathbf{z}_n^{(l)}$  and corresponding weights  $\omega_n^{(l)}$ . In the selection process, the samples with a weighting factor smaller than a fixed threshold  $\omega_{\text{th}}$  are dropped [32]. The remaining set of  $Q_n \leq L$  samples represents the so-called elitist particles  $\bar{\mathbf{z}}_n^{(q_n)}$  [32]



Fig. 4. Concept of the EPFES with an approximated posterior PDF

and is characterized by the normalized weights

$$\bar{\omega}_{n}^{(q_{n})} = \frac{p\left(d_{n}|\bar{\mathbf{z}}_{n}^{(q_{n})}\right)}{\sum\limits_{q_{n}=1}^{Q_{n}} p\left(d_{n}|\bar{\mathbf{z}}_{n}^{(q_{n})}\right)},$$
(16)

where  $q_n = 1, ..., Q_n$ . Based on (14), the estimate of the state vector is given as weighted superposition of the elitist particles

$$\hat{\mathbf{z}}_n = \sum_{q_n=1}^{Q_n} \bar{\omega}_n^{(q_n)} \bar{\mathbf{z}}_n^{(q_n)}.$$
(17)

Subsequently, the estimated state vector  $\hat{\mathbf{z}}_n$  is employed to approximate the PDF  $\hat{p}(\mathbf{z}_n|d_{1:n})$  as proposed for the Gaussian case in [30]. From this distribution, we draw  $L - Q_n$  samples to refill the set of L particles for the following time step. In the terminology of evolutionary strategies (ES), this introduction of innovation can be identified as mutation. Finally, the set of particles  $\mathbf{z}_n^{(l)}$  at the beginning of every iteration consists of  $Q_{n-1}$  elitist particles of the previous time step and  $L - Q_{n-1}$  new samples. This implies that we create two subsets of particles which intend to capture dynamic nonlinearities by mutation and time-invariant system components by the evolutionary selection with recursive weights.

An overview of the EPFES is shown in Fig. 4. In comparison to the classical particle-filter concept in Fig. 3, we replace the discrete distribution by an approximated PDF and integrate an evolutionary selection process which facilitates the introduction of innovation into the set of particles by taking samples from the approximated posterior PDF  $\hat{p}(\mathbf{z}_n | d_{1:n})$ .

# 4.2. Application to NL-AEC

In the following, we apply the EPFES to the task of NL-AEC. Based on the definition in (5), the impulse response vector of the acoustic path and the coefficients of the nonlinear function are modeled by the state vector  $\mathbf{z}_n$ . This implies that the samples  $\mathbf{z}_n^{(l)}$  represent different realizations of the nonlinear echo path. Consequently, the EPFES is applied to make an estimate for all unknown quantities in the physical system, except for the additive uncertainty  $v_n$ . Furthermore, the estimated microphone samples  $\hat{d}_n^{(l)}$  are calculated based on (2) and (5) to define the likelihoods  $p(d_n | \mathbf{z}_n^{(l)})$  with Laplacian PDFs

$$p\left(d_n | \mathbf{z}_n^{(l)}\right) = \frac{1}{\sqrt{2}\sigma_n} \exp\left\{-\frac{\sqrt{2}\left|\hat{d}_n^{(l)} - d_n\right|}{\sigma_n}\right\},\qquad(18)$$

where

$$\sigma_n = |d_n - \hat{\mathbf{h}}_{n-1}^T \cdot y(\mathbf{x}_n, \hat{\mathbf{a}}_{n-1})| + \varepsilon.$$
(19)

To provide numerical stability, the instantaneous estimate of the absolute error in (19) is augmented with a small positive constant  $\varepsilon$ . The approximation of the posterior PDF  $\hat{p}(\mathbf{z}_n|d_{1:n})$  is based on a uniform distribution with the estimated state vector  $\hat{\mathbf{z}}_n$  as mean vector. The absolute deviation from the entries of  $\hat{\mathbf{z}}_n$  is equal to  $\eta$  for the coefficients  $a_{\tau,n}$  and equal to  $\mu |\hat{h}_{\kappa,n}|$  for the filter taps  $h_{\kappa,n}$ . The latter choice is inspired by the proportionate normalized least mean square (PNLMS) algorithm, which includes an individual update of the filter coefficients dependent on their current value [33].

# 5. EXPERIMENTAL RESULTS

In this section, we evaluate the proposed realization of the EPFES by means of the echo return loss enhancement (ERLE)

$$\operatorname{ERLE}_{n} = 10 \log_{10} \left( \frac{\mathcal{E}\{d_{n}^{2}\}}{\mathcal{E}\{e_{n}^{2}\}} \right)$$
(20)

as indicator for the quality of the system identification, where  $\mathcal{E}\{\cdot\}$  is the expectation operator, which we approximate by averaging over intervals of 0.3 s. The evaluation is based on different smartphone recordings of male and female speech and durations of 18 s. Furthermore, the acoustic path is characterized by a reasonable time-invariance and the signal to noise ratio is approximately 40 dB. The filter adaptation stops at the time instant of 9 s, so that we can evaluate the online performance during the first half and the expected AEC performance in case of double-talk during the second half of the simulation. The estimated filter vector consists of M = 256 coefficients at a sampling rate of 16 kHz. Furthermore, the time-invariant parameters are set to  $\lambda = 0.9995$ ,  $\varepsilon = 10^{-5}$ ,  $\eta = 10$ , and  $\mu = 0.1$ . The observation model is chosen as weighted superposition of the linear term and the Legendre functions  $\mathcal{L}_{\varrho}\{\cdot\}$  of first kind and orders  $\rho = 3, 5$ :

$$d_n = \mathbf{h}_n^T \left( \mathbf{x}_n + a_{3,n} \mathcal{L}_3 \{ \mathbf{x}_n \} + a_{5,n} \mathcal{L}_5 \{ \mathbf{x}_n \} \right) + v_n.$$
(21)

As proposed in the previous section, the state vector of the EPFES models the impulse response vector of the acoustic path as well as the coefficients of the nonlinearity. This implies a high-dimensional search space and a bad system identification during the first seconds. For this reason, we integrate the estimated filter vector of the normalized least mean square (NLMS) algorithm as innovation into the set of particles  $\mathbf{z}_n^{(l)}$  at every time step *n*. This is realized with a stepsize of the NLMS algorithm equal to 0.5 and results in a more efficient system identification and the equivalence of EPFES and NLMS algorithm for L = 1.

Fig. 5 shows the resulting ERLE for a male (a) and female speech signal (b) for different numbers of samples L, where L = 1 corresponds to the NLMS algorithm. We can notice that the system identification improves with increasing number of samples L and that the EPFES performs remarkably better than the NLMS algorithm. Furthermore, the recursively determined particle weights lead to a better performance also in the case of frozen filter coefficients: The performance with converged, non-adaptive filters after the time instant of 9 s is improved with respect to the NLMS algorithm. Consequently, the simulation results verify the property of the EPFES to identify a time-invariant nonlinearity. Table 1 shows the ERLE for instantaneous filter adaptation and frozen filter coefficients averaged over the corresponding interval of 9 s.

**Table 1**. Average ERLE of smartphone recordings for instantaneous filter adaptation (0 s - 9 s) and frozen filter coefficients (9 s - 18 s).

Recorded speech	Male	Male	Female	Female
Time frame	0 s - 9 s	9 s - 18 s	0 s - 9 s	9 s - 18 s
L = 1	9.2 dB	8.7 dB	$14.5\mathrm{dB}$	13.2 dB
L = 10	$15.9\mathrm{dB}$	13.4 dB	20.2 dB	16.2 dB
L = 100	16.9 dB	16.3 dB	20.6 dB	17.2 dB



Fig. 5. Performance comparison in terms of ERLE between the EPFES and the NLMS algorithm which is represented by L = 1.

## 6. CONCLUSIONS

In this paper, we introduced the EPFES as novel approach for NL-AEC. Similar to the classical particle filter, this algorithm consists of a set of particles and corresponding weights which represent different realizations of the nonlinear echo path and their likelihood to be the solution of the optimization problem. To cope with conceptional disadvantages of the classical particle filter, the EPFES includes an evolutionary selection of elitist particles and a recursive calculation of the particle weights. As a consequence, we can minimize the instantaneous error signal and generalize the solution to identify time-invariant nonlinearities. These properties of the EPFES have been verified with real smartphone recordings of male and female speech: The system identification improves significantly relative to the NLMS algorithm for instantaneous filter adaptation and also generalizes better than an NLMS-adapted linear filter.

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